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On Systematics and their Mitigation in MAGIS-100 atomic interferometer Experiment to Explore the Dark Sector and Early Universe

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ON SYSTEMATICS AND THEIR MITIGATION IN MAGIS-100 ATOMIC INTERFEROMETER EXPERIMENT TO EXPLORE THE DARK SECTOR AND EARLY UNIVERSE

Jeremiah Thomas Mitchell, Ph.D.
Department of Physics
Northern Illinois University, 2020
Dr. Swapan Chattopadhyay, Director

Understanding the dark sector and early universe are of great importance to those in cosmology as well as physics in general. To probe these areas many large detector platforms and international experiments have come online to work in unison to test proposed models of dark matter and to provide direct detection of gravitational waves to explore what cannot be seen. Along these lines the Matter-wave Atomic Gradiometer and Interferometric Sensor (MAGIS-100) has set out to explore these fields of study using atom interferometry which provides a unique and scalable method for making sensitive measurements as well as being a test bed for future large scale atom interferometers. My work has investigated the capabilities of MAGIS-100 and future MAGIS detectors specifically focusing on systematics and noise sources that would limit the detector’s sensitivity and their mitigation. In addition simulations have been performed to guide the research and development of crucial systems in MAGIS-100, such as the optical imaging system, and to construct the base for more in depth interferometry simulations, data acquisition and analysis. Work was also done on prototype experimental systems for the laser system and the atomic sources. Investigations of possible future science are also outlined with a view towards kilometer and space based detectors.
This work will guide MAGIS-100 development and lead to a better understanding of the technical requirements for large baseline atom interferometers.
ON SYSTEMATICS AND THEIR MITIGATION IN MAGIS-100 ATOMIC INTERFEROMETER EXPERIMENT TO EXPLORE THE DARK SECTOR AND EARLY UNIVERSE

BY

JEREMIAH THOMAS MITCHELL
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A DISSERTATION SUBMITTED TO THE GRADUATE SCHOOL IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS

Dissertation Director:
Dr. Swapan Chattopadhyay
ACKNOWLEDGMENTS

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DEDICATION

To my family and parents Jerry Mitchell and Laura Krezel-Mitchell for your constant motivation to be open minded and to follow my curiosity to change the world.
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PREFACE

As cosmology moves into the realm of precision measurement a path has opened which allows us to test and explore the early universe and dark sector. Large apparatus’, such as the Large Hadron Collider at CERN, have solidified theories of the Standard Model of particle physics as well as make discoveries pushing our understanding forward with precise experimental data. For the Standard Model of Cosmology LIGO and VIRGO have opened a window for looking into the universe through gravitational waves. Some of the largest events in the universe create gravitational radiation, binary black hole and binary neutron star mergers, and possibly inflation and reheating after the big bang. All of this sets the stage for new experiments that can simultaneously explore the pieces missing from the Standard Model, i.e. the dark sector, and view the early universe through gravitational waves.

One such experiment the Matter-wave Atomic Gradiometer and Interferometric Sensor, MAGIS-100, was proposed and is now underway at Fermi National Accelerator Laboratory as the first step in the MAGIS program aiming to develop terrestrial and space based sensors. The detector is an atom interferometer which utilizes precise quantum mechanics control and decades of advances in atomic clock technology. What sets atom interferometry apart is the sensitivity many types of science signals while using a single baseline in an unexplored frequency range known as the “mid-band” from 0.01 Hz to 10 Hz. There is a large number of theorized and measured signals to search for in this range including: dark matter, dark energy, and gravitational waves. There also exists the possibility of early universe signals which will provide insights for understanding the beginning of our Universe.

The ability of atom interferometers to make sensitive measurements has been extensively studied. The majority of the systematics and noise backgrounds have been investigated
with vertical 10 m scale experiments such as the EP experiment [1–5] and the MAGIS-100 prototype tower [6] both at Stanford University. The relevant sources of noise for MAGIS-100 include: laser frequency noise [7], laser wavefront aberrations [1, 8–14], seismic vibration [2, 7], Coriolis effects arising from Earth’s rotation [1, 3, 10, 15], laser pointing jitter [1, 3, 10, 16, 17], AC Stark shifts [4], initial cloud kinematics [1, 3, 5, 10, 15], mean field shifts [18], magnetic fields [10, 17, 19, 20], blackbody radiation shifts [17, 19], and imaging aberrations [2, 5]. These effects are all well-understood. My research adds to this long list of systematics by extending studies of Coriolis, laser wavefront aberrations and gravity gradient noise for the longer baseline of MAGIS-100 and the expected advancements of atom interferometry. A primary focus of my research was developing mitigation schemes for gravity gradient noise as this is an absolute limit on any terrestrial based detector and characterizing laser wavefronts which are amplified by increasing the baseline and increasing the amount of light interacting with the atoms. A simulation has also been built that generates realistic images of the atom interference pattern and allows for understanding our phase extraction precision dependent on physics and design parameters. This analysis allows us to establish limits on the effect of Coriolis, laser wavefront aberrations, and gravity gradient noise.

I also study the possibility of MAGIS-100 or MAGIS-km testing extensions to the Einstein Equivalence Principle, quantum time dilation, and gravitational decoherence of quantum superpositions. In addition I explore probing the stochastic nature of the early universe using devices such as MAGIS-km and a future network of atom interferometers. Conservative bounds are placed on what MAGIS can measure for these theories.

The layout of my dissertation is as follows. In chapter 1 I present the historical significance and motivation for a detector that can explore the dark sector and the early universe. I also explain the implications on testing quantum mechanics at a macroscopic scale. Chapter 2 focuses on the science goals of MAGIS-100 as well as the concept and design of the detector. In order to realize those goals chapter 3 goes into the details of the systematics.
and noise backgrounds that need to be measured and mitigated. Chapter 4 contains possible measurements that can be done with MAGIS-100 outside of the main scope of the detector. Chapter 5 covers the experimental work I have done on prototype test setups for the MAGIS-100 laser system and the atom source trapping and shuttling. Chapter 6 surveys the future landscape of long baseline atom interferometers and what strategies will need to be explored to control systematics as well as the exciting possibilities that lie ahead. I conclude with chapter 7.
CHAPTER 1
INTRODUCTION

1.1 Dark Matter

In the past century we have learned what constitutes the matter all around us. The molecules are made of elements that are made of atoms like protons, neutrons, and electrons. These particles all interact through gravitational forces and coulomb forces. This ordinary matter, however, makes up a slim 5% of the universe, according to current observations. There is still about 95% of our universe that is unknown to us. Of this, 20% constitutes matter that only interacts through gravitational effects. These effects were first observed through experiments related to galaxy dynamics. Prior to our understanding of the intricacies of particle dynamics we had observed strange behavior of galaxy motions that did not work with our current understanding of Newtonian gravity. To account for the expected results of Newtonian gravity one would need to add some unobserved matter into the universe. The possibilities included large astronomical objects that were just dim, some particle that we have not detected before, or we would have to alter our century old theories to account for observations. These results were first observed in the 1930s and remained a mystery into the 1970s, and are still the focus of current research. This invisible matter is called dark matter.
1.1.1 Experimental Evidence

The first experiments to truly open the door to dark matter were those of galaxy cluster apparent velocity dispersion. These were astronomical experiments focused at measuring the redshifts of galaxies within clusters, following the work of Edwin Hubble. Observations of the Coma cluster and the Virgo cluster were the first analysis that lead to unexpected results. Though galaxy velocity measurements were nothing new, Fritz Zwicky was the first to apply the Virial theorem to the recorded velocities to derive the mass of the galaxy cluster. Using the Virial theorem, which equates the kinetic energy of a system to it’s potential energy, he stated the mass must be 50 times larger than what was measured using visible light coming from the cluster [21]. He proposed this discrepancy as the mass to light ratio, which was a metric to compare the mass derived from velocity measurements to the mass observed from visible light. A similarly large mass to light ratio was observed a couple years later in the Virgo cluster by Sinclair Smith [22]. In Zwicky’s paper he said that there must be some sort of dark object causing these discrepancies in the data. Both researchers alluded to the possibility of the existence of low luminosity matter existing in the interstellar space between galaxies, or within the galaxies themselves. Following Zwicky and Smith, many other experiments of specific galaxies started to show similar results. Measuring these galaxies with x-ray emission and long-slit spectra showed high mass to light ratios as well. At that time the term dark matter was more inclusive and allowed for any astrophysical object that was not visible with current technology.

The subject of what dark matter was remained a mystery until several decades later in the 1970s. Following the observations about galaxy cluster velocity dispersion, other experiments focused on galaxy rotation curves of single galaxies, shown in figure 1.1. Even in these experiments some unexpected results appeared. Comparing the velocity curves of
the galaxy as a function of radius to the mass density curves as a function of the radius gave a large discrepancy. The velocity fit a constant value after a certain distance whereas the mass dropped off. This corresponded to some extra mass that was not visible through current technologies giving rise to a near constant velocity. The advent of new technologies such as radio astronomy and photo imaging allowed for new ways of looking into space. Data precision was increased and made it possible to take measurements of galaxies out to 110 arcminutes from galactic centers. New image techniques by Vera Rubin and Kent Ford also allowed the comparison of data with that obtained from radio measurements [23]. Rubin and Ford showed with precise resolution that the dark unobserved portions of the galaxies were indeed moving with great velocity. This was counter to what one would assume by placing all of the mass of a galaxy where the majority of the light was radiating from. With this increase of experimental evidence the astrophysics community became convinced that there was a need for some undetected mass at the outer most points of galaxies.

