Interest on Reserves in a Partial Two-Sector Banking Model

Shawn A. Osell
Lossmoss@hotmail.com

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The Federal Reserve implemented a new monetary tool policy as it simultaneously conducted the first round of quantitative easing in 2008. At that time, the Fed began paying interest on a commercial bank’s required and excess reserves in order to prevent the federal funds rate from falling to zero. Before quantitative easing, reserves were scarce enough for the federal funds rate to be determined by the supply and the demand for reserves. Consequently, the Fed would use open market operations to manipulate the federal funds rate, and thereby influence other market interest rates. When the Federal Open Market Committee decided to raise the federal funds rate in December, 2015, open market operations were no longer viable to effect the overnight rate. Following three rounds of quantitative easing in which the Fed created trillions of reserves from buying long term assets and mortgage backed securities, reserves became so ample that the federal funds rate fell to its lower zero bound. To restore control over the federal funds rate, the Fed began paying interest on excess reserves, which sets a floor interest rate below where the federal funds will not trade. Open market operations became ineffective because of the plethora of funds in the reserve market. A market with ample reserve causes the federal funds rate to fall to the zero lower
bound. However, the interest on excess reserve rate can adjust the effective federal funds rate independently of the quantity of reserves.

In a regime of ample reserves, adjusting the interest on excess reserves rate became the Fed’s preferred approach to adjusting short-term market interest rates. In order to compare the Federal Reserve’s interest on reserve monetary policy tool with open market operations, this monograph develops a two-sector equilibrium model with interest on reserves in the banking sector. It also constructs structural vector autoregression models with empirical data including interest on reserves. The objective of this paper is to analyze the monetary policy transmission effects of the Federal Reserve’s interest on reserve policy. Impulse response functions are applied to both the theoretical and empirical models. The estimated impulse response functions are then compared to the theoretical model’s impulse response functions in order to see how well the model fits the data. This paper’s model finds that both monetary policy tools have the same effect on macroeconomic aggregated variables.

**Keywords**: Federal Reserve; monetary policy; interest on reserves.

**JEL Classification**: C11; C15; C32; E31; E32.
INTEREST ON RESERVES IN A
PARTIAL TWO-SECTOR BANKING MODEL

BY

SHAWN A. OSELL
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A DISSERTATION SUBMITTED TO THE GRADUATE SCHOOL
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE
DOCTOR OF PHILOSOPHY

DEPARTMENT OF ECONOMICS

Dissertation Director:
Carl Campbell III
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LIST OF SYMBOLS

$\alpha$  Capital share

$\beta$  Discount factor

$\eta$  Bank Screening cost

$\delta$  Depreciation rate

$\nu$  Shift parameter
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<td>FR</td>
<td>Federal reserve</td>
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<td>MP</td>
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<td>GSE</td>
<td>Government Sponsored Enterprise</td>
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<td>Federal Open Market Committee</td>
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<td>OMO</td>
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<td>IOR</td>
<td>Interest on reserves</td>
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<td>$f_{fr}$</td>
<td>Federal funds rate</td>
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<td>Non-borrowed reserves</td>
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e  Excess reserve ratio
r  Required reserve ratio
c  Currency reserve ratio
m  Money multiplier
FC  Fixed cost
LTA  Long term assets
GSE  Government sponsored enterprise
f  Firm’s profits
b  Bank’s profits
P  Price of output
x  Monetary injection
k  Capital
c  Consumption
l  Loans
d  Deposits
$R_f$  Interest rate for loans to firms
$R_{ior}$  Interest on reserve rate
CHAPTER 1
INTEREST ON EXCESS RESERVES

1.1 Introduction

The purpose of this monograph is to examine the dynamics of a DSGE model when a monetary authority has the ability to adjust the interest rate that it pays on a bank’s reserves. The experience of Canada, New Zealand, and Australia is that their respective central banks have been able to maintain tighter control over their target interest rates compared to the Federal Reserve by implementing a “channel system.” This is where the target interest rate’s ceiling is the penalty rate (commonly referred to as the discount rate) and the target rate’s floor is the interest paid by the monetary authority on a bank’s reserves. Until October of 2008, required reserves remained idle on a bank’s balance sheet and therefore did not generate income for a bank. Thus, required reserves were seen as an implicit tax on all financial intermediaries that were subject to balance requirements. Taxes, explicit or implicit, are considered a distortion to markets. By allowing the monetary authority to pay interest on a financial intermediaries reserves, there is not always an inverse relationship between the target rate and the aggregate money supply.

Since October 2008, the Federal Reserve has had the authority to pay interest to commercial banks for funds stored at the district banks. Simultaneously, the Federal Reserve significantly increased the country’s money supply, and thus its balance sheet, through an unprecedented process called quantitative easing (QE). Despite the significant increase in the
money supply, inflation in the US has remained under 2 percent—contrary to what economic theory would predict.

Because the Federal Reserves has only recently implemented interest on reserve policy, there have not been many academic papers analyzing this relatively new tool. Guzman (2008) analyzes price level determinacy from interest on reserve policy. This paper utilizes a two-period overlapping generations model in order to find that paying interest on reserves results in a greater amount of steady state equilibria compared to a state without an IOR policy. The conclusion of this paper is that IOR policy increases welfare. Neyer (2009) analyzes bank reserves management where banks are paid interest on reserves in the Eurozone. This paper applies a model that reflects various options of remunerating a commercial bank’s required reserves. Neyer (2009) finds that the European Central Bank has more monetary policy flexibility when it pays Eurozone banks interest on their required reserves.

Williamson (2015) describes a two-sector model in order to analyze the redistribution effects of IOR policy. The model includes two types of banks: conventional and unconventional. Conventional banks receive interest on reserves where unconventional banks do not. Both types of banks are allowed to lend in the interbank loan market. This paper compares various equilibria that result from different combinations of asset holdings by banks, including excess reserves which are influenced by the interest on reserves rate. Bech and Klee (2009) also model the differences between traditional banks and other types of financial intermediaries such as government sponsored enterprises (GSEs). The objective of their model is to explain why the effective federal funds rate is below the interest on reserve rate floor. Their paper focuses on the bargaining power of the heterogeneous agents who participate in the federal funds market. Bargaining power exists since the traditional FIs are allowed to earn interest on reserves while the GSEs are not.

This paper is interested in comparing the transmission effects of IORs with that of the traditional open market operation tool. It is important for policy makers to know the
different ramifications of each tool. This paper also uses structural vector autoregression (SVAR) in order to compare the model with aggregated data. The empirical analysis finds that the paper’s model is partially successful at replicating the empirical data.

Only half of the estimated coefficients match the predictions of the paper’s theoretical model. The other variables are either ambiguous or give the opposite sign. This is to be expected since there is a lack of data to utilize heretofore. The IOR rate has only been increasing on a regular basis since December, 2015 as described below. Thus, the calibration of the model produces results that are somewhat different than what we would expect with a larger sample period. Moreover, the IOR tool was implemented simultaneously with the first round of quantitative easing. That is, the IOR tool has been in place along with a plethora of excess reserves in the reserves market. The purpose of the paper’s analysis to examine IOR policy in a state without quantitative easing or its aftermath. As more empirical evidence becomes available, the following model can be retested and modified.

1.2 Implementation of IOR Policy by the Federal Reserve

Throughout each business day, there are deposits and withdrawals among financial intermediaries (FIs). Before 2008, banks strived to keep their excess reserves (er) at a minimum because there is was opportunity cost to maintaining them. That cost was the interest that could have been earned if the bank had lent out its excess reserves. Concurrently, some banks are subject to maintain a certain amount of required reserves (RR). Banks attempted to avoided insufficient reserves at the end of the business day since it was costly to make up the difference. Holding at least some excess reserves was necessary to protect against unexpected withdrawals. Hence, maintaining excess reserves acted as a safety margin, or insurance, against a dearth of required reserves.
Required reserves have been considered an implicit tax on financial intermediaries because of the opportunity cost of not being able to lend them out. An additional cost of reserves is the time and expense of rearranging the bank’s balance sheet by keeping the actual reserves as close to the required reserves. By implementing IOR policy, the opportunity cost of holding both required and excess reserves is either reduced or eliminated. In addition, the monetary authority has additional leverage over monetary policy when it pays interest on a bank’s reserves.

In 2008 and afterward, several changes to monetary policy took place in order to provide liquidity to US financial markets. As a result, the Federal Reserves balance sheet increased to $2 trillion in that year from $800 billion just three months earlier (Hornstein, 2010). One such change was that the Federal Reserve began paying interest to financial intermediaries for both required reserves and excess reserves that were held at a Federal Reserve district bank.

On October 13, 2006, Congress gave permission to the Federal Reserve to pay interest on reserves (IORs) starting in October, 2011. However, this date was moved back to October 2008 because of the Emergency Economic Stabilization Act in order to facilitate bank liquidity. The interest rate set on required reserves was initially set at 140 basis points. The original policy was to pay an interest rate 10 basis points below the federal funds rate ($ffr$). The policy regarding excess reserves was to pay an interest rate of 75 basis points below the target $ffr$. A few weeks later, the spread was adjusted to 35 basis points. By the end of 2008, the rates on both types of reserves were set to 25 basis points and were held constant until December, 2015. These IOR rates were increased again on December, 2016 and March, 2017. All three increases were by 25 basis points.

Changing the interest on reserve rate is loosely analogous to changing the required reserve ratio. That is, by adjusting the IOR rate, banks will choose to also adjust the amount of reserves to hold rather than being forced to hold a required amount. Another implication is
that the monetary authority can adjust the policy rate without having to also manipulate the money supply via open market operations.

The remainder of this chapter explains the purpose and theory of why a monetary authority would want to pay depository institutions to hold excess reserves.

1.2.1 IOR Policy Tool Theory

The traditional federal funds market is shown in Figure 1.1. Before October, 2008, the Fed could adjust the federal funds rate with open market operations. Figure 1.2 shows how an open market operation purchase will increase the amount of reserves in the reserve market, and the federal funds rate decreases from point A to point B. By imposing an interest rate floor by paying interest on reserves, the lower bound is no longer zero. The federal funds rate is prevented from decreasing to zero and the federal funds rate increases because of the interest on reserve rate as shown in Figure 1.3. Equilibrium in the federal funds market moves from point B to point C.

By paying interest on a bank’s excess reserves, the Federal Reserve can now narrow the channel from below without affecting the market federal funds rate. The difference between federal funds rate and the interest on reserve rate is referred to in this paper as the interest rate spread. As shown in Figure 1.4, when the interest on reserve rate increases from IOR rate\(_1\) to IOR rate\(_2\), the interest rate spread decreases from \(\Delta_1^*\) to \(\Delta_2^*\). In this paper’s theoretical model, the interest rate spread initiates the monetary policy transmission process through bank lending. The intuition is that as the interest rate spread increases, the opportunity cost for banks to hold excess reserves increases, so banks will increase lending. As the interest rate spread decreases, so does the opportunity cost of holding excess reserves. As a result, bank lending will decrease.
If the equilibrium takes place on the horizontal segment of the demand curve like points C in Figure 1.5, the monetary authority can lift the ffr without implementing open market operations by using the IOR tool. Unlike the pre-2008 scenario in Figure 1.1, the Fed can adjust the federal funds rate without changing the amount of reserves in the reserve market.

As shown in Figure 1.5, an increase in the IOR will also increase the ffr from equilibrium point C to equilibrium point D. In fact, the relatively new IOR tool became necessary to increase interest rates in the aftermath of quantitative easing. As we will see below, IOR behavior in theory is not exactly IOR in practice. Though the two may eventually become the same when the reserve market resembles pre-quantitative easing conditions.

![Diagram](image)

**Figure 1.1: Equilibrium in the Market for Reserves.** A channel system describes a reserve market with the upper bound at the discount rate and the lower bound at the interest on reserve rate. The effective federal funds rate, *fed funds rate*, is determined by the supply of reserves, *R,s* and demand for reserves, *R,d* at point A. The discount rate is the ceiling since a bank would not borrow from another bank at a rate above the discount rate. The interest on reserve rate is the floor since a bank would not lend in the federal funds market at a rate below the interest on reserve rate.

With IORs, monetary policy (MP) can now achieve various combinations of quantities of reserves and *ffrs*. The benefit of this independence is that the FR can target financial mar-
Federal Funds Rate

\[ \text{discount rate} \]

\[ R^s \]

\[ \text{fed funds rate}_1 \]

\[ A \]

\[ \text{fed funds rate}_2 \]

\[ B \]

\[ IOR \text{ rate} \]

\[ R^d \]

\[ NBR_1 \]

\[ NBR_2 \]

Figure 1.2: **An open market operation purchase in the federal funds market.** When the vertical segment of the supply of reserve curve intersects the downward sloping segment of the demand for reserve curve, the federal funds rate can be adjusted with open market operations. An open market operation purchase shift the supply for reserve curve to the right. The equilibrium moves from point A to point B as the FFR decreases from \( \text{fed funds rate}_1 \) to \( \text{fed funds rate}_2 \) while the non-borrowed reserves increases from \( NBR_1 \) to \( NBR_2 \).

Markets and the macroeconomy separately where this was not possible before. More specifically, the FR can now adjust aggregate bank reserves independently of the FFR. This would be particularly useful to counter any future shocks to the financial system without disrupting the overall economy (Goodfriend, 2002). On the perfectly elastic segment of the \( R^D \) curve, it is now possible to adjust reserves while holding the FFR constant.

Alternatively, by adjusting IORs the central bank has the option of adjusting the FFR while holding reserves constant (See Irland, 2011; Goodfriend, 2002). The next section analyzes these variables along with other related monetary aggregates.
Figure 1.3: Adjusting the federal funds rate rate with open market operations in the presence of an interest on reserve rate. An interest on reserve rate creates a floor for the federal funds rate such that the lower bound is greater than zero. Without an interest on reserve rate greater than zero, the floor is the zero lower bound. An open market operation purchase will cause the federal funds rate to fall until the federal funds rate, $fed\ funds\ rate_3^*$, is equal to the interest on reserve rate, $IOR\ rate_1$. The equilibrium moves from point B to point C. The demand for reserves on the horizontal segment of the demand for reserves curve is perfectly elastic.

1.3 Empirical Monetary Aggregate Behavior Pre and Post Quantitative Easing

The purpose of this section is to analyze the empirical behavior of IORs and excess reserves. With the option to influence excess reserves, we can then surmise the monetary authority’s ability to manipulate other monetary aggregates. Figure 1.6 ostensibly shows that there is a correlation between IORs and excess reserves. Unfortunately we cannot say if, or how much, of this relationship is causation. However, we can see what could happen to monetary aggregates by analyzing the behavior of excess reserves in the context of quantitative easing. The first implementation of quantitative easing occurred in September
Figure 1.4: Increasing the interest on reserve rate below the federal funds rate narrows the channel with no impact on the federal funds rate. Increasing the interest on reserve rate from \( IOR \ rate_1 \) to \( IOR \ rate_2 \) does not effect the federal funds rate as long as the interest on reserve rate remains below the effective federal funds rate, \( fed \ funds \ rate^* \). The advantage of increasing the interest on reserve rate below the federal funds rate is that it narrows the channel in which the federal funds rate can fluctuate. This paper also makes the case that decreasing the spread between the two interest rates from \( \Delta_1^* \) to \( \Delta_2^* \) also decreases the opportunity cost of holding reserves. As a result of a smaller interest rate spread, lending will decrease.

2008, while IORs were introduced in October of the same year. At that time, excess reserves were $1.8 billion dollars. Excess reserves reached a peak of $2.7 trillion dollars in August, 2014, and fell to $1.3 trillion by October, 2019. As a result of the Fed’s Covid-19 stimulus, excess reserves hit a new peak of $3.2 trillion in May of 2020.

Other interesting changes in the behavior regarding excess reserve aggregates have also taken place since the implementation of QE. Until October 2008, the nominal amount of excess reserves rarely increased above $2 billion. The most notable exception was September, 2001, when it jumped to $19 billion. However, excess reserves fell back to $1.3 billion the next month and, for the most part, remained around or below $2 billion. The two exceptions were August, 2003, and August, 2007, where excess reserves temporarily jumped to $3.77 billion
Figure 1.5: Adjusting the federal funds rate with the interest on reserve rate. Initially, the equilibrium takes place on the vertical segment of the supply for reserve curve and the horizontal segment of the demand for reserve curve $R^d_1$. This takes place at point C. Interest on reserve policy allows the monetary authority to raise the federal funds rate by increasing the interest on reserve rate from $IOR_1$ to $IOR_2$. As a result, the vertical segment of the demand for reserves curve shifts up from $R^d_1$ to $R^d_2$ and the federal funds rate simultaneously increases from $fed\ funds\ rate^*_3$ to $fed\ funds\ rate^*_4$.

and $4.8$ billion, respectively. Since November, 2009, excess reserves have been around or above $1$ trillion. Figure 1.7 shows excess reserves in levels and Figure 1.8 shows the percent change.

Figure 1.9 shows both er and RR as a percentage of TR from 1960 until 2007. With few exceptions, RRs made up between 97% and 99% of TRs. Again, the brief but notable exception was the year 2001 when RRs made up only 66% of TRs. We can see in Figure 1.10 that in the year 2008 is when excess reserves went from 4% of TRs to over 90% of TRs and have vacillated between 91% and 95% ever since.

Historically, the monetary base has been smaller than the M1 money supply. Before 2008, the amount of excess reserves within the monetary base was practically zero. Despite the significant increase in the monetary base by just under 380%, M1 has not increased at the same pace. The reason for this is because over half of the monetary base is composed
Figure 1.6: Excess reserves and interest paid on reserves, 2008-2016. Quantitative easing and IOR policy were both implemented in the last half of 2008. This figure shows the rapid increase in excess reserves. After a few initial adjustments, the IOR rate was held at 25 basis points until December, 2015. The IOR rate was raised again on December, 2016 and March, 2017.

Figure 1.7: Excess reserves, 2008-2020. Excess reserves increase from $1.8 Billion to $2.7 Trillion from October, 2008 to August, 2014. Excess reserves subside until the Covid-19 Pandemic and then reaches a new peak in early 2020.

of excess reserves. In fact, since 2008 the monetary base is now larger than M1. The base
Figure 1.8: Excess reserves - percent change. There are two significant changes in the percent change of excess reserves. The first occurred during the month of September in 2011. The most significant occurred with the implementation of QE 1 in September, 2008.

Figure 1.9: Excess reserves and required reserves as a percent of total reserves, 1960-2007. is not larger than M2, however. The behavior of the base and M1 are presented together in Figure 1.11.
The money multiplier is calculated by dividing the money supply by the monetary base. Figure 1.12 shows how there was a significant decrease in the M1 money multiplier during the last half of the year 2008 and has remained below one ever since. The large increase in the excess reserve ratio caused the money multiplier to decrease quickly.
Figure 1.12: The ratio of the M1 money supply and the monetary base is the M1 money multiplier. Historically, the money multiplier is greater than one. However as the excess reserve ratio increased, the money multiplier was driven below one. Since the money supply equals the money multiplier times the monetary base, a money multiplier of less than one causes the money supply to be a fraction of the monetary base instead of a multiple of the base.

In contrast to the excess reserve ratio, a decrease in the currency reserve ratio causes the money multiplier to increase. Thus, the money multiplier would have been even lower if the currency ratio had not also decreased. Total Reserves also surpassed total checkable deposits as shown in Figure 1.13.

As the money supply increased while lending decreased, the excess reserve ratio increased, which is common during recessions as banks consider lending risky during recessions. Also, bank regulation contributed to the decrease in lending after 2008. Furthermore, low interest rates provided banks with a disincentive for lending –especially while the Federal Reserve was paying IORs. At the same time, the interest rate on riskless securities decreased to almost zero such that the opportunity cost of holding excess reserves became marginal. As a result of these factors, the excess reserve ratio increased dramatically in 2008 from essentially zero and has fluctuated between 1.5 and just under 3 ever since. The excess reserve ratio is shown in Figure 1.14. The currency ratio is also included for comparison purposes. Even though
we see abnormal behavior in the currency ratio post-2008, the decrease in the currency ratio is explained by the use of debit cards rather than because of any type of monetary policy (Mishkin, 2015).

![Chart](chart.png)

Figure 1.13: Total checkable deposits and total reserves. As a result of QE 1, total reserves actually surpassed the amount of checkable deposits.
Figure 1.14: Currency and er ratios. The excess reserve ratio increased by a factor of 1,000 because of the implementation of QE and has fluctuated around two ever since.
1.4 Federal Reserve Policy in Practice Since 2008

Before QE, traditional OMOs were effective at manipulating interest rates because of a scarcity of reserves in the federal funds market. However, this has not been the case since the implementation of QE. As we saw above, by December 2014 there were $2.6 trillion of reserves in the banking system, the bulk of which were excess reserves. Other tools will therefore be necessary until the federal funds market can be drained of the plethora of Long Term Assets (LTAs). The process of returning to a pre-QE state is referred to as “normalization,” which the FOMC said it will do gradually. This is because the FOMC wants to focus on the ffr rather than the quantity of reserves. An aggressive sell-off of LTA’s would distort money markets by creating unintended and unpredictable consequences. See Frost, Logan, Martin, McCabe, Natalucci, and Remache (2015).

Beginning in December 2008, the FOMC declared a target range of 0-25 basis points instead of a specific target rate. As the FR began to raise the ffr in December 2015, the FOMC continued to target a range with a 25 point basis point spread. The process of increasing interest rates after QE has been named “liftoff.”

A few months after IOR policy was implemented, the IOR rate has been set to the upper bound of the target range. Since “liftoff” began in December 2015, the overnight reverse repurchase (ON RRP) agreement rate has been set to the lower bound of the target range. The FOMC intends to temporarily offer ON RRPs as a complementary tool to IORs until ON RRPs are no longer needed to support the ffr. Eventually IORs will become the sole tool for maintaining a floor for the ffr (Frost et al., 2015).