Figure 1.1: Galaxy rotation curves for selected galaxies, M31, M101, M81, and the Milky Way (dashed line). Measurements by Roberts and Rots in 1973 [24].
Modern measurements of galaxy collisions and the cosmic microwave background (CMB) have added to the growing collection of supporting evidence for dark matter. By looking at the lensing of light passing through colliding galaxies one can map out the dark matter that may be present. The curvature of spacetime that causes the lensing will show an enhancement with respect to the expected curvature arising from the measured visible mass [25]. An example of this is the bullet cluster as shown in figure 1.2. In addition to these precise measurements strong constraints on the presence of dark matter and the amount it contributes to the universe energy density budget have been made by observations and analysis on the CMB [26].

![Figure 1.2: Weak lensing map of the bullet cluster. Color scale shows the mass distribution determined by visible light measurements. Green contour lines show the reconstructed weak lensing signal proportional to the cluster mass density [25].](image)

### 1.1.2 Nature of Dark Matter

A considerable effort began in search of dark matter now that other experiments had verified the results using differing techniques. There were three contending interpretations
for dark matter. In terms of physical objects, which can not be observed through light, there were either astrophysical objects that were compact and difficult for us to detect or some new particle. The third option was to modify the actual theories — Newtonian gravitation — that we have leaned on for so many years. For the past 30–40 years we have searched for an answer to what dark matter is, and we still have yet to come to a conclusion. During this search we have discovered many interesting phenomena and have been able to set constraints on both the mass as well as the time of creation for dark matter with respect to specific models.

For astrophysicists an obvious guess would be that there were compact yet faint objects accumulated on the outer halo of galaxies. This density would explain the large mass to light ratio as well as the galaxy rotation curves. Possible candidates include: white dwarfs, red dwarfs, brown dwarfs, neutron stars, and black holes. In order to see these objects astronomers and astrophysicists needed to use the relativistic effect of microlensing. Thanks to Einstein’s theory of general relativity (GR) we know that mass can bend light’s path through space-time. With this fact alone they could indirectly detect invisible masses by measuring the amount of lensing occurring near the edge of galaxies. Experiment collaborations such as, MACHO, EROS, and OGLE began searching for these massive compact objects through microlensing surveys hoping to match the mass discrepancies \[27–31\]. After 5.7 years of measurements the MACHO collaboration had only flagged 14 to 17 candidate events as being caused by these faint compact objects. This corresponded to 8% – 50% of the mass of the Milky-Way being attributed to compact objects. Later the EROS collaboration further bounded this range to the 8% value as they only saw a single event. With these observations it was clear that compact objects could not possibly make up for the large increase in mass at the outer halo of galaxies. Some modern theories include primordial black holes that could have been formed around the time of the big bang \[32–34\]. Observational constraints
have left a very small mass range for their existence but they still have not been entirely excluded.

Since the late 70s there have been quite a few particle candidates that have come to the forefront of what we believe dark matter to be. The first particle to be investigated was the neutrino [35]. At the time we knew little about the neutrino other than its neutral charge, light mass, and the fact that it weakly interacted with the weak force. These made it a viable candidate so long as it could match the necessary abundance after the big bang and could lead to the structure formation we observe in the universe. Although the neutrino appeared a plausible solution, issues arose because of the low mass and high speeds of neutrinos formed during the early universe. Computer simulations showed that depending on the temperature/energy of the dark matter during structure formation different patterns would appear. Primarily hot dark matter simulations gave rise to large scale structure that was not consistent with observational data [36, 37]. This led to a move away from neutrinos and to considerations of more exotic, non standard model, particles. Following the neutrino came a slew of particles from the super symmetric theories. These included super partners to the electroweak gauge bosons [38]. A concern with these super symmetric variants was whether they could be stable enough to have an effect on the long time structure formation. This problem could be solved by invoking R-parity which allowed the lightest neutralinos to become stable. Currently super symmetry has not been experimentally verified and we are still on the hunt for super symmetric particles. The next theoretical particle to be considered is the axion. Originally this particle was theorized to solve the strong CP problem [39–42]. This problem arose from the fact that the strong force could break charge and parity symmetry in Quantum Chromodynamics, but it does not. Some requirements for the axion are a limited mass less than \(10^{-3}\) eV. If the mass were heavier than 10 keV rates for exotic meson decays would increase, and if it were greater than 1 eV red giant stars would cool rapidly. There are two possibilities for how the axions could be produced; thermal
production, which would not create a large enough dark matter density, and misalignment of the Pecciei-Quinn field [43–45]. The common feature to all of these particles is that they need to be either cold or warm in order to exist in the abundance levels we see today. More generally, there is a category of ultralight scalar fields that can be candidates for dark matter [46–48]. They would act as a classical field at a large enough number density that could account for the extra mass seen in the universe. This type of dark matter can also be generated by inflation, so a measurement would help to strengthen models of the early universe as well [49].

The last explanation for the effects of dark matter may not be a physical object at all. Our theories may just need adjusting for new precise observations at large intergalactic scales. Modified Newtonian dynamics — MOND for short — is one possibility to explain the experimental effects that we have observed. During the first observations of galaxy rotation curves Milgrom thought maybe the data could be fit with a modification to the equations being used rather than postulating some new type of matter [50–52]. He suggested a weak field approximation to alter Newton’s equations at different length scales,

\[ F = \mu(a)ma. \] (1.1)

He put this forward as a first order approximation of what may be a larger underlying theory. The problem with this approach was that it only matched the data for galaxies and dust, but broke down for galaxy cluster scales. Another issue was the incompatibility of this formalism with GR. To fix this issue Milgrom and Bekenstein proposed a relativistic variant of modified Newtonian gravity, which used an altered lagrangian, known as AQUAL and eventually RAQUAL [53, 54]. Other versions have been developed since then such as TeVeS (Tensor, Vector, Scalar gravity) [55]. This theory uses a modified GR metric and two
additional fields to become relativistic, however, it is still incompatible with data on galaxy cluster scales.

These candidates potentially answer the question of what dark matter is. Currently the experimental evidence has not favored one over the other, rather the computer simulations and researchers have put significant interest into which should be explored leaning on the effectiveness of quantum field theory techniques and the standard model of particle physics. In order to finally make a concrete statement we need more data by using more advanced detectors and technologies.

1.1.3 Measuring Dark Matter

The effort to detect dark matter has been ongoing with no solid conclusions as of yet. Current research has narrowed down the mass range for particle and astrophysical object dark matter to increasingly smaller margins. Since the discovery of gravitational waves by LIGO [56] efforts have been strongly refocused on primordial black holes (PBH) as a source of the dark matter in the universe. For PBHs in the mass ranges of order $10 \, M\odot$ have been excluded as making up 100 percent of the dark matter abundance by Radio and X-ray observations using the Very Large Array and Chandra X-ray [57]. This mass of PBHs has also been restricted by 21-cm spectrum measurements to have $f_{PBH} < 10^{-3}$ [58]. Pulsar timing arrays have also been used to measure the dark matter percentage residing in PBHs with $1–30 \, M\odot$ [59]. This study found the fraction to be no more than 1%–10%. The CMB is also a source of new constraints on PBH dark matter. The merger rate and effect on polarization constrains the dark matter fraction to be less than 1% and also rules out PBHs with mass larger than $10^2 \, M\odot$ [60–62]. With all of this analysis the possibility that PBHs make up all of the abundance of dark matter in the universe is unlikely.
Massive astrophysical compact objects (MACHOs) have also had increasingly strong constraints placed on them. They have all but been eliminated as a possibility to make up all of the dark matter abundance. Most recently new microlensing, dwarf galaxies, and supernova have been used to place some strong limits on MACHO dark matter. Using quasar microlensing compact objects outside of mass range $0.05 \, M_\odot < M < 0.45 \, M_\odot$ are negligible. Within the mass range abundance is found to be around 20% in accordance with stellar abundance [63]. Supernova magnification and lensing data has also been used to set abundance limits [64]. On top of these searches ultra-faint dwarf galaxies have been used as models of being dominated by their dark matter content. They are heated by MACHOs and set limits on abundance. They have excluded all MACHO masses greater than $10^{-7} \, M_\odot$ [65]. With these current observations the total search area for the nature of dark matter is shrinking.

1.1.4 Dark Matter and Atoms

Atoms serve as an excellent tool in measuring the precise effects of some dark matter models. For example, ultra light dark matter would clump together and become dense enough to treat as a field of dark matter. That field may interact with the known energy levels or other fundamental constants of atoms such as strontium. This interaction can cause the levels to be pushed apart or fluctuate in time and change the time for a level to emit a photon or react to an interaction with laser light. By comparing a difference in the energy levels of two atoms in free-fall in a vacuum one can test dark matter model couplings. Other characteristics of the atoms can be altered by dark matter as well. The mass of the atom may be shifted by the dark matter field or the fine structure constant may pick up additional terms. This can be thought of as an additional force acting on the atoms caused by gradients
in the dark matter field. Another possible feature of dark matter would be any coherent oscillation of the energy levels caused by the interaction with a dark matter field. Using atoms as a differential measurement to test for these alterations will further the search for dark matter.

1.2 Dark Energy

1.2.1 Einstein’s Constant and Hubble’s Acceleration

After Einstein’s work on formulating the theory of GR he began working out the equations to describe the evolution of the universe. He had worked out an initial set of equations assuming the universe was static and unchanging, however these equations required an extra term denoted as the cosmological constant that was necessary for his incorrect assumption [66]. While not entirely unsettling this constant began to cause troubles in the late 1920s because of the contradictory measurements made by Hubble. Einstein’s views would soon change once the conclusions of Hubble’s research were verified.

Hubble was working on measuring the distance and Doppler shift of the farthest visible galaxies. Using his observational data he compared the galaxy velocities with distance and found a strong linear relationship now known as the Hubble law, \( v = H_0d \), where the radial velocity of the galaxy is proportional to the distance from Earth. The constant of proportionality is the Hubble constant. Compiling his data revealed that galaxies in the universe were moving away from us in all directions and that the universe could not be static. All of the light that was arriving at the telescope was redshifted at different levels the farther out he observed implying the universe was actually expanding in all directions [67]. This revelation led to a shift in how we perceive the cosmos. Modern research that has
spawned off of Hubble’s has led to more precise measurements of the Hubble law, and a search to nail down the value of the Hubble constant.

Further conflicts were caused by the developments of quantum field theory (QFT). Comparing the size of the predicted vacuum energy from GR and QFT led to a disagreement at the order of $10^{121}$. Clearly this caused great tension in the field of cosmology and still to this day remains an open question [68]. Einstein had decided to throw away the cosmological constant for it gave him nothing but grief. However in 1998–2000 the cosmological constant would make a necessary return to the cosmology stage to answer new observations but give rise to even more questions.

### 1.2.2 Expanding Universe Measurements

Dark energy as we know it today was first discovered by two international teams consisting of Adam Reiss, Saul Pelmutter, and Brian Schmidt. They used a method of observing type Ia supernova which are known as standard candles for distance measurements because of their well defined luminosity [69, 70]. Their measurements use the known luminosity to track the rate of expansion of the universe as predicted by Hubble. They ended up showing that even more puzzling the universe was accelerating in its expansion at faster and faster rates. Their measurements along with the assumed flatness of the universe from cosmic microwave background (CMB) measurements constrained the energy density of matter, $\Omega_m = 0.28^{+0.09}_{-0.08}$, and the rest attributed to dark energy. Measurements of supernovae would go on to further support this measurement and earn the team a Nobel Prize in 2011. Current day measurements from the PLANCK Collaboration have set tight constraints of $\Omega_m = 0.315 \pm 0.007$ [26].
The existence of dark energy is also predicted and constrained by measurements of the CMB and the age of the universe through the WMAP experiment. For example a flat universe with no dark energy can only be 8–10 Gyr whereas a non-zero value for dark energy leads to approximately 13.1 Gyr which is what the data from WMAP fits. The CMB till now prefers a flat universe. The most recent data establishes the age of the universe to be $13.772 \pm 0.059$ Gyr \cite{71}. With the existence of dark energy highly preferred from almost a decade of measurements we are now at a point where understanding the true nature of the dark energy is required.

\subsection{1.2.3 Theoretical Models}

Since the discovery of dark energy theorists have laid out an extensive list of models. These models can be separated into two groups. We shall discuss first the cosmological constant representations of dark energy where the constant introduced by Einstein is reinstated in the equations of motion and interpreted as a universal parameter set by observation. Then we shall briefly review some of the dynamic dark energy models. These assume a dynamic parameter that either fluctuates in time or has an energy dependence.

Dark energy led to the revival of the cosmological constant, however, this time it is not used for balancing the universe in a static state but for providing an internal (negative) pressure that accelerates the expansion of spacetime. Some of the contending theories include \Lambda derived from: quantum gravity, degenerate vacua, higher dimensional gravity, supergravity, string theory, spacetime foam, and vacuum fluctuations. Compiled reviews of these can be found in \cite{72}. Assuming that the dark energy originates purely from vacuum energy leads to the “greatest error in physics”. The energy density as derived from QFT as a sum of the zero-point energies leads to a value $\rho_{\text{vac}} = 10^{74}$ GeV$^4$. This value is much larger than the
current observed value of $\rho_\Lambda \approx 10^{-47} \text{GeV}^4$. To avoid such a large discrepancy some theories have considered treating the cosmological constant to be zero and using a light scalar field approach to explain dark energy.

Some models theorize that dark energy is actually a dynamic quantity that evolves in time or under interactions with other matter. These dynamic theories include: Quintessence, K-essence, ghost fields, chameleon fields, and time varying cosmological constants [73]. In these models dark energy is treated as a scalar field and each theory places different constraints on the equation of state of the field. What makes these theories interesting is their potential for interacting with matter in the lab. This provides a means other than astronomical observation for verifying the candidate models.

### 1.2.4 Dark Energy and Atoms

Atom interferometers could play a crucial role in constraining some models of dark energy. The primary models that can be explored are those with screening mechanisms that hide a fifth force coupling above certain length thresholds and those that alter atomic quantities such as the fine structure constant [74, 75]. Of the models mentioned above the chameleon fields present an opportunity to test the effect of spatially differing potentials across a relatively small distance scale. The screening mechanism of the chameleon field would mask all but the shortest range couplings. Efforts have already been made towards constraining this theory through torsion-balance experiments [76, 77], Bose–Einstein condensates [78], and neutron interferometry [79–82]. A measurement of the effects of local chameleon fields would require a gradiometer measurement of two different atom clouds with different local densities. One would then search for screening differences in the compared phase measurement of the atom interference patterns.
1.3 Gravitational Physics

Effects of gravity have been pondered since before Galileo. Why do objects fall when tossed up? Why can we not leave the ground without great effort? How do the stars, Sun, and Moon hang in the sky effortlessly while we are seemingly fixed to the Earth? Theories included metaphysical arguments about the natural state of like elements attracting each other such as rocks remaining on the surface of the Earth like other rocks, and the stars are fixed in the sky like being glued to the surface of a sphere encasing the globe. It was not until Newton that the local and distant effects were brought together and made into a whole theory that was obeyed universally. Even then there seemed to be something lacking from the description of gravity. Although it allowed for unprecedented measurement and predictive powers nature found ways to avoid description. One example was the retrograde motion of Mercury. This could not be fully explained using Newton’s laws of gravity and orbital motion.

When the works of Faraday and others were sewn together by Maxwell even more discrepancies in gravitation arose. Space and time in Newton’s formulation were deemed absolute and unchanging and all motion could reference a specific ruler and watch, however, in electrodynamics light has a maximum finite speed. Newton’s law of gravitation implies that the gravitating effect of one mass on another is an instantaneous force. How can this be if electrodynamics sets a maximum speed for observable interactions? Should gravity not follow the same rules? Einstein hoped to settle this argument through the development of special relativity in which he put space and time on equal footing and found the speed one moves influences how they make measurements. Special relativity imposed the maximum speed from electrodynamics on the laws of gravitation. Some surprising results were that space and time could no longer be thought of as absolute in the Newtonian sense. Observers
then would measure different values for the location and moment of events happening. Gravitation could therefore not be instantaneous. This was only one step towards explaining the nature of gravity and spacetime.

Einstein did away with the concepts of absolute space and absolute time with special and general relativity [83]. To further generalize his theory he allowed for spacetime to be curved. In flat spacetime Euclidean geometry and laws hold for the description of how objects move, i.e. parallel lines never crossing and the sum of a triangle’s angles being 180 degrees. By applying differential geometry as the underlying framework of motion Einstein would discover the intimate connection between spacetime and energy. This expansion led to Einstein’s theory of GR which describes the effects of matter and light on spacetime and simultaneously the effect of spacetime curvature on the trajectories of matter and light. Using the methods developed by Einstein led to the beginning of cosmology and the study of the early universe. The largest predictions made by GR were the explanation of Mercury’s retrograde motion, bending of light around massive stars and planets (lensing), black hole singularities in locations of enormous mass, and finally the radiation of gravitational waves by massive moving objects. All of these predictions have now been observed with the newest being the discovery of gravitational waves in 2016 [56]. These waves can now be measured using extremely sensitive interferometers.

Gravitational waves will be useful in answering how the universe evolved into the state we perceive today and may shed light on the correct model of the early universe. The leading theories begin with the hot big bang model [84], which states the universe started in an extremely hot and dense state and expanded. Expansion of the universe was shown by Hubble to be a fact based on observations of Doppler shifted light from stars [67]. The mechanisms that took place after the big bang have evolved in order to address the discrepancies of modern observations. The leading description is the inflationary model. There are also more exotic explanations such as cyclic models of the universe, and bouncing cosmology models.
To understand the need for theories of the dynamics of the universe we must consider the physical conflicts that arose from the hot big bang theory. These can be captured as 1) the horizon problem and 2) the flatness problem [85].

The horizon problem is so called because the observations of the CMB show a homogeneous universe despite the causally disconnected regions of spacetime from our perspective. There exist disconnected regions because of the finite speed of light. Photons observed today would have past time-cones that do not overlap meaning there is no physical mechanism by which they could communicate. This becomes puzzling when looking at the temperature of the CMB. Over large arc angles the temperature is strikingly uniform. This leads to the question of what mechanism provides such a uniform and isotropic universe.