The 25 basis point spread between the two rates intentionally encourages arbitrage in order to increase the ffr. As can be seen in Figure 1.15, the ffr consistently stays within the target range. Unlike what theory described above predicts, the IOR rate has not provided
a floor. This is because arbitrage has not been complete. There are a few reasons for this. One reason is that participants who can earn IORs are only a subset of those who can lend in the federal funds market i.e. government sponsored enterprises and money market funds. Moreover, banks already have a plethora of excess reserves and there are costs associated with arbitrage. Adding additional reserves increases the required amount of FDIC insurance and requires banks to hold more capital.

Figure 1.15: Federal Funds Rate, Interest on Reserve Rate, and Federal Funds Rate Target Range, 2008-2021. The FOMC established a federal funds target range of 0-0.25% in December, 2008. Beginning in December 2015, the FOMC raised the target range by 0.25% increments. The upper bound of the range coincided with the interest on reserve rate until June, 2018. At that time, the target range was increased by 0.25% while the IOR rate was increased by only 0.20% in order to push the federal funds rate down towards the center of the target range. This approach was effective for just a few months until the funds rate moved back towards the upper bound of the target range. The FOMC significantly decreased the IOR rate twice during the beginning of 2020 because of the recession that resulted from the Covid-19 virus.  
Source: St. Louis Federal Reserve District bank FRED data.
1.5 Chapter Conclusion

As a result of the financial crises that began in 2007, the Federal Reserve implemented additional tools to stimulate the US economy and to increase liquidity to financial markets. One of the new tools, IORs, was originally intended to go into effect in 2011, but was put in place in 2008 because of the 2007-2009 recession. Textbook discussions of monetary policy have traditionally described three tools. IOR policy is now considered the fourth tool of monetary policy.

Why is IOR policy important? First, it creates a non-zero lower bound. A second benefit of IOR policy is that the monetary authority can now adjust the ffr rate, and thus interest rates in general, without following the conventional negative relationship between interest rates and non-borrowed reserves. That is, the ffr can be adjusted by changing the IOR rate while holding non-borrowed reserves fixed. A third reason for IOR policy is that in the aftermath of quantitative easing, IOR policy is necessary for lifting the ffr since open market operations are no longer effective and will not be until the massive amounts of reserves are drained out of the reserve market.

Despite the implementation of IOR policy in 2008, the effective federal funds rate has fluctuated below the IOR rate. That is, IOR’s have not created a ffr floor as economic theory predicts. One explanation for this counter-intuitive fact is that non-bank participants in the federal funds market are able to lend in the federal funds market but by law are not eligible to receive IORs. Also, financial intermediaries do not borrow as much as they did before quantitative easing because of the plethora of excess reserves they already hold. Thus, the difference between the ffr and the IOR rates are not arbitraged away.

Beginning in December 2008, the Federal Open Market Committee began setting a federal funds range instead of a specific target rate. Also, the IOR rate was set at the upper
bound on the target range. Since December 2015, the ON RRP implied rate is used as the lower bound of the range. The ON RRP pushes the $ffr$ up by encouraging arbitrage and by creating an increase in the scope of influence on money markets. Arbitrage is possible because there is a different set of financial intermediaries that can participate in the overnight reverse repurchase agreement market then those that can earn interest on reserves. The next chapter develops a model in which we can compare the interest on reserves monetary policy tool with open market operations.
CHAPTER 2
INTEREST ON RESERVES AND FORWARD GUIDANCE IN A DETERMINISTIC MODEL

2.1 Theoretical Model

This chapter describes a deterministic model. The model economy is a two-sector banking model where the firms and banks are owned by the households. The firm’s output can be used to add to the capital stock, or used as a consumption good. A central bank influences bank lending by changing the interest on reserve rate or by adjusting a monetary injection. Firm and bank profits are paid to the households in the form of a dividend, which is deposited into the banks. This model does not have an exogenous stochastic process. In order to model the Fed’s forward guidance policy, agents correctly anticipate all changes to monetary policy by the monetary authority.

Firms

Firms’ output is determined by a Cobb-Douglas production function, and they borrow funds from the banks in order to finance their investment. Production depends on capital from both the current and previous time period. Each time period is long enough such that any capital produced in the beginning of time period $t$ can also be used to produce output at the end of the same time period $t$. Since capital is built throughout each time period, any capital built at the beginning of each time period can then be used throughout the time
period to build more capital.

Loans are taken out at the beginning of a time period and then are paid back at the end of the same time period. Thus, the firm’s profits can be expressed as

$$ f_t = P_t k_t^{\alpha \phi} \cdot k_{t-1}^{\alpha(1-\phi)} - P_t (k_t - k_{t-1} + \delta \cdot k_{t-1})(1 + R_{F,t}) - FC_f$$

$$0 < \alpha < 1, \forall t. \quad (2.1)$$

Output is determined by a weighted average of capital produced in the current time period, $t$, and the previous time period, $t-1$. The parameter $\phi$ is the weight on the capital produced in time $t$ that is used in time $t$ production. The output elasticity of capital is denoted by $\alpha$ and the depreciation rate of capital is $\delta$. Firm profits over the two time periods are described as:

$$ f = P_1 \cdot k_1^{\alpha \phi} \cdot k_0^{\alpha(1-\phi)} - P_1 (k_1 - k_0 + \delta \cdot k_0)(1 + R_{F,1})$$
$$+ \beta \left[ P_2 \cdot k_2^{\alpha \phi} \cdot k_1^{\alpha(1-\phi)} - P_2 (k_2 - k_1 + \delta \cdot k_1)(1 + R_{F,2}) \right] - FC_f (1 + \beta), \quad (2.2)$$

where $\beta$ is a discount factor. The bank’s profit function is subject to a lending (or resource) constraint. The dividends that households receive from firms are equal to firm’s non-retained profits since the households own the firms. We assume that all agents have perfect foresight.

The first term of the profit equation is the revenue that the firm earns from its output. The level of output is determined by the production function, $k_t^{\alpha \phi} \cdot k_{t-1}^{\alpha(1-\phi)} \cdot n^{1-\alpha}$. The amount of labor that households provide to their own firms is normalized to unity, $n = 1$. The second term is the amount that the firms must pay on its loans from the banks. Loans are one period loans such that firms borrow in each time period and pay back at the end of the time period. The second term shows that the firms borrow money for gross investment between the current time period and the previous time period, and they pay back the loan with interest. Gross
investment is the change in the capital stock plus depreciation, \((k_t - k_{t-1} + \delta \cdot k_{t-1})\), where the parameter \(\delta\) is the depreciation rate. The last term in equation (2.2) reflects that firms also pay a fixed cost, \(FC_f\), each time period. This fixed term is included to ensure that there are a finite number of firms.

Since historically the capital stock grows over time, we can assume that \(k_t > k_{t-1} (1 - \delta)\), implying that there is no resale value for capital and assuring that the value of loans is always positive.

Firms borrow money to finance capital purchases each time period according to a lending constraint. The demand for loans by firms is equal to the price level times gross investment. The analysis considers only circumstances when gross investment is positive. The demand for loans is specified as:

\[
  l_t = P_t \cdot (k_t - k_{t-1} + \delta \cdot k_{t-1}).
\]  

(2.3)

In the first time period, the demand for loans is:

\[
  l_1 = P_1 \cdot (k_1 - k_0 + \delta \cdot k_0),
\]

and in the second time period

\[
  l_2 = P_2 \cdot (k_2 - k_1 + \delta \cdot k_1)
\]

Capital is also used as collateral in case of a loan default. Therefore, this equation is also a resource constraint. This means that loans cannot be more than the value of investment (see Kiyotaki & Moore (1997)).
**Marginal Product of Capital**

This subsection determines the marginal product of capital in each time period, which is equal to the bank’s lending rate for the two time periods. Firms maximize their profit function (2.2) subject to the capital constraint \( k_t > k_{t-1} (1 - \delta) \) in each time period. The firm solves the Lagrangian problem:

\[
\mathcal{L} = P_1 \cdot k_1^{\alpha \phi} \cdot k_0^{\alpha (1 - \phi)} - P_1 (k_1 - k_0 + \delta \cdot k_0) (1 + R_{F,1}) \\
+ \beta \left[ P_2 \cdot k_2^{\alpha \phi} \cdot k_1^{\alpha (1 - \phi)} - P_2 (k_2 - k_1 + \delta \cdot k_1) (1 + R_{F,2}) \right] - FC_f (1 + \beta) \\
- \lambda_1 [k_1 - k_0 (1 - \delta)] - \lambda_2 [k_2 - k_1 (1 - \delta)].
\]  

Take \( \mathcal{L} \) with respect to \( k_1 \):

\[
\frac{d\mathcal{L}}{dk_1} = 0 : \quad \alpha \phi P_1 \cdot k_1^{(\alpha \phi - 1)} \cdot k_0^{\alpha (1 - \phi)} - P_1 \cdot (1 + R_{F,1}) \\
+ \beta \left[ P_2 k_2^{\alpha \phi} (\alpha (1 - \phi)) \cdot k_1^{\alpha (1 - \phi) - 1} + (1 - \delta) P_2 (1 + R_{F,2}) \right] - \lambda_1 + \lambda_2 (1 - \delta) = 0.
\]  

In order to determine the optimal level of capital in period 2, we take the FOC of equation (2.4) with respect to \( k_2 \):

\[
\frac{d\mathcal{L}}{dk_2} = 0 : \beta \left[ P_2 \alpha \phi \cdot k_2^{\alpha \phi - 1} \cdot k_1^{\alpha (1 - \phi)} - P_2 (1 + R_{F,2}) \right] = \lambda_2.
\]  

**Case I.** In the first case, neither constraint is binding, such that \( \lambda_1 \) and \( \lambda_2 \) are both equal to zero. Moving all the \( k \)'s in equation (2.5) to the LHS,

\[
\alpha \phi P_1 \cdot k_1^{(\alpha \phi - 1)} \cdot k_0^{\alpha (1 - \phi)} + \beta P_2 k_2^{\alpha \phi} (\alpha (1 - \phi)) \cdot k_1^{\alpha (1 - \phi) - 1} = P_1 (1 + R_{F,1}) - \beta (1 - \delta) P_2 (1 + (R_{F,2})).
\]
Solving for $R_{F,1}$ to get:

$$R_{F,1} = \frac{\alpha \phi P_1 k_1^{(\alpha \phi - 1)} k_0^{\alpha (1 - \phi)} + \beta P_2 k_2^{\alpha \phi (1 - \phi))} k_1^{(1 - \phi)^{-1}} - P_1 + \beta (1 - \delta) \cdot P_2 (1 + R_{F,2})}{P_1}.$$ 

Solve equation (2.6) for $R_{F,2}$:

$$R_{F,2} = \alpha \phi k_2^{(\alpha \phi - 1)} \cdot k_1^{(1 - \phi)} - 1.$$ 

**Case II.** In the second case, $\lambda_1 > 0$ and $\lambda_2 = 0$, thus:

$$k_1 = k_0 \cdot (1 - \delta).$$

**Case III.** For the third case, $\lambda_1 = 0$ and $\lambda_2 > 0$, thus:

$$k_2 = k_1 \cdot (1 - \delta).$$

**Banks**

The bank profit function is specified as the interest the bank earns from loans and the interest it earns from holding deposits minus the cost of producing loans:

$$b_t = R_{F,t} \cdot l_t + R_{stor,t+1} (d_t + x_{t+1} - l_t) - \frac{\eta \ l_t^{\nu} d_{t+1} x_t}{d_t + x_{t+1}} - FC_b.$$  \hspace{1cm} (2.7)

The parameters $\eta$ and $\nu$ are the loan production’s shift parameters. The parameter $\nu$ reflects that there are increasing costs $\nu > 1$ to loan production.