The flatness problem is slightly more technical to explain. From Einstein we know that the geometry of spacetime is directly related to the matter content of the universe. The more energy there is the stronger the fabric of spacetime is contorted. The geometry of the universe can be described as concave (closed), convex (open), or flat. These are related to the value of the curvature term arising in the Friedmann equations describing the dynamics of the universe and the evolution of its energy content. From current measurements the universe appears flat, however this implies that the initial value of the Hubble constant needs to be fine-tuned otherwise small deviations in the early universe lead to a non-flat spacetime today. That would disagree with current observation.
1.3.1 The Early Universe

Inflationary Cosmology

As observational data from the CMB began coming in the hot big bang theory gained strong support. This lead to questions about the mismatch between current observations of the homogeneity and isotropy of the universe and the expected initial conditions from the big bang. In order to answer these discrepancies many versions of early inflation theories put forth ideas of super-cooling, i.e. an extremely rapid expansion, of a vacuum state with large energy density (false vacuum), and chaotic inflation where the universe eternally grows [85–89]. These ideas put forward by Linde, Chibisov, and Starobinsky in the late 1970s and early 1980s laid the ground work for the current theory of inflation, however they solved the horizon and flatness problems at the cost of having problems of their own. For example the old inflation models suffered from “graceful exit” problems related to the question of how the universe stopped inflating and ended up in its current homogenous state.

Modern inflation was built upon the old inflation theories with the goal of finding a theory that solved the horizon and flatness problems, and did not suffer from a graceful exit, or a false vacuum state problem. In the late 1980s Guth and Linde had worked on a model of inflation which allowed for a smooth exit by imposing a simple effective potential and allowing the representative scalar field (the inflaton) to slowly roll down the potential like a ball rolling through a thick fluid [90]. This inflation model required some fine tuning constraints on the initial conditions for inflation to begin and thus was modified to allow for a larger range of initial conditions that would all lead to an inflation scenario.

Current cosmological measurement methods have not yet determined the validity of inflation as there exist only a few parameters to test for including: the tensor to scalar ratio,
the spectral tilt (the slope of the power spectral density), and the power spectrum of tensor perturbations, also known as gravitational waves [91]. With advancing telescope technologies and the evolution of the PLANCK experiment the constraints on the tensor to scalar ratio as well as the spectral index are becoming tighter still and the best possible method for verifying inflation may be through the use of gravitational waves to measure the early universe [26]. Standard inflationary models predict a flat spectral energy-density. The expected dimensionless strain for detection would have an amplitude of $10^{-22}$ to $10^{-32}$ for ambient gravitational waves with frequencies from $10^{-5}$ Hz to $10^5$ Hz. The spectral index will also be red shifted. Another possibility of probing inflation is through a detection of ultralight scalar dark matter. The production of such dark matter requires an inflationary model to have existed [92]. These measurements would vary significantly from other more exotic models of the early universe dynamics.

**Cyclic and Exotic Models**

Other lines of thought were also followed for answering the fundamental cosmological questions. Of these alternative models the most successful are the cyclic models of the universe. The variations of cyclic models include the ekpyrotic model, and bounce model. These models differ from inflation by having multiple regimes of changing spacetime scale but the main difference, conceptually, is a history before the big bang. These models are not as well accepted as the inflationary models however still warrant an examination and testing as we wish to remain agnostic to the true nature of the early universe.

Cyclic models developed by Steinhardt and Turok provide solutions for the horizon and flatness problem and do not suffer from the multiverse problems of modern inflation. The ekpyrotic model, the first developed by Khuroy, Steinhardt, and Turok in 2001, was based
Heterotic M-theory led to the use of branes which are abstract higher dimensional manifolds that spacetime is embedded in. In the ekpyrotic model there is a visible brane, representing the visible universe, and a hidden brane. Each brane is 5 dimensional with 4 spatial dimensions and time. One can imagine a bulk volume between the two branes that allows for interaction of the branes along the 4th spatial dimension. A potential between these two branes leads to oscillatory motion of the boundaries and collisions of the branes act as big bangs. Although the overall description lacks an inflationary period the same homogenous and flat nature of the universe can be regained during a period of contraction. Although the ekpyrotic model answered some of the issues of the big bang it left one major hole.

Original cyclic models of the universe suffered from problems of traversing the big bang singularity. If the universe really did go through cycles of contraction and expansion how are we to understand the singularity at the heart of the big bang model? This question lead to a revised cyclic model of the universe by Steinhardt that updated the 5-D brane picture by allowing the potential between the visible and hidden branes to be represented by Dark Energy [94]. This naturally allows Dark Energy to explain the periods of expansion as the time after collision where brane kinetic energy is being smoothed out and added back to the potential energy of the bulk along the fourth spatial dimension. Another solution was recently worked on with the bouncing models of the universe where a simpler model is used, say one cycle of contraction, big bounce, and expansion. Then the bounce can be treated as a non-singular bounce which allows the prior information to pass through without being destroyed [95, 96]. Another form of traversing the bounce has also been worked on using vortexes to move through the non-singular bounce [97]. These models have matured to the point of providing some measurable observations.

The gravitational wave background expected from Cyclic models has a different form than that expected from inflation. The primordial gravitational waves produced by such
cyclic models would be blue shifted and have a much smaller strain amplitude, on the order of $10^{-38}$ to $10^{-34}$ over frequencies $10^{-5}$ Hz to $10^5$ Hz, than those predicted by inflation [98]. Another discernible difference is the value of the spectral tilt measured by the CMB [99, 100]. To verify the cyclic models predictions more direct measurements of the early universe gravitational wave background are needed.

Using the expected gravitational wave power spectrum as well as the tensor to scalar ratio and spectral index any direct measurement of the primordial gravitational wave background would allow us to discriminate between which early universe model is correct. This suggests that extremely precise gravitational wave detectors in a new frequency band would provide out best hope of learning more about the early universe.

1.3.2 Classical and Quantum Equivalence Principle

Weak Equivalence Principle

The weak equivalence principle is the equivalence between inertial mass and gravitational mass. It can equally be stated as the tendency for all matter to fall at the same rate in a uniform gravitational field. This is the same principle tested as far back as Galileo. Unlike his predecessor Isaac Newton, Galileo did not formulate his hypothesis in a mathematical framework and tested the theory for various objects at great heights. The mass of an object is considered in two of the equations described by Isaac Newton. He showed the gravitational force between two objects of mass is proportional to the product of their masses. Newton’s second law of motion also involves the mass of an object and states that the force is equal to the mass times the object’s acceleration. As is taught in introductory physics courses the acceleration caused by the Earth’s gravitational field is, $g \approx 9.81 \text{ m/s}^2$. According to the
weak equivalence principle the all forms of matter should fall with this acceleration. The modern day challenge has been to put the weak equivalence principle to the test at the smallest and largest scales and see if it persists.

Testing the weak equivalence principle requires high vacuum, precisely known masses, and a large change in position. If not all of the systematic effects on a falling mass are taken into account then the maximum sensitivity to set constraints on the equivalence principle falls greatly. Current tests have used various methods. Some of them include extremely delicate torsion balances or large drop towers reminiscent of Galileo’s original experiments as well as lunar laser ranging [101–103]. Some more modern methods also involve making measurements of galaxies and other astrophysical objects such as the MICROSCOPE experiment, or by using gravitational waves [104, 105]. There are even tests of the weak equivalence principle using trapped antimatter [106]. Atoms also serve as an extremely sensitive tool for measuring mass differences. One simply compares the trajectory of two isotopes of the same atom by putting them into a quantum superposition and letting them free-fall under gravity.

Atom interferometry has matured rather rapidly and will be able to operate close to the most sensitive classical tests of the equivalence principle which have set bounds on the Eötvös parameter \( \eta = 2(M_a - M_b)/(M_a + M_b) \), where \( M = m_g/m_i \) the ratio of gravitational to inertial mass, of \( \eta < 1.3 \times 10^{-14} \) [107]. Current experiments from Mark Kasevich’s lab have set a lower bound of \( \eta < (1.6 \pm 1.8\text{(stat)} \pm 3.4\text{(sys)}) \times 10^{-12} \) for the weak equivalence principle using isotopes of Rubidium atoms [5, 108]. Future atom interferometers will be able to push these tests to a maximum sensitivity.
Quantum Equivalence Principle

When considering quantum mechanics it should be natural to ask whether the mass of a particle is associated with a specific energy state or with some macroscopic averaging of all the possible states. We should consider the weak equivalence principle as applied to the masses in a quantum system as well the Einstein Equivalence Principle which states there is no discernible way to differentiate between an accelerating reference frame or being stationary in a uniform gravitational field. To ask whether these quantum equivalence principles hold we should begin by specifying the framework we need in order to ask the question in the first place.

There are three categories of a quantum version of the Equivalence Principle that we would like to consider. These are: quantum formulation of the Weak Equivalence Principle, quantum violations of the Einstein Equivalence Principle, and a formal Quantum Equivalence Principle [109–112]. The difficulty of constructing a quantum equivalence principle comes directly from the difference between classical and quantum mechanics. In classical mechanics the locality of the considered objects and their trajectories allow for an explicit discussion of the gravitating of a massive object and the path the mass takes through spacetime. In quantum mechanics, however, the wavefunction is nonlocal and does not explicitly state where the mass of the gravitating object is with complete certainty. Thus to discuss the equivalence of inertial mass and gravitating mass we need to be able to assign a mean value path for the quantum particle in order to satisfy the equivalence principle. The theories mentioned above attempt just that. They explore the breaking of the classical equivalence principle by the existence of superposition states for massive quantum particles and devise ways this breaking can be measured using atom interferometers and the matter-wave nature of atoms.
1.3.3 Atoms Feel The Universe

As with the dark sector of the universe being visible through small effects on atoms, the early universe can also influence the phase signal of an atom interferometer. The early universe can now be explored through gravitational waves. Different models of the dynamics and beginning of the universe lead to different models of expected gravitational waves. An atom interferometer set in a long baseline configuration would then be able to measure the time lag of the laser beam used to interact with the atom clouds. This interaction delay would effect the phase readout of the atom interference and provide limits on early universe models.

The classical equivalence principle applies to the free-fall of different mass objects such as two isotopes of atoms which can be used to test the acceleration difference precisely through the phase accumulation. Putting atoms into a superposition such as in an atom interferometer would also allow for testing the quantum effect on the acceleration of free-fall by comparing the phase at difference points along the atoms trajectory and state.

1.4 A New Detector

All of the above science goals can be explored using atom interferometry. Atom interferometers are relatively new as the requisite components have been in development since the late 1990s and have since been built in various different configurations in order to test different theories [113]. The main science goal comes from a measurement of the phase difference between two “arms” of the atom interferometer. This is in analogy to Mach–Zehnder laser interferometers which use two arms to split a coherent light source, send the beams down different paths and recombine them at an output imaging port. The resulting effect is the
laser light interfering with itself either constructively or destructively. Any small difference between the paths taken, whether it be a distance change or other force acting on the light, will be measurable from the phase difference of the interference peaks. In a similar way the atoms are treated quantum mechanically as matter waves and then put into a superposition state where the ground energy level state and the excited energy level state act as the different paths for the matter-wave to take. Under free-fall the two paths will interact with gravity and any other force fields present and once recombined and caused to interfere the phase shift will contain the information of the trajectory and environment the atoms moved through.

With a robust error model and well controlled systematics atom interferometers are a next generation tool for sweeping out the space of dark matter candidates and testing dark energy models, gravitational waves, and early universe models.

### 1.4.1 Dark Matter Detection

A subset of the dark matter candidates referred to as ultra light scalar dark matter are testable by an atom interferometer [48, 114]. Since the particles are ultra light, meaning they have an energy on the order of $10^{-22}\text{eV} - 10^{-3}\text{eV}$, we treat them collectively as a classical field as long as they have a high enough number density. This field will have an oscillation frequency base on the Compton frequency associated with the mass of the dark matter candidate. For a sensitive enough quantum detector this oscillation will coherently influence the internal energy states of an atom. Such an internal oscillation is detectable using an atom interferometer where the atom energy states are well known and we can compare differential measurements of the atom wavefunction interference.
Two effects that can be present in the atom phase signal are a variation in the fundamental constants of the atom: mass, \( m \), or fine structure constant, \( \alpha \). These effects take the form [114]

\[
\frac{m_{\text{eff}}}{m} = 1 + \left( \frac{\sqrt{\hbar c} \phi(\vec{r}, t)}{\Lambda_{n,p}} \right)^n,
\]

\[
\frac{\alpha_{\text{eff}}}{\alpha} = 1 + \frac{(\sqrt{\hbar c} \phi(\vec{r}, t))^n}{\Lambda_{n,p}},
\]

\[
\phi(\vec{r}, t) = \phi_0 \cos \left( n\omega_{\phi} t - \vec{k}_\phi \cdot \vec{r} \right),
\]

where \( \phi \) is the dark matter field wavefunction, \( \Lambda_{n,x} \) is the high-energy renormalization cut-off for \( x \), and correction orders \( n = 1 \) linear or \( n = 2 \) quadratic terms.

The second possible phase shift comes from the fluctuations of the Earth’s gravitational field

\[
\Delta g_n = \left( \frac{2\rho_{\text{DM}} \hbar^3}{m_{\phi}^2 c \Lambda_n^2} \right)^{n/2} \frac{1}{2^{n-1}} \cos \left( n\omega_{\phi} t - \vec{k}_\phi \cdot \vec{r} \right).
\]

Here the dark matter field density and mass are \( \rho_{\text{DM}} \) and \( m_\phi \) respectively, and \( \omega_{\phi} \) the Compton frequency. For dark matter fields with spatial variations, such as a gradient in the rest mass of the atom \(-\nabla mc^2\), a network of atom interferometers across the planet may be able to make correlated measurements to extract this gradient dependent phase shift. Such a network has been proposed by the Atom Interferometer Observatory Network (AION) [115].

The effect of the parameter oscillations will be imprinted directly in the phase signal extracted from the atom cloud interference patterns of an atomic interferometer [116].

### 1.4.2 Dark Energy Detection

Various screening models and fifth force dark energy models have the possibility of being measured with an atom interferometer thanks to the precision of the atom clouds sensitivity to the surrounding environment while in free-fall. One way of probing chameleon fields
is by using an atom interferometer with an extreme source mass between the separated wavefunctions of the matter-wave [75, 117]. Once interfered any fifth force applied to one of the arms would cause a phase shift that could be measured through the trajectory phase shift. For example a chameleon field model has a Lagrangian

$$L_{\text{chameleon}} = -\frac{1}{2} (\partial \phi)^2 - V(\phi) - \frac{\phi}{M} \rho_m.$$  

(1.4)

The potential $V(\phi)$ can take on various forms associated with the type of screening effect present.

### 1.4.3 Early Universe Signals

There are many interesting signals that can be sensed through gravitational waves and are of great interest to fully understanding our universe. They include gravitational waves coming from cosmological sources, detection of phase transitions, and exploring the pre big bang era [118–120]. With an atom interferometer with a baseline on the order of 100 m strain measurements will be able to increase sensitivity in the mid-band (0.1 Hz–10 Hz) which is currently unexplored. One would hope that there exist many possible signals to detect in this frequency band coming from proposed theoretical models such as extreme solar mass binary black holes, cosmic string networks, and white dwarf binaries [121–124]. Once atom interferometers are scaled up to the kilometer baseline scale a differential measurement across the entire baseline will be sensitive to strain fluctuations of currently observed sources such as binary black hole mergers detected by LIGO [17].

The ability for atom interferometers to measure gravitational waves in the mid-band frequency range allows for the ability to perform multi-messenger astronomy. For the sources that were detected by LIGO an atom interferometer would be able to make measurements of
the binary black hole system well before the eventual coalescence. This makes sky localization possible months to years ahead of time. All telescopes, gravitational and optical, will be able to align their measurements and make a joint effort in performing the signal detection [125]. With correlated measurements of networks of atom interferometers it may also be possible to measure stochastic gravitational wave backgrounds which would allow a direct probe of the early universe and allow discrimination between inflationary and cyclic models which predict different spectra of background gravitational radiation.
CHAPTER 2
MAGIS-100

2.1 Atom Interferometry

Atom interferometers take advantage of the developments of ultra cold atoms, precise
atom-laser interactions, and quantum superpositions. These components allow for treating
a large number of atoms in an atom cloud as a single wave packet which can be split into
a superposition by undergoing Rabi oscillations when interacting with a laser pulse. This
superposition then builds up phase differences similar to the phase difference in the two
paths of a classical laser interferometer. The separate wavepackets can then be interfered in
order to readout this phase difference from an image of the matter wave interference pattern.

Ultra cold atoms allow for using and manipulating a large number of atoms in a pure
and coherent state. If we begin with atoms in a state $|1\rangle$ a tuned laser pulse can provide a
momentum kick of $\hbar k$ where $k$ is the wavevector of the light putting the atom into state $|2\rangle$
by absorbing a photon and providing a recoil velocity $v_r = \hbar k/m$. Once an atom is in state
$|2\rangle$ it can be transitioned back to state $|1\rangle$ via stimulated emission of a photon again applying
a frequency tuned light pulse and slowing down by the same recoil velocity. In addition to
giving the atoms momentum kicks and changing their internal states with light pulses we
can also achieve superpositions of states $|1\rangle$ and $|2\rangle$. This is accomplished by applying tuned
light known as a beamsplitter ($\pi/2$-pulse) which causes Rabi oscillations of the atom having
a equal probability of being in either state. A mirror pulse ($\pi$-pulse) transitions the atom
between $|1\rangle \leftrightarrow |2\rangle$ by either absorbing a photon or emitting a photon.
The simplest atom interferometry configuration is the Mach-Zehnder interferometer. In this configuration 3 light pulses \( \left( \frac{\pi}{2} - \pi - \frac{\pi}{2} \right) \) are used to complete a single loop of the atom interferometer. A beamsplitter pulse, with a tuned frequency, intensity and duration, is used first to separate the atoms into a superposition of states \( |1\rangle \) and \( |2\rangle \) at time \( t = 0 \) where the upper trajectory has the added momentum from the absorption of a photon. After the wavepackets are given time to separate at time \( T \) a mirror pulse is applied to interact with both arms and to reverse the direction of the atoms \( (|1\rangle \leftrightarrow |2\rangle) \) by a recoil kick again with the light tuned to cause emission and absorption in the proper arms. At time \( 2T \) when the wavepackets are overlapped a final beamsplitter pulse is applied that causes the wavepackets to interfere with mixed contributions from the upper arm and lower arm. These two locations with mixed populations are called the ports of the atom interferometer and are then imaged by resonantly scattering photons causing the atoms to fluoresce. Figure 2.1 shows a diagram of the Mach-Zehnder interferometer sequence.