There are several reasons why the loan production function exhibits increasing costs. First, in order to increase lending, banks would need to either increase advertising or weaken...
their credit standards. The former requires increased marketing costs, while the latter would entail an increase in expected default losses or higher screening costs to limit defaults. As a result, there are increasing costs to loan production. A second and related explanation is that since the credit risk of borrowers is heterogeneous, a weakening of credit standards implies an increase in riskier borrowers, who often require more costly monitoring than safer borrowers. Third, regulators typically require higher bank capital ratios for banks with higher loan losses or higher loan-to-asset ratios. Since the cost of funding loans with equity capital exceeds the cost of deposit funding, this implies that the funding cost of loans is increasing in the amount of loans extended. Finally, it may become even more difficult to expand lending in a high interest rate environment because increasing loan interest rates beyond a certain point can push up default losses enough that expected loan revenue can be decreasing in loan interest rates (as argued, for example, by Stiglitz and Weiss (1981), inter alia).

The bank’s profit, \( b_t \), is paid to the households as a dividend each period since the households own the banks. The term \((d_t + x_{t+1} - l_t)\) is the bank’s excess reserves, which is earns a interest rate of \( R_{ior} \). The variable \( x_{t+1} \) is a monetary injection given to the banks by the monetary authority. Like the firms, banks also pay a fixed cost each period. This fixed cost ensures a finite number of banks. Take the first order condition:

\[
\frac{db_t}{dl_t} : R_{F,t} - R_{ior,t+1} - \frac{\eta \nu l_t^{\nu-1}}{d_t + x_{t+1}} = 0.
\]

Solving for the optimal amount of bank lending, \( l_t \), we get:

\[
l_t = \left[ \frac{1}{\nu \eta} (R_{F,t} - R_{ior,t+1}) (d_t + x_{t+1}) \right]^{\frac{1}{\nu-1}} \text{ where } R_{F,t} \geq R_{ior,t+1}.
\] (2.8)
If $\nu = 2$, then

$$l_t = \frac{1}{\nu \eta} (R_{F,t} - R_{ior,t+1})(d_t + x_{t+1}) \text{ with } l_t \leq (d_t + x_{t+1})$$

The term $(R_{F,t} - R_{ior,t+1})$ is the interest rate spread. The intuition is that as the spread increases, banks will have a greater incentive to lend. Notice that once the interest rates become equal, then the banks will no longer have an incentive to lend. Since loan supply is a function of the interest on reserve rate and the monetary injection, both monetary policy tools initiate the transmission mechanism through bank lending.

### 2.2 Equilibrium

**Product Market Equilibrium**

The firm’s output is determined by the law of motion for capital. The firm’s output can either be used as a consumption good this period or as capital. The amount of capital produced is the difference between output and consumption. The goods market clearing are described by

$$c_t + k_t - k_{t-1} + \delta \cdot k_{t-1} = k_t^\alpha \phi \cdot k_{t-1}^{\alpha(1-\phi)},$$

(2.9)

where consumption plus investment equals output. Clearing in the goods market in period 1 is represented by:

$$c_1 + k_1 - k_0 + \delta \cdot k_0 = k_1^\alpha \phi \cdot k_0^{\alpha(1-\phi)},$$

and in period 2 as

$$c_2 + k_2 - k_1 + \delta \cdot k_1 = k_2^\alpha \phi \cdot k_1^{\alpha(1-\phi)}.$$
Money Market Equilibrium

The cash-in-advance constraint is price times consumption and investment equals the money supply. Rotemberg & Woodford (1997) show that the cash-in-advance constraint implies constant velocity. Since there is no cash in this model, the money supply equals deposits. Clearing in the goods market in period 1 is described by:

\[ P_t \cdot (c_t + k_t - k_{t-1} + \delta \cdot k_{t-1}) = d_t. \]  \hspace{1cm} (2.10)

Thus, the period 1 clearing condition is

\[ P_1 \cdot (c_1 + k_1 - k_0 + \delta \cdot k_0) = d_1, \]

and in period 2 by

\[ P_2 \cdot (c_2 + k_2 - k_1 + \delta \cdot k_1) = d_2. \]

It is also assumed that nominal consumption, \( P_t \cdot c_t \), equals nominal income in each period:

\[
P_t \cdot c_t = f_t + b_t = P_t \cdot (k_t^\alpha \cdot k_{t-1}^{\alpha (1-\phi)}) - P_t (k_1 - k_{t-1} + \delta \cdot k_{t-1})(1 + R_{F,t}) - FC_f
+ R_{F,t} \cdot l_t + R_{ior,t} \cdot (d_t + x_{t+1} - l_t) - \frac{\eta l_t^\nu}{d_t + x_{t+1}} - FC_b.
\]

For time period 1 the consumption market can be described by:

\[
P_1 \cdot c_1 = f_1 + b_1 = P_1 \cdot (k_1^\alpha \cdot k_0^{\alpha (1-\phi)}) - P_1 (k_1 - k_0 + \delta \cdot k_0)(1 + R_{F,1}) - FC_f
+ R_{F,1} \cdot l_1 + R_{ior,1} \cdot (d_1 + x_2 - l_1) - \frac{\eta l_1^\nu}{d_1 + x_2} - FC_b.
\]
Then in time period 2:

\[ P_2 \cdot c_2 = f_2 + b_2 = P_2 \cdot (k_2^\alpha \cdot k_1^\alpha (1-\phi)) - P_2 (k_2 - k_1 + \delta \cdot k_1)(1 + R_{F,2}) - FC_f 
+ R_{F,2} \cdot l_2 + R_{ior,2} \cdot (d_2 + x_3 - l_2) - \frac{\eta l_2^\nu}{d_2 + x_3} - FC_b. \]

\[ P_1 \cdot c_1 = f_1 + b_1 = P_1 \cdot (k_1^\alpha \phi \cdot k_0^\alpha (1-\phi)) - P_1 (k_1 - k_0 + \delta \cdot k_0)(1 + R_{F,1}) - FC_f 
+ R_{F,1} \cdot l_1 + R_{ior,1} \cdot (d_1 + x_2 - l_1) - \frac{\eta l_1^\nu}{d_1 + x_2} - FC_b. \]

\[ P_2 \cdot c_2 = f_2 + b_2 = P_2 \cdot (k_2^\alpha \phi \cdot k_1^\alpha (1-\phi)) - P_2 (k_2 - k_1 + \delta \cdot k_1)(1 + R_{F,2}) - FC_f 
+ R_{F,2} \cdot l_2 + R_{ior,2} \cdot (d_2 + x_3 - l_2) - \frac{\eta l_2^\nu}{d_2 + x_3} - FC_b. \]

### 2.3 Twelve Equations to be Solved

The twelve unknown endogenous variables to be solved are \( l_1, l_2, k_1, k_2, d_1, d_2, P_1, P_2, R_{F,1}, R_{F,2}, c_1, \) and \( c_2. \) The system of equations consists of the following twelve equations:

\[
\begin{align*}
l_1 &= P_1 \cdot (k_1 - k_0 + \delta \cdot k_0) \\
l_2 &= P_2 \cdot (k_2 - k_1 + \delta \cdot k_1) \\
k_1^\alpha \phi \cdot k_0^\alpha (1-\phi) &= c_1 + k_1 - k_0 + \delta \cdot k_0 \\
k_2^\alpha \phi \cdot k_1^\alpha (1-\phi) &= c_2 + k_2 - k_1 + \delta \cdot k_1 \\
d_1 &= P_1 \cdot (c_1 + k_1 - k_0 + \delta \cdot k_0) \\
d_2 &= P_2 \cdot (c_2 + k_2 - k_1 + \delta \cdot k_1) \\
l_1 &= \left[ \frac{1}{\nu \eta} (R_{F,1} - R_{ior,1})(d_1 + x_2) \right]^{\frac{1}{\nu - 1}} \\
l_2 &= \left[ \frac{1}{\nu \eta} (R_{F,2} - R_{ior,2})(d_2 + x_3) \right]^{\frac{1}{\nu - 1}} \\
\end{align*}
\]
\[ R_{F,1} = \alpha \phi P_1 k_1^{(\alpha \phi - 1)} k_0^{\alpha (1 - \phi)} + \beta P_2 k_2^{\alpha \phi} (\alpha (1 - \phi)) k_1^{\alpha (1 - \phi) - 1} - P_1 + \beta (1 - \delta) \]

\[ \cdot P_2 (1 + R_{F,2}) \div P_1 \]

\[ R_{F,2} = \alpha \phi k_2^{(\alpha \phi - 1)} \cdot k_1^{\alpha (1 - \phi)} - 1 \]

The parameters are calibrated as follows. The capital share of national income, \( \alpha \), is calibrated to 0.40, and the capital depreciation rate, \( \delta \), is set to 0.07, which is common in the literature. The two parameters in the loan production function, \( \eta \) and \( \nu \). The parameter \( \eta \) is set to 1.10 and \( \nu \) is calibrated to 1.10. The discount factor \( \beta \) is set to 0.8, which reflects empirically that 20 percent of wage earners to not have any savings. The share of current capital stock that is used in production \( \phi \) is also set to 0.8. In the following analysis, we are interested to see how changes in the monetary policy tools will change the values of the endogenous variables from their initial values. The initial values are chosen such that labor reflects that workers spend a third of the day working and that all variables are positive. A nonlinear Newton-type solver is used to compute the initial values. The values are presented in Table 2.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>8.367</td>
</tr>
<tr>
<td>( c )</td>
<td>1.753</td>
</tr>
<tr>
<td>( R_F )</td>
<td>0.140</td>
</tr>
<tr>
<td>( l )</td>
<td>0.357</td>
</tr>
<tr>
<td>( d )</td>
<td>1.425</td>
</tr>
<tr>
<td>( P )</td>
<td>0.609</td>
</tr>
</tbody>
</table>

Table 2.1: Endogenous Variable’s Initial Values.

The initial values of the exogenous variables are chosen as follows. The interest on reserve rate is set to 0.03 and the monetary injection is set to zero. The deterministic positive interest on reserve shock is simulated by increasing the IOR rate to 0.04, and a negative shock is simulated by lowering the IOR rate to 0.02. In order to simulate an increase in the monetary
injection, the monetary injection variable, $x$, is increased to 2.6. A negative shock to the monetary injection is simulated by lowering $x$ to 2.4. In addition, it was verified that the inequality constraints in equation (2.4) were not violated.

2.4 Exogenous Monetary Policy Shocks

The first shock we analyze is a permanent increase in the interest on reserve rate. The optimal amount of lending decreases as the interest rate spread decreases. As a result, real investment, $k_t$, falls and the lending rate increases. Empirically, market interest rates tend to move with the monetary authorities administered rates. That is what the model also exhibits. The decease in capital also means that there is less output available for consumption. Therefore, the level of the consumption good decreases. The lower total output, along with the higher interest rate, causes firm profits to fall. The decrease in output and prices causes firm and bank profits to fall. The effects of a permanent shock to the IOR rate is shown in Figure 2.1.

When the monetary authority permanently decreases the IOR rate, the variables move in the opposite direction compared to the previous case. A reduction in the IOR rate will increase the interest rate wedge, which increases the supply of lending. The increase in lending causes investment in capital to grow. The addition to the capital stock also increases the amount of output that is available for consumption.

The increase in output and the initial higher price level increases profits for firms. At the same time, an increase in lending is profitable for banks. The permanent IOR expansionary policy impulse response functions are shown in Figure 2.2.

The next step in the analysis is to purview the effects of a permanent change in the monetary injection. A permanent increase in the money supply, $x$, causes the supply of loans
Figure 2.1: **Permanent Positive IOR Shock.** The monetary authority permanently increases the interest on reserve rate, which decreases the interest rate wedge. As a result, lending decreases and causes investment in capital to fall, and the lending rate to increase.

Figure 2.2: **Permanent Negative IOR Rate Shock.** The monetary authority permanently decreases the interest on reserve rate. As a result, the interest rate wedge initially increases, which increases the supply of loans. Firms increase their investment in capital, which causes the lending rate to decrease.
to increase. A lower interest rate causes the demand for loans to increase. An increase in borrowing for capital purchases causes real investment to increase, which also contributes to an increase in the consumption good. The higher price level increases the profits of firms, which leads to an increase in deposits. This scenario is presented in Figure 2.3.

A permanent decrease in the money supply, x, causes lending to decrease and the capital stock to fall. A higher interest rate also causes borrowing to decrease. An decrease in borrowing for capital purchases causes the capital stock to decrease. This scenario is presented in Figure 2.4.

Figure 2.3: **Permanent Positive Shock to the Monetary Injection, x.** The monetary authority increases the monetary injection, which increases lending. The capital stock increases over time, which causes the lending rate to decrease. In addition, the stimulative monetary injection increase causes an increase in consumption, lending, deposits, and the price level.
Figure 2.4: **Permanent Negative Shock to the Monetary Injection, x.** A permanent decrease in the monetary injection causes the endogenous variables to decrease except for the lending rate, which increases. Theory predicts that a decrease in the money supply will increase the market interest rate. The model makes the same prediction. Economic theory and the theoretical model also coincide regarding the other endogenous variables. A contractionary monetary injection have the same effect on the capital stock, consumption, lending, deposits, and the price level.
The two tools tend to mimic each other when we perform either a contractionary or stimulative policy by adjusting the monetary injection or by adjusting the interest on reserve rate. An adjustment of the monetary injection in the model is a proxy for tool for open market operations. From the above analysis, we can see that the relatively new interest on reserve rate tool is a good substitute for open market operations.

US monetary policy has drastically changed since the recession of 2007-2009. The Fed has introduced several new tools in order to conduct monetary policy in the aftermath of the global financial crises. The result of implementing three rounds of large-scale asset purchase programs, also known as quantitative easing, was that the traditional tool. Open market operations were no longer an effective operating tool because of the ample supply of reserves in the reserve market. The primary tool that replaced open market operations is the interest on reserve tool. Since October 2008, the Fed now pays certain financial intermediaries interest on the funds that are held at Federal Reserve district banks. Another tool is referred to as forward guidance. This is where the FOMC announces its conduct of future monetary policy.

This chapter is interested in comparing the open market operation tool with the interest on reserve tool in the context of a credible central bank that provides forward guidance. The paper’s analysis finds that the interest on reserve rate tools is a good substitute with open market operations.
CHAPTER 3
VECTOR AUTOREGRESSION ANALYSIS

This chapter conducts impulse response functions on empirical variables with a stationary vector autoregressive model. The VAR model is developed in order to conduct structural analysis. The objective of this chapter is to develop an empirical model based on US aggregated data that includes the interest on reserve rate.

3.1 SVAR Procedure

Reduced form VARs are described by

\[ y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \]

where \( p \) is the VAR order. Contemporaneous restrictions are imposed based on the transmission process described above. Let \( K \) equal the number of endogenous variables. Then \( y_t \) is a \( K \times 1 \) random vector such that \( y_t = (y_{1t}, \cdots, y_{Kt})' \). The coefficient matrices are denoted by \( A_i \)'s, where \( i = 1, \cdots, p \). The \( A \) matrices consist of coefficients that model the instantaneous relationships among variables in the VAR system. The matrix \( u_t \) are the reduced form disturbances. This term reflects the white noise innovations such that \( E(u_t) = 0 \), where \( E \) is the expectations operator. Furthermore, \( E(u_t u_t') = \Sigma_u \) implies that we expect...
the innovations to be contemporaneously correlated with the full rank matrix $\Sigma_u$. However, the innovations are expected to be uncorrelated with their lead and lagged innovations:

$$E(u_t u_s') = 0 \text{ where } t \neq s.$$  

In a structural VAR, the parameters are not identified. Identification restrictions are therefore required to recover the structural shocks. The theoretical model’s transmission process is used to impose contemporaneous restrictions on the structural VAR. The structural representation includes contemporaneous independent variables and is represented as

$$A_0 y_t = A_1^* y_{t-1} + \cdots + A_p^* y_{t-p} + B \varepsilon_t,$$

where $\varepsilon_t \sim (0, I_k)$ and is assumed to be orthogonal. The matrix $A_0$ is an instantaneous response matrix, which reflects the contemporaneous relationships among the endogenous variables. The matrices $A_p^*$ are the lag coefficients that are to be estimated.

The structural shocks are

$$\varepsilon_t = (\epsilon^c \; \epsilon^k \; \epsilon^p \; \epsilon^{RF} \; \epsilon^d \; \epsilon^l \; \epsilon^{er} \; \epsilon^{Rcor})'$$

and the reduced form disturbances are

$$u_t = (u_t^c \; u_t^k \; u_t^P \; u_t^{RF} \; u_t^d \; u_t^l \; u_t^{er} \; u_t^{Rcor})'.$$

The reduced form errors and structural form errors are related according to

$$u_t = A^{-1} B \varepsilon_t.$$
Multiply both sides by A in order to get an AB model:

\[ Au_t = B \varepsilon_t \]

described in Amisano and Giannini (1997). The covariance matrix of the AB model is defined by:

\[ \Sigma_u = A^{-1} B^{-1} B' A^{-1}'. \]

The restrictions imposed on matrix A reflect the transmission mechanism described in the theoretical model. The restrictions assumed on the B matrix reflect the correlation of the errors. See also Lütkepohl (2004) for a detailed explanation of the linear restrictions placed on the A and B matrices. In order to analyze the monetary policy shocks to the system of equations, the system includes the restriction

\[ u_t^R_{tor} = b_{11} \cdot \varepsilon^R_{tor}. \] (3.1)

This reflects that the interest on reserve rate is not contemporaneously impacted by any of the other variables in the model.

Restrictions are imposed on the A and B matrices such that the A matrix is a lower diagonal with ones along the main diagonal and the B matrix is a diagonal matrix of coefficients. The model for innovations is specified as:
\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{43} & 1 & 0 & 0 & 0 & 0 \\
a_{51} & a_{52} & a_{53} & a_{54} & 1 & 0 & 0 & 0 \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & 1 & 0 & 0 \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 1 & 0 \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & 1
\end{pmatrix}
\begin{pmatrix}
u_t^{R_{ior}}
\end{pmatrix}
\begin{pmatrix}
u_t^R \\
u_t^c \\
u_t^{RF} \\
u_t^d \\
u_t^P \\
u_t^{er}
\end{pmatrix}
\]

\[
\begin{pmatrix}
b_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & b_{33} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b_{44} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & b_{55} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & b_{66} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & b_{77} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{88}
\end{pmatrix}
\begin{pmatrix}
\epsilon_t^{R_{ior}}
\end{pmatrix}
\begin{pmatrix}
\epsilon_t^R \\
\epsilon_t^c \\
\epsilon_t^{RF} \\
\epsilon_t^d \\
\epsilon_t^P \\
\epsilon_t^{er}
\end{pmatrix}
\]

where the matrix \( u \) are the structural shocks. The a’s and b’s reflect the unrestricted coefficients. The ones and zeros are the identifying restrictions. The matrix \( \epsilon \) on the right is the residuals from the reduced form equations. These shocks reflect the unexpected changes of each variable.

The identification scheme reflects the interest on reserve monetary policy shock beginning with the first row. Loans are contemporaneously determined by the change in the interest on reserve rate, which is implied by the second row. The third row tells us that capital is then determined by the amount of loans provided by banks. The capital investment then determines the bank lending rate in row four. The consumption good is then determined
by the amount of capital as described by row five. All of these variables contribute to firm
and bank profits in the theoretical model, and profits determine deposits. Hence, all of the
above variables determine bank deposits as shown in row six and the price level in row seven.
Finally, excess reserves are the remaining part of deposits that are not lent back into the
economy. Row eight shows that excess reserves are contemporaneously determined by all
other variables in the model. The estimated coefficients of matrix $A$ is presented in Table
A.5 in the appendix. The results are used to create the impulse response functions described
in the next section.

### 3.2 Empirical Data

The time series variables used in the following empirical analysis are described in Table
3.1 and are shown graphically in Figure 3.1. The sample years are from 1960 to 2020. Since
the Capital Formation variable is reported quarterly, the monthly variables are averaged
into quarterly data. Six of the nine variables clearly show a trend, which must be dealt with
before we can perform any analysis. The implementation of quantitative easing is apparent
in panels (1), (2), and (3).

Deposits in panel (1) shows two distinct trends. From 1960 until October, 2008, deposits
gradually increase over time. The second trend takes place with the structural change of
quantitative easing as deposits increase significantly along with the money supply.

Excess reserves has a very subtle trend that cannot be seen before quantitative easing
takes place in panel (2). However, the trend after quantitative easing is significant.

In 2008, the Fed increased reserves by approximately $1.5$ trillion. Excess reserves reached
a peak of $2.7$ trillion in August, 2014 until a fourth round of quantitative easing took place
in early 2020. The FOMCs implementation of IOR policy can also be seen in panel (4). The
money supply in panel (3) also shows a structural change at the time of quantitative easing. At the same time, the three month Treasury bill interest rate becomes almost zero as seen in panel (7).