![Figure 2.1: Schematic of 3-pulse Mach-Zehnder atom interferometry sequence. Wavy lines represent the laser light pulses with infinite speed. Beam-splitter pulses are denoted \( \pi/2 \) and mirror pulses \( \pi \).](image)

Phase builds up in each arm of the atom interferometer from three main areas: the propagation of the atoms through space-time, the laser phase as imprinted upon interaction with atoms during light pulses, and the final separation of the ports of the atom interferometer.
during interference. The contributing portions to the total phase shift in a non-relativistic semi-classical calculation take the form

\[ \Delta \phi = \Delta \phi_{\text{prop}} + \Delta \phi_{\text{laser}} + \Delta \phi_{\text{sep}}, \]

\[ \Delta \phi_{\text{prop}} = \left( \sum_{\text{upper } t_i}^{t_f} (L_c - E_i) \, dt \right) - \left( \sum_{\text{lower } t_i}^{t_f} (L_c - E_i) \, dt \right), \]

\[ \Delta \phi_{\text{laser}} = \sum_{j} \pm \phi_L(t_j, \vec{x}_u(t_j)) - \sum_{j} \pm \phi_L(t_j, \vec{x}_l(t_j)), \]

\[ \Delta \phi_{\text{sep}} = \langle \vec{p} \rangle \cdot \Delta \vec{x}. \]

In equations 2.1 upper and lower refer to the atom’s wavepacket trajectory depicted in figure 2.1, \( L_c \) is the classical Lagrangian for the atom of mass \( m \), \( E_i \) is the internal energy of the atom, \( \phi_L(\vec{x}, t) = k_{\text{eff}} \cdot \vec{x} - \omega_{\text{eff}} t + \phi_{\text{eff}} \) is the laser phase imprinted during interaction with the atoms at the points \((\vec{x}, t)\), \( \langle \vec{p} \rangle \) is the canonical average momentum, and \( \Delta \vec{x} \equiv x_l - x_u \) is the center of mass distance between the output port wavepackets. With knowledge of the effect of various systematics and noise sources on the lasers, the atom trajectories, and the final separation we can compute the associated phase shifts in the atom interferometer.

### 2.2 Science Goals

As a stepping stone experiment MAGIS-100 has many routes for advancing fundamental physics and pushing the boundaries of our understanding. One of the key searches for the MAGIS-100 sensor is time-dependent dark matter signals that can modify the fundamental atom parameters, cause acceleration variations, or lead to spin precession.

The discovery of gravitational waves by LIGO has caused great excitement in searching a larger frequency range for gravitational wave signals. MAGIS-100 is a test bed system
to establish the abilities of atom interferometers in this realm and to prepare for longer baseline systems capable of detecting more sensitive strains from currently observed sources and searching for new forces in the mid-band frequency range (0.3 Hz–10 Hz).

With a large quantum sensor such as MAGIS-100 we can experimentally push quantum mechanics to its limits on wavepacket superposition separation in space and time. The successful operation of MAGIS-100 will itself be a test of quantum mechanics at the macroscopic level.

### 2.2.1 Dark Matter Searches

Current observational lower bounds have set a mass limit of $10^{-22}$ eV for particle dark matter candidates. Experiments have shown null results in only a very narrow band of masses [126], so the search must be expanded. In the range $10^{-22}$ eV–$10^{-3}$ eV potential candidates include the QCD axion, axion-like particles, and the relaxion. Dark matter with such a light mass has a high number density and can be treated as a classical wave with a frequency set by the candidate’s mass. Such a time dependent field interacting with a quantum sensor can give rise to time dependent shifts in the detectors properties such as the energy levels of the atoms, their spin, and mass. These shifts then become time dependent signals we can search for with the MAGIS-100 sensor [48, 127, 128]. Theoretical dark matter candidates in this range can be detected through accelerations on the test masses, precession of spin, and changes in fundamental constants. Each of these paths can be searched for with MAGIS.

Effects that alter the fundamental constants would directly alter the energy levels and quantum states used in the atom interferometer. To search for these modulations two atom interferometers would be compared along the baseline. The constants would then have time
dependent oscillations arising from the frequency of the dark matter field set by the mass of the candidate dark matter. These shifts would appear in the quantum phase difference of the two interferometers. This measurement would be sensitive from 0.1 Hz–10 Hz and be able to search for candidates in the mass range of $10^{-15} \text{eV} < m_\phi < 10^{-13} \text{eV}$. MAGIS-100 will be able to make an improvement of up to two orders of magnitude in the range.

Dark matter models that cause accelerations of test masses can be looked for by using the atom interferometer in an accelerometer mode and comparing the acceleration phase shift signals of two different strontium isotopes. This utilizes the fourth MAGIS configuration shown in 2.5 and has been demonstrated in previous experiments [5, 129–131]. An example of such a new force is that generated by a B-L coupled vector boson. Potential sensitivities are shown in figure 2.2.

![Figure 2.2: Plot of MAGIS-100 dark matter sensitivity (red curve) and future kilometer length experiments (blue curve) to ultra-light scalar field dark matter coupled to electron mass with strength $d_{me}$. Also shown are current bounds from equivalence principle (EP) tests and fifth-force experiments.](image)
The final possibility for dark matter detection comes from candidates that cause the nuclear spin of atoms to precess. These include the class of general axions. Such a search would compare simultaneous co-located atom interferometers with each strontium cloud having different spins [128].

### 2.2.2 Gravitational Wave Detection

Although MAGIS-100 will not have sufficient sensitivity for detecting currently known and measured sources of gravitational radiation, binary black hole mergers and neutron star mergers, it will improve upon current bounds [121] in the mid-band by many orders of magnitude. There is however always a possibility when searching in an unexplored frequency range to discover something unexpected. Most importantly MAGIS-100 is a testing ground experiment aiming to advance the technical requirements for future long baseline atom interferometers i.e. MAGIS-1000 with a kilometer baseline and a future space based detector. With this extended baseline it may be possible to detect currently known gravitational wave sources [56]. MAGIS-100 fits in between the LISA experiment which is sensitive from 1 mHz–50 mHz, and LIGO which is sensitive above 50 Hz. This also adds to the ability of doing multi-messenger astrophysics by observing sources over different cycles of their lifespan.

Other interesting sources of gravitational waves lie in the mid-band at frequencies too low for LIGO to access. In this range there is the possibility of observing cosmological sources of gravitational waves such as inflation and reheating perturbations [91, 132] as well as axion inflation signals. Other possible sources that can be detected are early universe phase transition signals and cosmic string networks [118, 120].

MAGIS and future long baseline detectors also add the ability to discriminate between astrophysical and cosmological sources of black holes. Since binary black holes spend a
longer time in the mid-band their gravitational wave signals can be measured over many cycles giving extra information about the spin of the binaries as well as the ability to localize the sources in the sky [122].

The required ingredients for making a gravitational wave measurement are inertial references or test masses and a precise measurement of time using a clock. In the case of MAGIS the freely falling inertial references are the atoms where an incoming gravitational wave would perturb the baseline distance between two atom interferometers. This is then measured by the atoms internal clock modes to time the laser light travel time from one atom interferometer to the next. MAGIS can be used as a single baseline gravitational wave detector since much of the noise is common along the baseline and is subtracted out through the gradiometer measurement of the quantum phase. The light traveling down the baseline is used to excite the atomic clock mode of the strontium atoms. The rate at which these transitions occur is then linked to the travel time of the laser beam. Any modulation in the time it takes the laser light to travel down the baseline would result in a phase shift of the atom interferometer [7, 10, 123, 133].

2.2.3 Quantum Mechanics Tests and New Forces

With the technologies and quantum nature of the MAGIS-100 sensor quantum mechanics can be tested by the successful operation and measurement of the quantum phase difference of the atom’s DeBroigle wavepackets. The superpositions of the atom interferometers can be separated up to 10 m pushing superpositions into a truly macroscopic state. Current state of the art has seen these superpositions extended to 54 cm [4]. MAGIS will also push to maintain these separated states for up to a maximum of 9 s. These limits would set a new bound on quantum control with an atom being delocalized and present in two locations at
once. Other advances this quantum sensor can make include utilizing quantum entangled sources and spin squeezed states to push the sensitivity of the phase measurement below the quantum shot noise limit [134, 135].

MAGIS-100 can also serve as a test bed for future tests of the transition between quantum and classical mechanics. These include testing various theories of Quantum Equivalence Principles which are extensions of the Einstein Equivalence Principle. Some of these have already begun being explored by atom interferometry [110]. There is also great interest in bridging the gap between quantum mechanics and gravity through quantum gravity. A possible starting point is through exploring the effect of gravitation and space-time on the superposition of massive quantum states. These models focus on gravitational decoherence and quantum time dilation. Some of these will be further discussed in Sec. 4.2.

In addition to searching for wave and field signals in the phase of the atom interference pattern new dark sector particles could be searched for through new forces and interactions with the atoms [136]. These new forces can either arise from the Earth or from test masses located near the atom interferometer. For short range forces the distance between the sensor and the test mass can be fluctuated while running the atom interferometer sequence leading to a phase shift between the atom interference. Longer range forces other than gravity can be searched for by differential acceleration measurements of a dual strontium isotope drop and comparing the phase difference between them.

2.3 MAGIS-100 Concept and Design

Over the past several decades, atom interferometers have evolved from a tightly constrained lab based experiment into a deployable field based detector platform. Current experiments include using atom interferometers in space, underground, on moving vehicles,
and being dropped from great heights [10, 116, 137–140]. Current atom interferometry experiments have shown the extreme sensitivity of making fundamental physics measurements [4, 75, 141–143]. Many atom interferometry research groups have begun pushing the limits on the terrestrial length and time scales that these detectors can be operated such as the MIGA collaboration [144], the Bremen drop tower [145], and ZAIGA [146]. In addition long term vision experiments such as ELGAR, and AION are being developed with international networking in mind [115, 147, 148].

The Matter-wave Atomic Gradiometer Interferometric Sensor (MAGIS-100) is a large baseline atomic interferometer being built at Fermilab with three atom sources capable of generating concurrent atom interferometers along the baseline. MAGIS-100 aims to be a next generation probe for dark matter searches, tests of quantum effects, and gravitational wave detection [149, 150]. The detector uses advances in atomic interferometry while simultaneously pushing the limits of baseline length, time duration, and large momentum transfer (LMT) [6, 151–158] to the atoms. We will also be using the MAGIS apparatus as a stepping stone and pathfinder for examining the effects of the environment at much larger scales and in less controlled sites such as access and mining shafts. All of this effort will be put towards development of future terrestrial extreme baseline atom interferometers at the 1 km scale.

MAGIS-100 will utilize the capabilities and advancements of atom interferometers and atomic clock physics. Advances in light-pulse atom interferometry such as large momentum transfers, multi loop sequences, and long lived superpositions will be tested and implemented. The use of atomic clock atoms for keeping precise timing information during the interferometry sequence will be used to keep track of signals that distort time as seen by the atoms like gravitational waves. By incorporating a larger baseline, multiple atom sources, and precise atomic clock atoms, the MAGIS detector will be a dynamic multipurpose experiment with many avenues for discovery. MAGIS-100 will be located at Fermi National Accelerator Laboratory in Batavia, Illinois and installed in a 100 m shaft, see figure 2.3.
2.3.1 The Detector

Extending the capabilities of current atom interferometers the MAGIS detector will have multiple configurations using an extremely precise atomic clock atom, strontium in our case, a gradiometer measurement, amplified atom optics, and three atom sources along the baseline. This setup provides: suppression of common laser noise and vibrational noise, insensitivity to magnetic field noise sources, enhanced phase shift sensitivity, and four different operating modes.

Previous atom interferometers have used rubidium, cesium, and other alkali atoms. The use of strontium, an alkaline atom, has many advantages over the alkali atoms in MAGIS-100. It is currently used in the most precise atomic clocks [159, 160], and has already
been implemented in atomic interferometers [161]. Strontium also allows for a new way of implementing the “beamsplitter” and “mirror” pulses described above for light-pulse atom interferometry. In alkali based interferometers two-photon transitions (Raman or Bragg) are required to implement the atom optics [162]. This requires the use of two counterpropagating laser beams which increases sensitivity to laser noise. For strontium atoms a single photon transition is needed to move the atom into an excited state with a long lifetime. Since a single laser beam is required for these transitions common-mode laser noise can be rejected more effectively. Another benefit of using strontium atoms is their lower sensitivity to magnetic fields. Previous atom interferometers used magnetically insensitive states that still have a second order Zeeman energy shift which needs to be controlled at the milligauss level. The clock transition for strontium atoms has a magnetic susceptibility 1000 times lower than that of the alkali atoms [163]. Using strontium atoms in conjunction with a gradiometer measurement allows for excellent noise suppression.

As demonstrated in section 2.1 the phase shift of a single atom interferometer has a contribution from the laser beam that interacts with the atoms. Any noise in the laser beam arising from laser jitters or vibrations of the main laser system’s optics will affect the phase measurement. To suppress these noise sources we can run the MAGIS-100 in a gradiometer configuration which takes two or more atom interferometer phase shift measurements made simultaneously and the phase difference measurement is made between two atom interferometers subtracting out the common-mode noise sources across the baseline [7, 124]. This differential signal can then be used to search for new physics signals. The gradiometer configuration and effectiveness has been used in multiple atom interferometry experiments [164].

The MAGIS detector sensitivity can be increased by implementing various advanced atom optics techniques. One enhancement that MAGIS-100 will incorporate is LMT [6, 151–158]. By using many light pulses between the beamsplitter and mirror pulses the sensitivity of the phase shift measurement can be amplified by $n$ the number of light pulses used to
separate the atom wavepackets by adding to the momentum of one of the arms of the interferometer. In addition, the sensitivity can be enhanced by using multi-loop sequences, as shown in figure 2.4, where the arms of the interferometer cross many times with a specified time separation $t$. This provides increased resolution at the frequency $1/t$ [123]. For time dependent signals the phase shift built up over each loop of the atom interferometer will resonantly add as long as the period of the signal matches the chosen loop period. MAGIS-100 will allow for testing of these advancements and provide crucial knowledge about their future implementation for long baseline terrestrial atom interferometers.

![Figure 2.4: Conceptual diagram of a 3-loop atom interferometry sequence for resonant enhancement and suppression of rotational and Coriolis systematic effects. Wavey lines represent the laser pulses labeled by $\pi/2$ (beam-splitter) and $\pi$ (mirror). The blue path is the upper arm path for quantum state $|2\rangle$ and the red line is the lower arm path for state $|1\rangle$.](image)

By using three atom sources, located at the top, bottom and middle of the detector’s baseline, MAGIS-100 will have multiple choices for operation modes. In figure 2.5 some of the possible configurations are shown. The first utilizes the maximum gradiometer baseline measurement (50 m) with a drop time on the order of 3 s. Atom clouds are dropped from the top and middle atom sources and fall to the next source for detection. This mode offers measurements in the lower frequencies of the midband. The next takes the top and
bottom atom interferometers and simultaneously launches them 10 m to then be detected at their original launch locations. With this mode the baseline spans 100 m and fall times less than 1 s allowing for measurements below 1 Hz. For terrestrial detectors an important systematic arises from ground motion known as gravity gradient noise, which is further discussed in section 3.6. To characterize this noise source and provide suppression we can run the detector with all three atom sources simultaneously launched 10 m to map out the vertical dependence of the seismic noise. The final operation mode is a maximum launch of two isotopes of strontium for equivalence principle tests and other possible tests that require comparisons between differing masses.

Figure 2.5: The MAGIS-100 detector design consists of three atom sources (blue clouds) placed along the vacuum pipe at the bottom, middle, and top of the shaft. Light pulses (red beam) travel along the vacuum pipe and interact with atoms at each of these locations. (a) Maximum drop time gradiometer. Atoms from the atom sources at the top and middle are dropped 50 m and detected at the middle and bottom locations respectively. (b) Maximum baseline gradiometer. Atoms from the bottom and top sources are launched on short (∼10 m) trajectories and detected at the initial launch positions. (c) GGN characterization. All three sources can be used with short launches in order to explore Newtonian noise variation along the baseline. (d) Dual-species launch for an alternate dark matter detection mode.
2.3.2 The Subsystems

The main systems that make up the MAGIS-100 detector are the laser system, the atom sources, and the vacuum pipe where the atoms will undergo interferometry and imaging. Below I will describe the main purposes of each of these systems and the current research and development progression.

Laser System

The detector relies on multiple sets of high power lasers with a fine tuned stability for the atom interferometry, and for implementing the required atom optics. As a collaborator for the MAGIS-100 experiment at Fermi National Accelerator Laboratory, Northwestern is taking the lead on developing the main laser systems and timing controls for running the interferometric sequences. Currently a test setup has been established at Northwestern in order to optimize the methods for Bragg single photon transitions, maximizing laser stability and pointing and suppressing other noise sources that affect the laser beams. These efforts feed directly into making MAGIS-100 an advanced long baseline atom interferometer.

Laser light requirements for the MAGIS-100 interferometry and detection modes include the ability to generate light at three wavelengths. For the atomic clock transition of strontium light at a wavelength of 698 nm is required to excite the atoms into this long lived (150 s) state using single-photon transitions and requires a power of 8 W. The other useful single-photon transition is far more narrow at 689 nm ($^{1}S_{0} \rightarrow ^{3} P_{1}$) and can be used with dual-species measurements, unlike the 698 nm atomic transition which only exists for the fermionic isotope $^{87}$Sr. Finally the two-photon transition of 679 nm can also be used with proper correction.

$^{1}$The bosonic clock transition is weakly coupled to the 698 nm transition and requires a large magnetic field to have a usable Rabi frequency.
for noise sources coming from the need for stimulated emission. All of these wavelengths will be used for Bragg pulses. The main benefit from using the single-photon transitions is less interaction between the laser light and the atoms during interferometry lowering the need for increased stability controls. There is however still a need for optimizing this control.

To increase the efficiency of transferring photons to the atoms during the interferometry MAGIS-100 needs high power lasers that can reach Rabi frequencies in the kHz range while also providing a beam waist on the order of cm's. The pulse efficiency for the atom interferometry depends on the Rabi frequency as $P_e \approx 1 - \frac{\delta^2}{\Omega^2}$ for a small laser detuning $\delta$, and the Rabi frequency $\Omega$ [165]. Because the phase of the laser interacting with the atoms during interferometry gets imprinted in the overall quantum phase we seek to measure, see equation (4.24), the stability of the laser beams will directly add noise to the measurement. The lasers pulses need to have an absolute frequency control down to 10 Hz which will be accomplished by the use of a frequency comb locked on the resonance of the 698 nm clock transition as well as easily allowing the locking of other wavelengths of light required for the experiment providing long stability duration. Detailed examinations of the effects of laser pointing and intensity stability can be found in section 3.1. In addition to these noise sources a large contributing factor comes from the laser wavefront of the beam at the time of interaction with the atoms. Wavefronts arise from impurities or distortions in optical elements as the laser makes its way to the atoms. For smaller scale experiments these mostly diffract out of the beam however with a longer baseline these perturbations can build up and severely affect the phase of the interference pattern. These can be minimized in many ways. MAGIS-100 will use in situ characterization and free space propagation of the beam to mitigate laser wavefront aberrations. A full derivation of the effect and mitigation of laser wavefront aberrations can be found in subsection 3.3.

MAGIS-100 aims to push the limits of LMT to orders of 100 and 1000 pulses of light in order to maximize the separation of the deBroglie wavepackets. These pulses are applied
from both directions up and down the vacuum pipe in order to realize Bragg transitions of the atoms [152]. This allows for highly efficient transfer of momentum to the atoms. Other methods of optimizing the pulses of the laser beams include composite pulses [166–168], and optimal quantum control [169, 170]. Composite pulses increase the coherence of the processes of atom interferometry allowing for larger area between the upper and lower arms of the interferometer and increasing the time the laser and atoms can interact.