Macroeconomic aggregates frequently have an increasing trend. As a result, these variables are not stationary. Thus, it is necessary to transform the data into a stationary process by removing the unit root. The empirical sample data used in this paper are filtered by differencing the data so that the data becomes stationary and the unit root is removed. Additional information regarding transforming the empirical data into time invariant data is described in the appendix.

Vector autoregressions (VARs) are multivariate linear time series systems of equations. VARs are designed to reflect the joint dynamics of multiple time series variables. Each of the endogenous variables in the system are functions of lagged variables of various orders. VARs are utilized in order to analyze shocks to the system of equations\(^1\).

Policy innovations are not easily identified with the model’s other variables. One remedy is to apply restrictions in order to identify the shock in question. Structural VARs (SVARs) are applicable for such policy applications. As a result, we are then able to analyze the reactions of the remaining variables from a monetary policy shock. See Kilian (2012).

Table 3.1: Empirical Variables

<table>
<thead>
<tr>
<th>Model Variable</th>
<th>c</th>
<th>k</th>
<th>P</th>
<th>( R_f )</th>
<th>d</th>
<th>I</th>
<th>er</th>
<th>( R_{m-r} )</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable Description</td>
<td>Consumption</td>
<td>Real Investment</td>
<td>Core CPI</td>
<td>Three month T-Bill Rate</td>
<td>Deposits</td>
<td>Loans</td>
<td>Excess Reserves</td>
<td>Interest on Excess Reserves Rate</td>
<td>Money Supply (Injection)</td>
</tr>
<tr>
<td>FRED Variable Code</td>
<td>PCE</td>
<td>USAGFCFQDSMEI</td>
<td>CORECPI</td>
<td>TB3MS</td>
<td>DEMDEPSL</td>
<td>LOANS</td>
<td>EXCSRESNS</td>
<td>IOER</td>
<td>M1SL</td>
</tr>
</tbody>
</table>

**Descriptive Statistics of Nominal Data**

| Mean Growth Rate | 1.63 | –    | 0.922 | –    | 1.173 | 1.92 | 64.4 | –   | 0.01 |
| Mean Interest Rate | –    | 9.618 | 3.65  | –    | 21.786 | 6.5685 | 13.87265 | –   | 0.07 |
| Maximum Growth Rate | 5.04 | –    | –    | 4.61 | –    | –    | –    | 0.08 | –   |
| Minimum Growth Rate | –1.91 | –6.64 | 0     | –    | –6.72 | –3.36 | –54.81 | –   | –0.01 |
| Std. Dev. of Growth Rate | 0.977 | 1.8378 | 0.674939 | –    | 3.048 | 1.428 | 910.918 | –   | –0.01 |
| Skewness of Growth Rate | 0.44 | –0.20 | 1.72  | –    | 2.62 | –0.15 | 15.11 | –   | 0.78 |
| Kurtosis of Growth Rate | 4.97 | 6.40  | 6.49  | –    | 16.984 | 3.73 | 15.11 | –   | 5.46 |

**Descriptive Statistics of Real-Differenced Data**

| Mean | 0.18 | 0.137 | 0.981 | 3.55 | 0.008 | 0.0338 | 0.04478 | –0.768 | 0.2559 |
| Maximum | 4.38 | 0.213 | 2.966 | 12.69 | 1.47 | 2.05 | 5.42 | 1.85 | 31.95 |
| Minimum | –5.29 | –0.27 | –1.17 | –0.86 | –1.17 | –0.667 | –1.20 | –3.69 | –0.18 |
| Std. Dev. | 0.50 | 0.50 | –0.749 | 2.709 | 0.161 | 0.170 | 0.495 | –3.697 | 2.28 |
| Skewness | –3.11 | –1.43 | 2.37 | 0.562 | 3.13 | 5.48 | 7.32 | –0.314 | 12.31 |
| Kurtosis | 79.84 | 11.69 | 4.017 | 3.483 | 53.531 | 73.45 | 81.53 | 5.326 | 161.50 |
| Observations | 244 | 244 | 244 | 244 | 244 | 244 | 244 | 244 | 244 |

Note: Consumption is personal consumption expenditures. Real investment is gross fixed capital formation, also known as capital formation. Core CPI is an index of urban prices that excludes food and energy. The three month treasury bill rate is the interest rate of 3-month treasury bills in the secondary market. Deposits are total demand deposits at commercial banks. Loans are loans and leases of commercial banks; Excess Reserves are total reserves minus required reserves of depository institutions. The interest on excess reserves rate is the interest rate that Federal Reserves district banks pay depository institutions on excess reserves held at Federal Reserve district banks.

† Consumption, capital formation, deposits, loans, excess reserves, and the money supply is converted into real data by dividing by the CPI. The three month T-bill rate and the interest on excess reserves are converted into real terms by subtracting the inflation rate. In order to remove the unit root from the real variables, all variables are first differenced except for deposits, which is twice differenced. See appendix for details.

Source: St. Louis Federal Reserve FRED data.
3.3 Impulse Response Functions to the Empirical SVAR Model

The remainder of this chapter applies a standard SVAR approach to analyze the short-run dynamics of the macroeconomic variables used in the theoretical model. A shock to the SVAR system is referred to as an impulse response function (IRF). We are then able to see the effects of the shock on the VAR system. Moreover, the response of the shock on the endogenous variables can then be analyzed. Below we apply a unit shock to several variables in order to observe the effects on the VAR system. A IRF shock is also referred to as the impulse, residual, or the innovations\(^2\). In this analysis, there is a one standard shock to this paper’s SVAR model in order to see the reactions of the variables. The data set are quarterly variables, so each time period represents one quarter. The data set used is 1960-2020, with 235 observations. After running diagnostic tests for the appropriate number of lags, four lags are included for each variable. Various lag order selections tests are shown in the Appendix. Eviews 10 was used to perform the SVAR computations. The reduced form VAR was conducted by OLS with differenced variables in order to check for stationarity as described above.

3.4 Shock to Empirical Data

The purpose of this monograph is to study the effects of IOR policy. A positive innovation to the IOR rate represents a contractionary policy by the monetary authority. Figure 3.2 shows the IRF of the SVAR model when there is a positive unit shock to the IOR rate. Since

\(^2\)Some papers do not consider these terms as synonyms. See Ramey (2016) for an explanation of the different definitions.
the data is quarterly, each time step represents a quarter. Here, the empirical SVAR IRFs are examined in Figure 3.2. The estimated coefficients are presented in the appendix.

When comparing the sample data to the theoretical model in Figure 2.1, we can see that the theoretical model has a few successes. The model predicts that the IOR rate and the lending rate will coincide. When comparing panels (1) and (4) in Figure 3.2, the behavior of the empirical interest rates are also very similar. The three month treasury bill interest rate increases as the interest on reserve rate increases. The sample data shows real investment increase in panel (3). The model predicts a negative relationship with the higher interest rates. Empirically, consumption initially jumps up and then decreases as shown in panel (5). Deposits increase, decrease, and then increase in panel (6). The model predicts that deposits will decrease.

The price level decrease in the sample data as shown in panel (7) and also in the theoretical model. Excess reserves decrease in the data as shown in panel (8).

Next, we compare the monetary injection in the SVAR model. This is done by replacing equation (3.1) with a monetary shock. The model for innovations is then specified as:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{43} & 1 & 0 & 0 & 0 \\
a_{51} & a_{52} & a_{53} & a_{54} & 1 & 0 & 0 \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & 1 & 0 \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 1 \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & 1 \\
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
\begin{pmatrix}

\begin{pmatrix}
\begin{pmatrix}
u^{x}_t \\
u^{k}_t \\
u^{R}_t \\
u^{d}_t \\
u^{P}_t \\
u^{e}_t \\
u^{r}_t
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
= 
\begin{pmatrix}
\begin{pmatrix}
\begin{pmatrix}

\begin{pmatrix}
\begin{pmatrix}
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
\[
\begin{pmatrix}
  b_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & b_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & b_{33} & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & b_{44} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & b_{55} & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & b_{66} & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & b_{77} & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{88}
\end{pmatrix}
\begin{pmatrix}
  \epsilon^x_t \\
  \epsilon^i_t \\
  \epsilon^k_t \\
  \epsilon^R_t \\
  \epsilon^c_t \\
  \epsilon^d_t \\
  \epsilon^P_t \\
  \epsilon^{ex}_t
\end{pmatrix},
\]

The IRFs for the sample data are presented in Figure 3.3. In order to compare the sample data to the model, we compare the SVAR model IRFs to the model IRFs in Figure 2.3. The estimated coefficients for the A matrix are presented in Table A.6. The monetary shock in Figure 3.3, shows the money supply increase in panel (1). Business loans mostly increase in the sample data as shown in panel (2) and also increase in the model.

Real investment initially decrease, but then temporarily increase, and then decrease again in panel (3), but temporarily increases in the model. Interest rates decrease in panel (4), and temporarily decrease in the model. The model shows what macroeconomic theory predicts, which is a temporary decrease in the interest rate. Then interest rates are expected to increase with the price level as the monetary injection is expected to cause inflation.

Panel (5) shows consumption decreasing in the sample data. The model shows consumption increases and then decreases. Deposits increase and then fluctuate ambiguously in the sample data as shown in panel (6). Deposits temporarily increase in the model. Panel (7) shows the behavior of the price level, which initially decreases but then increases and then decreases again in the sample data, but unambiguously increases in the model. We can see that excess reserves increase when the interest on reserve rate increases in the sample data from panel (8).
Figure 3.1: **Time series empirical variables, 1960-2020.** The vertical line in each panel shows the implementation of the first round of quantitative easing. The interest rates in panel (4) and (7) are reported daily. All other variables are monthly except for capital formation, which is quarterly. Panel (2) shows that excess reserves increased from $2.2T to $2.7T by September, 2014 as a result of three rounds of quantitative easing. Excess reserves then receded until the Fed implemented another round of quantitative easing in response to the Covid Pandemic, where excess reserves the increased from $1.6T to $3.2 between March and May of 2020. We can see a structural change in deposits and the money supply in panels (1) and (3) respectively as a result of the first round of quantitative easing and the Fed’s Covid Pandemic Stimulus. Deposits gradually increase over time until quantitative easing is implemented. Deposits then increase at a faster rate between the 2007-2009 and 2020-2021 recessions from $361B in October, 2008, to $1.6T in February, 2020. Then we can see a sharp increase to $3.36T by January, 2021. Because of the Covid Pandemic, the Fed stimulus caused the M1 money supply to increase from $4T in February, 2020 to $18T by January, 2021 as shown in panel (3). Also, the definition of M1 changed in May, 2020. As the Board of Governors raised the IOR rate in December, 2015, the three month Treasury Bill rate followed. The IOR rate peaked at 2.4% in March, 2019. The IOR was significantly lowered in February and March in 2020 to 0.10% in response to the Covid Pandemic. The IOR rate and the Three Month Treasury Bill rate tended to mimic each other ever since the IOR rate tool was implemented in 2008. The two interest rates are presented in panels (4) and (7). Business loans, panel (5), consumption spending, panel (6), capital stock formation (real investment), panel (8), fluctuate together. We can see a significant decrease in both variables in 2008. However, business loans significantly increase while the capital stock formation, and consumption spending quickly fall at the time of the Covid Pandemic.
Figure 3.2: Response of the sample data to the IOR rate innovation. Empirically, interest rates tend to move together. In panels (1) and (4), the Three Month Treasury-Bill rate increases as the IOR rate is increased. Business loans initially decrease, but then increase in panel (2). Real investment and consumption initially are positive but then decrease in panels (3) and (5) respectively. Deposits fluctuate over time in panel (6). Core-CPI is negative in panel (7). Excess reserves decrease as a result of an increase in the IOR rate in panel (8).
Figure 3.3: Response of the sample data to the money supply innovation. The innovation to the money supply in panel (1) causes an decrease to the interest rate on the Three Month Treasury-Bill and the price level in Panels (4) and (7). However, the price level increases and then decreases again. The increase in the money supply also results in more loans, an initial decrease and then increase in real investment, panel (3), and deposits to fluctuate, panel (6). Real investment, consumption, and the price level initially decrease, then increases and then decreases again in panels (3), (5), and (7) respectively. Panel (8) shows that excess reserves increase as a result of a monetary injection.
CONCLUSION

This dissertation studies the transmission effect of a relatively new Federal Reserve monetary policy tool. The interest on reserve interest rate has been an important tool for raising interest rates in the aftermath of quantitative easing. Since December, 2015, IORs have been used to raise the federal funds rate in a floor system. The Federal Open Market Committee will continue to do this for some time until it unwinds its Long Term Asset Purchases and thereby reduce the supply of reserves so that supply and demand can determine the federal funds rate. Until the Fed unwinds its quantitative tightening, it will use the IOR rate as a floor interest rate in its corridor system. Hence, an interest rate of zero will no longer be the lower bound when the interest on excess reserve rate is greater than zero.

Before the implementation of quantitative easing, open market operations were the conventional tool for manipulating interest rates. The goal of this paper is to compare the effects of these two monetary policy tools. In order to achieve this objective, a two-sector partial equilibrium model is developed. The theoretical model includes a firm sector and a banking sector. The bank’s balance sheet includes excess reserves. The Interest on Reserve rate influences a commercial bank’s balance sheet, which has important implications on lending and therefore on aggregate variables. The model is then analyzed by applying impulse response functions in order to examine the transmission process of each tool. The purpose of this paper’s analysis is to understand the transmission mechanism of each monetary policy tool impacts the macroeconomy.

The theoretical model is then evaluated by applying US data in a structural vector autoregression model. The theoretical model is successful at emulating most of the variable shocks. When there is a contradiction, economic theory is utilized in order to explain the
difference. An important distinction between the model and the empirical data is that the model’s objective is to create a scenario without quantitative easing. However, IOR policy was implemented at the same time as the first round of quantitative easing in order to prevent the federal funds rate from falling to zero.

The results of this analysis are a preliminary attempt to model propagation mechanisms from IOR policy. As IOR policy continues to be utilized by the FOMC, more US data will become available to facilitate research regarding this new monetary policy tool. In this study, the structural model is tested by analyzing how well it is able to replicate empirical fluctuations of US aggregate data. This was done by comparing the structural model’s dynamics with the impulse response functions of an restricted structural vector autoregression model. The conclusion is that the structural model is partially successful at replicating the empirical dynamics.

The conclusion of the vector autoregression analysis is that the structural model is partially successful at replicating the empirical dynamics. As more data becomes available during quantitative loosening, the model can be re-estimated in order to reflect interest on reserve policy.
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Ireland, P. N. (2014). The macroeconomic effects of interest on reserve *Macroeconomic Dynamics*, 18(6), 1271-1312.


APPENDIX
A.1 Stationarity

This appendix explains the process of making the empirical variables stationary in order to conduct VAR modeling. A system of equations in a VAR consists of a vector $y_t$ with $n$ variables:

$$y_t = [y_{1t}, y_{2t}, y_{3t}, \ldots, y_{pt}]'$$

An auto regressive process of order $p$ is described by

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + \epsilon_t$$

where $\Phi$ is an $m \times m$ matrix. The number of variables is $m$ and $p$ is the number of lags. The shock term is $\epsilon_t \sim iid \ N(0, \Sigma_e)$ and is an $m \times 1$ matrix. The associated companion matrix is checked to see if its eigenvalues lie within the unit circle. If so, then the VAR is stationary. See Pesaran (2015) and Becketti (2013).

If the eigenvalues of $\Phi$ lie in the unit root circle while $M$ approaches infinity, then the solution is called covariance stationary. This section utilizes the augmented Dicky-Fuller test to test for stationarity. That is, we want to test the null hypothesis that the variable has a unit root. The variables that are used in the VAR analysis below is the first difference of the variables except for the core CPI, which is second differenced. Since all the roots lie inside the unit circle, the VAR will be stable. The ADF statistics for each variable are shown in Table A.1 and the unit root circle is presented in Figure A.1.
Table A.1: Test for Stationary on Differenced Variables

<table>
<thead>
<tr>
<th>variable</th>
<th>Augmented DF test</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>-10.5</td>
</tr>
<tr>
<td>k</td>
<td>-6.7</td>
</tr>
<tr>
<td>P</td>
<td>-16.4 (2)</td>
</tr>
<tr>
<td>$R_F$</td>
<td>-13.9</td>
</tr>
<tr>
<td>d</td>
<td>-3.07</td>
</tr>
<tr>
<td>l</td>
<td>-11.5</td>
</tr>
<tr>
<td>er</td>
<td>-11.7</td>
</tr>
<tr>
<td>$R_{ior}$</td>
<td>-6.49</td>
</tr>
</tbody>
</table>

Note: The 1%, 5% and 10% critical values for first set (level) of variables are -3.46, -2.88 and -2.57 respectively. All values were first differenced except for P, which was second differenced and indicated by a (2).

Table A.2: Lag Order Selection Criteria for Quarterly Data.

Sample: 1960Q1 2020Q1
Included observations: 229

<table>
<thead>
<tr>
<th>Lag</th>
<th>LogL</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-7513.21</td>
<td>NA</td>
<td>4.91e+17</td>
<td>66.27560</td>
<td>66.330</td>
</tr>
<tr>
<td>1</td>
<td>-6799.76</td>
<td>1364.17</td>
<td>1.87e+15</td>
<td>60.70275</td>
<td>61.25069</td>
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<tr>
<td>2</td>
<td>-6613.22</td>
<td>341.8505</td>
<td>7.38e+14</td>
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<td>60.81397</td>
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<tr>
<td>3</td>
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<td>236.0019</td>
<td>4.64e+14</td>
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<td>60.83483</td>
</tr>
<tr>
<td>4</td>
<td>-6332.5</td>
<td>244.4596*</td>
<td>2.66e+1*</td>
<td>58.764*</td>
<td>60.75500*</td>
</tr>
</tbody>
</table>

Note: * indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
HQ: Hannan-Quinn information criterion
Figure A.1: Unit Root Circle After Differencing.
Table A.3: Vector Autoregression Estimation of Quarterly Data

<table>
<thead>
<tr>
<th></th>
<th>INVESTMENT</th>
<th>CONSUMPTION</th>
<th>CPI</th>
<th>DEPOSIT</th>
<th>ER</th>
<th>IOR</th>
<th>LOANS</th>
<th>3-Month T-Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>INVESTMENT(-1)</td>
<td>0.325**</td>
<td>2.147***</td>
<td>0.002</td>
<td>−0.486***</td>
<td>−416.928</td>
<td>−0.002***</td>
<td>−1.354***</td>
<td>0.042***</td>
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<tr>
<td>INVESTMENT(-2)</td>
<td>−0.033</td>
<td>−1.587**</td>
<td>−0.007</td>
<td>0.496***</td>
<td>2347.345*</td>
<td>0.0006</td>
<td>0.187</td>
<td>0.002</td>
</tr>
<tr>
<td>INVESTMENT(-3)</td>
<td>0.194**</td>
<td>−0.326</td>
<td>0.007</td>
<td>−0.431**</td>
<td>−1545.139*</td>
<td>0.0002</td>
<td>2.624***</td>
<td>0.001</td>
</tr>
<tr>
<td>INVESTMENT(-4)</td>
<td>0.369***</td>
<td>1.724</td>
<td>0.006</td>
<td>0.207</td>
<td>−1909.170*</td>
<td>−0.0003</td>
<td>−1.095***</td>
<td>−0.008</td>
</tr>
<tr>
<td>CONSUMPTION(-1)</td>
<td>0.050***</td>
<td>0.103</td>
<td>0.001</td>
<td>0.028</td>
<td>−4.1</td>
<td>−0.000002</td>
<td>0.456***</td>
<td>0.0008</td>
</tr>
<tr>
<td>CONSUMPTION(-2)</td>
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<td>0.368*</td>
<td>0.0001</td>
<td>0.049</td>
<td>−68.139</td>
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<td>0.329***</td>
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<tr>
<td>CONSUMPTION(-4)</td>
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<td>−0.146***</td>
<td>−0.0006</td>
<td>0.014</td>
<td>380.0821***</td>
<td>−0.0002</td>
<td>−0.319***</td>
<td>−0.001</td>
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<tr>
<td>CPI(-1)</td>
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<td>−0.652***</td>
<td>7.038***</td>
<td>−3139.513</td>
<td>0.012</td>
<td>−19.342*</td>
<td>−0.476*</td>
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<tr>
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<td>−12.53</td>
<td>−0.443***</td>
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<td>CPI(-3)</td>
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<td>−23091.89**</td>
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<td>6310.502</td>
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<td>10.127</td>
<td>0.336*</td>
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<tr>
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<td>−0.619***</td>
<td>0.004*</td>
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<td>0.685**</td>
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<td>−0.569**</td>
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<td>−0.0003</td>
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<tr>
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<td>0.0000006</td>
<td>0.00007***</td>
<td>0.408***</td>
<td>0.0000008***</td>
<td>−0.0002***</td>
<td>0.0000008</td>
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<tr>
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<td>0.00003**</td>
<td>0.0002***</td>
<td>−0.000004</td>
<td>0.00002</td>
<td>−0.109**</td>
<td>−0.000009***</td>
<td>0.0004***</td>
<td>−0.0000001</td>
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<tr>
<td>ER(-3)</td>
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<td>0.0001***</td>
<td>0.0000008*</td>
<td>−0.00009</td>
<td>−0.1461442**</td>
<td>0.0000007</td>
<td>−0.002***</td>
<td>−0.0000007</td>
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<tr>
<td>ER(-4)</td>
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<td>0.00008**</td>
<td>−0.0000009**</td>
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<td>0.054</td>
<td>−0.0000009*</td>
<td>−0.0004</td>
<td>−0.0000005</td>
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<tr>
<td>IOR(-1)</td>
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<td>110.131*</td>
<td>0.09</td>
<td>110.675***</td>
<td>200964.8</td>
<td>0.651133***</td>
<td>14.516</td>
<td>0.422</td>
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<tr>
<td>IOR(-2)</td>
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<td>176.301**</td>
<td>−1.277</td>
<td>−63.918*</td>
<td>−729876.1***</td>
<td>−0.366***</td>
<td>186.748</td>
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<tr>
<td>IOR(-3)</td>
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<td>209.436**</td>
<td>1.174</td>
<td>144.889***</td>
<td>−123271.8</td>
<td>0.4487***</td>
<td>−540.926***</td>
<td>−0.576</td>
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<tr>
<td>IOR(-4)</td>
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<td>16.822</td>
<td>−0.717*</td>
<td>−114.664***</td>
<td>15853.45</td>
<td>−0.145*</td>
<td>57.17</td>
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<tr>
<td>LOANS(-1)</td>
<td>−0.028***</td>
<td>−0.169***</td>
<td>−0.005</td>
<td>0.041*</td>
<td>144.974</td>
<td>0.0003**</td>
<td>0.158**</td>
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<tr>
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<td>0.109</td>
<td>0.0002</td>
<td>0.107***</td>
<td>56.495</td>
<td>0.0002**</td>
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<tr>
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<td>0.107***</td>
<td>6624.45**</td>
<td>−0.001**</td>
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<td>0.003</td>
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<tr>
<td>LOANS(-4)</td>
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<td>−0.03</td>
<td>0.0003</td>
<td>0.104***</td>
<td>−6154.686</td>
<td>−0.005</td>
<td>5.517</td>
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</tr>
<tr>
<td>3-Month T-Bill(-1)</td>
<td>0.163</td>
<td>−1.405</td>
<td>0.103*</td>
<td>−3.662*</td>
<td>−1654.686</td>
<td>−0.005</td>
<td>5.517</td>
<td>0.029</td>
</tr>
<tr>
<td>3-Month T-Bill(-2)</td>
<td>0.311</td>
<td>−3.785</td>
<td>0.057</td>
<td>0.22</td>
<td>4802.421</td>
<td>−0.002</td>
<td>0.943</td>
<td>0.027</td>
</tr>
<tr>
<td>3-Month T-Bill(-3)</td>
<td>0.576</td>
<td>2.597</td>
<td>0.053</td>
<td>−1.475</td>
<td>3447.260</td>
<td>−0.002</td>
<td>7.733*</td>
<td>0.078</td>
</tr>
<tr>
<td>3-Month T-Bill(-4)</td>
<td>0.034</td>
<td>3.266</td>
<td>−0.02</td>
<td>0.614</td>
<td>−1371.237</td>
<td>−0.002</td>
<td>−2.79</td>
<td>−0.048</td>
</tr>
<tr>
<td>Constant</td>
<td>0.816**</td>
<td>10.943***</td>
<td>−0.006</td>
<td>−2.074</td>
<td>−1201.410</td>
<td>−0.0008</td>
<td>−11.475**</td>
<td>0.051</td>
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</tbody>
</table>

Note: The top row is the dependent variable of each regression equation. The left column lists the lagged independent variables. The numbers in parenthesis refer to lags 1 through 4 respectively. CPI is the core consumer price index; ER is the excess reserves; IOR is the interest on reserves; 3-Month T-Bill is the interest rate on the three month treasury bill. *** is significant at the 0.01 level; ** is significant at the 0.05 level; * is significant at the 0.10 level. Source: St. Louis Federal Reserve FRED Data.
Table A.4: Vector Autoregression Summary Statistics.

Quarterly Data: 1960 - 2020
Observations: 227

<table>
<thead>
<tr>
<th></th>
<th>Investment</th>
<th>Consumption</th>
<th>CPI</th>
<th>Deposits</th>
<th>ER</th>
<th>IOR</th>
<th>Loans</th>
<th>3-Month T-Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-squared</td>
<td>0.698</td>
<td>0.681</td>
<td>0.428</td>
<td>0.621</td>
<td>0.549</td>
<td>0.558</td>
<td>0.649</td>
<td>0.168</td>
</tr>
<tr>
<td>Adj. R-squared</td>
<td>0.649</td>
<td>0.629</td>
<td>0.33</td>
<td>0.55</td>
<td>0.47</td>
<td>0.48</td>
<td>0.59</td>
<td>0.03</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>−657.62</td>
<td>−1093.07</td>
<td>−29.47</td>
<td>−859.85</td>
<td>−2762.96</td>
<td>467.26</td>
<td>−1141.18</td>
<td>−247.89</td>
</tr>
<tr>
<td>Akaike AIC</td>
<td>6.084782</td>
<td>9.921396</td>
<td>0.55</td>
<td>7.865</td>
<td>24.63</td>
<td>−3.858</td>
<td>10.3</td>
<td>2.49</td>
</tr>
<tr>
<td>Schwarz SC</td>
<td>6.2</td>
<td>10.0</td>
<td>1.25</td>
<td>8.25</td>
<td>25.94</td>
<td>−3.58</td>
<td>10.4</td>
<td>2.49</td>
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<tr>
<td>Mean dependent</td>
<td>4.78</td>
<td>59.7</td>
<td>0.2</td>
<td>6.7</td>
<td>9194.3</td>
<td>0.08</td>
<td>39.8</td>
<td>−0.77</td>
</tr>
</tbody>
</table>

ER are excess reserves, IOR is the interest on reserve rate, and 3-Month T-Bill is the three month treasury bill interest rate. Source: St. Louis Federal Reserve FRED Data.
### Table A.5: IOR Rate Shock Estimated Contemporaneous Coefficients of the A matrix

| Restriction on IOR Rate | Estimate | \( a_{11} \) | \( a_{12} \) | \( a_{13} \) | \( a_{15} \) | \( a_{16} \) | \( a_{19} \) | \( a_{1,10} \) | \( a_{10,1} \) | \( a_{10,2} \) | \( a_{10,3} \) | \( a_{10,4} \) | \( a_{10,5} \) | \( a_{10,6} \) | \( a_{10,7} \) | \( a_{10,8} \) | \( a_{10,9} \) | \( a_{10,10} \) |
|------------------------|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Parameter              |          | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |
| Parameter              |          | 3.86        | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |
| Parameter              |          | 1.57        | 0.68        | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |
| Parameter              |          | 1.15**      | 0.60        | 10.45*      | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |
| Parameter              |          | 0.32**      | 0.75        | 0.57        | 12.15**     | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |
| Parameter              |          | 0.05***     | 0.69***     | 0.64        | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      | 0.59**      |
| Parameter              |          | 0.94**      | 1.07**      | 0.818*      | 1.78**      | 0.23*       | 0.33***     | 0.33***     | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |
| Parameter              |          | −0.33***    | 0.54***     | 0.88***     | 0.76***     | 0.59**      | 0.89**      | 0.53***     | 0.53***     | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |
| Parameter              |          | 1.71*       | 1.10***     | 0.96**      | 0.27***     | 0.69**      | 0.97***     | 0.97***     | 10.28***    | 1.002       | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |
| Parameter              |          | 136.68***   | 4.26***     | 18.43**     | 1.28***     | 3.44**      | 1.18***     | 3.23***     | −2.88       | 8.37        | 1.00        | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |

**Note:** Entries correspond to the contemporaneous-coefficient matrix, \( A \).

*** is significant at the 0.01 level; ** is significant at the 0.05 level. * is significant at the 0.10 level.

Source: St. Louis Federal Reserve FRED Data.
Table A.6: Monetary Shock Estimated Contemporaneous Coefficients of the A matrix

<table>
<thead>
<tr>
<th>Restriction on IOR Rate</th>
<th>Parameter</th>
<th>a_{11}</th>
<th>a_{12}</th>
<th>a_{13}</th>
<th>a_{15}</th>
<th>a_{16}</th>
<th>a_{17}</th>
<th>a_{18}</th>
<th>a_{19}</th>
<th>a_{1,10}</th>
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</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>1.00</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<th>a_{22}</th>
<th>a_{23}</th>
<th>a_{24}</th>
<th>a_{25}</th>
<th>a_{26}</th>
<th>a_{27}</th>
<th>a_{28}</th>
<th>a_{29}</th>
<th>a_{2,10}</th>
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<tbody>
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<th>a_{32}</th>
<th>a_{33}</th>
<th>a_{34}</th>
<th>a_{35}</th>
<th>a_{36}</th>
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<th>a_{3,10}</th>
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<tbody>
<tr>
<td>Estimate</td>
<td>25.37***</td>
<td>9.41***</td>
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<th>a_{48}</th>
<th>a_{19}</th>
<th>a_{1,10}</th>
</tr>
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<tbody>
<tr>
<td>Estimate</td>
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<td>7.52***</td>
<td>3.51*</td>
<td>1.00</td>
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<tbody>
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<td>Estimate</td>
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<td>13.90***</td>
<td>2.34***</td>
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<th>a_{1,10}</th>
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</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>9.95**</td>
<td>8.30***</td>
<td>1.66***</td>
<td>1.71**</td>
<td>0.95**</td>
<td>1.00</td>
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<thead>
<tr>
<th>Parameter</th>
<th>a_{71}</th>
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<th>a_{73}</th>
<th>a_{74}</th>
<th>a_{75}</th>
<th>a_{76}</th>
<th>a_{77}</th>
<th>a_{78}</th>
<th>a_{19}</th>
<th>a_{1,10}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>30.35*</td>
<td>26.44**</td>
<td>6.01*</td>
<td>1.93**</td>
<td>1.75**</td>
<td>3.86***</td>
<td>1.00</td>
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<th>a_{83}</th>
<th>a_{84}</th>
<th>a_{85}</th>
<th>a_{86}</th>
<th>a_{87}</th>
<th>a_{88}</th>
<th>a_{19}</th>
<th>a_{1,10}</th>
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</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>13.31*</td>
<td>12.20***</td>
<td>2.87**</td>
<td>1.16***</td>
<td>0.61**</td>
<td>2.37*</td>
<td>1.12</td>
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<th>a_{93}</th>
<th>a_{94}</th>
<th>a_{95}</th>
<th>a_{96}</th>
<th>a_{97}</th>
<th>a_{98}</th>
<th>a_{99}</th>
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</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>11.11</td>
<td>4.11*</td>
<td>1.35**</td>
<td>2.64*</td>
<td>1.32**</td>
<td>2.37***</td>
<td>0.70***</td>
<td>1.31</td>
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<table>
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<tr>
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<th>a_{10,2}</th>
<th>a_{10,3}</th>
<th>a_{10,4}</th>
<th>a_{10,5}</th>
<th>a_{10,6}</th>
<th>a_{10,7}</th>
<th>a_{10,8}</th>
<th>a_{10,9}</th>
<th>a_{10,10}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>39.77***</td>
<td>109.27</td>
<td>16.01**</td>
<td>11.91***</td>
<td>6.91**</td>
<td>34.71*</td>
<td>7.96***</td>
<td>6.20</td>
<td>22.46</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: Entries correspond to the contemporaneous-coefficient matrix, $A$.

*** is significant at the 0.01 level; ** is significant at the 0.05 level. * is significant at the 0.10 level.

Source: St. Louis Federal Reserve FRED Data.