The laser system also requires sensitive optical mirrors with fine tuning controls for compensating the Earth’s rotation, allowing for tight controls over the laser beams pointing, and expanding the beam to a usable size through a telescope. Figure 2.6 shows the general layout of the MAGIS-100 laser system. Laser light generated and coherently combined will travel down a fiber to clean the modes of the light while then entering a free space vacuum pipe allowing for extra cleaning of the beam. In this region the laser wavefronts diffract out leading to a clean beam for interacting with the atoms. After propagation the beam is steered via a tip-tilt mirror with fine actuation control and is expanded through a telescope. From here it travels down the length of the vacuum pipe to reflect off a retroreflector mirror that has piezo motors for rotation control. This setup gives us the ability to direct light from the bottom and top of the interferometry pipe to implement the advanced atom optics discussed above.
Figure 2.6: Simplified conceptual schematic of the MAGIS-100 interferometry laser system. (Left) shows the coherent combination and fiber noise cancellation scheme preparing the laser for free space propagation to the vertical shaft. Acousto-optic modulators (AOMs) are used for frequency control and pulse shaping, and an electro-optic phase modulator (EOM) is used to put weak sidebands on one of the laser outputs for the purpose of generating an error signal to be used for coherent combination. The laser frequency is referenced to a frequency comb with an offset controlled by a double-passed AOM. This enables rapid frequency tuning over $\sim 200$ MHz, which is required to account for Doppler shifts of the atoms. A path for generating a continuous fiber noise cancellation (FNC) error signal bypasses the AOM used for temporal pulse shaping of the main beam. In an alternate operating mode, the two lasers will be separately offset locked to the comb (instead of coherently combined) to drive AC-stark-shift-compensated Bragg transitions [4] at 679 nm for dual isotope dark matter searches. (Right) represents the interferometry region where the laser light is guided by a tip-tilt mirror through a 1:30 telescope and down the baseline where a retroreflecting mirror is positioned for rotation compensation and to allow pulses of light from above and below the atom clouds.
Laser beam frequencies can shift based on external conditions such as thermal changes or drift in the lasing pump laser beam. To examine these effects a test laser system was built at Northwestern. In order to have an optimally stabilized laser beam for this test system we lock the laser to the resonance of a Fabry-Pérot cavity. Since the cavity has a well known resonance frequency this allows us to know precisely what frequency the laser beam is at. This makes the stability of the laser beam equivalent to the stability of the cavity. The method for locking requires characterization of the cavity finesse and decay time. Details of this work are included in section 5.1. In the full experiment laser system the lasers will be locked using a frequency comb which enhances the long term stability of the frequencies necessary for MAGIS-100.

**Atom Generation**

MAGIS-100 uses ultra cold atomic clouds of strontium shuttled into the vacuum pipe for the interferometry. The atom source, as defined for MAGIS-100, is an environmentally sealed enclosure containing the necessary lasers for the atom cooling, the electronics for control and timing of the lasers, the vacuum chamber for holding the ultra-cold atom cloud before shuttling into the interferometry vacuum pipe, and magnetic bias coils.

The atom generation process consists of heating an oven of strontium metal that generates a high-flux atomic beam which is then slowed down and cooled via: a Zeeman slower, a 2D magneto-optical trap (MOT), and finally brought into an ultra cold state by a 3D magneto-optical trap (MOT) in the atom source vacuum chamber. This all requires a precise optomechanics system to control the required lasers for each step of the atom cloud creation as well electronics for timing and frequency control. The 3D MOT captures atoms with the 461 nm strontium transition until roughly $10^9$ atoms are cooled to the mK range. The atoms
are then moved by the 689 nm transition into the μK temperature range. A final step of applying a dipole trap laser to the atom cloud brings the temperature down to the nK range via evaporative and delta-kick cooling techniques [171]. All of this will be incorporated into the atom sources being built at Stanford.

Some of the requirements on the atom source are the ability to generate up to $10^6$ atoms/s as an initial goal in order to acquire enough measurement statistics to probe the relevant science goals listed below. Because of the extremely long baseline that the atom clouds will be traveling along the cooling requirements and kinematic controls of the initial distribution have increased over previous shorter baseline atom interferometers. The atoms have to be cold enough as to not thermalize and expand beyond the interferometry laser beam waist or we will lose contrast in the phase shift measurements. This has lead to the need for pushing the temperatures of the atom clouds even lower using enhanced evaporative cooling and matter-wave lensing techniques to reach the pK level [171]. MAGIS-100 will also serve as a test bed for advancing these techniques.

Once the atom cloud is prepared in the atom source vacuum chamber it then needs to be moved into the interferometer region’s connection node where the atom source meets the vacuum pipe. From here it can either be dropped or launched. All of these operations require precise laser control for moving the atom clouds that are in a dipole trap over to an optical lattice generated at the connection node by an optical potential lattice using 689 nm light with two paths slightly detuned from each other. The lattice can then transfer momentum to the atoms through coherent acceleration and creating large Doppler shifts between the atoms and the lattice lasers by acousto-optic or electro-optic frequency adjustments [1, 165].

An important element of the atom sources is the ability to tightly constrain the strontium atom cloud and to shuttle it horizontally from the atom source chamber over to the connection node where the atoms may be freely dropped or launched in the 100 m vacuum tube. To accomplish both tasks in a refined way R&D done on possibly utilizing a spatial
light modulator (SLM). An SLM is a liquid crystal display device that allows one to alter the effective index of refraction of the liquid crystals by displaying images on the pixel array. This in turn allows one to imprint a phase on a reflected laser beam and upon passing through a Fourier lens projecting a desired intensity profile on the imaging surface. By using iterative Fourier algorithms we can extract the required phase image that takes us from an initial intensity, say a Gaussian beam, to a target intensity such as an array of trapping lattice sites. In order to shuttle the atoms a set of phase transitions can be displayed on the LCD to smoothly shift the trapping sites towards the connection node. Further details of work on this can be found in section 5.2.

**Interferometry Vacuum Pipe and Detection**

Enclosing the interferometry is the vacuum pipe installed along the shaft wall in the MINOS service building which will provide a 100 m baseline, ultra high vacuum, and magnetic shielding and bias coils.

The 100 m baseline allows for the positioning of three atom sources down the shaft equally spaced. With one at the top, one in the middle and one at the bottom the different configurations explained in 2.5 are realizable. Along with the atom sources the shaft will be fitted with a vacuum pipe down the shaft wall with connection nodes for interfacing with the atom sources. Each of these nodes provides shuttling access for the atoms into the optical lattice and viewports for imaging the atom interference pattern after fluorescence. The vacuum pipe will require an ultra high vacuum on the order of $10^{-10}$ Torr in order to keep atom loss from recoiling off residual gas minimal. This level of vacuum will be implemented by ion pumps down the length of the vacuum pipe as well as baking the interferometry region before operation. The expected diameter of the pipe will be 10 cm to 20 cm providing enough room
for the expanded interferometry laser beam while also giving room for rotation compensation of the laser beam.

Stray magnetic fields and the Earth’s magnetic field can affect the phase of the atom interferometer. In order to suppress these effects and create a uniform magnetic environment an advanced magnetic shield is being developed to enclose the vacuum pipe along with bias coils running the length of the pipe. The use of strontium atoms also acts to suppress the influence of stray fields on the atomic phase as was previously discussed. Even if their is an effective phase shift caused by magnetic fields there are well study measurement methods using Zeeman shifts to compensate [172]. There will also be temporal monitoring of the magnetic environment down the shaft to allow for measurement and mitigation.

Imaging of the atom clouds and measurement of the phase difference between the two ports is achieved by resonant scattering and applying a phase shear to the atom cloud at the final beam splitter [3]. This enhances the phase readout. The cloud is then allowed to drift for some extra time before applying the 461 nm imaging beam causing the atom cloud to emit photons allowing for imaging of the separate ports. These photons are then collected by an in-vacuum optics system and imaged by cameras located at the viewports of the three connection nodes. A full simulation of the precision of different cameras and fitting methods is underway and further described in section 3.7.
In order to precisely manipulate the atoms, by laser cooling and atom optics, the environmental effects and noise backgrounds modifying the atom interferometer phase needs to be well measured and analyzed. This includes understanding the active dynamics of temperature, humidity, vibrations, local seismic activity, and magnetic fields in and around the shaft that contribute to various noise sources in the subsystems of MAGIS-100. The temperature fluctuations play a role in both the effects on the materials of the vacuum tube and atom source chambers as well as the lasers. Vibrations and seismic activity play two roles in affecting the atom phase. The preliminary effect is that of direct vibration of the atom sources and lasers used to interact with the atoms. This vibration will cause laser jitter and thus imprint fluctuations on the atom clouds. The secondary effect is indirect fluctuation of the local gravitational potential by mass density fluctuations of the ground known as GGN. Magnetic fields can cause noise in the atom phase by the Zeeman shift of the atomic levels. There is also a need to characterize the possible noise sources to increase our sensitivity to science signals in the phase of interference pattern. For this purpose a simulation has been established with ongoing work that simulates atom interferometry image data of the two ports of the interferometer and fits the phase.

Work in this chapter was worked on in collaboration with MAGIS-100 collaborators Timothy Kovachy at Northwestern University and Jason Hogan at Stanford University.
3.1 Experimental Systematics

This section describes anticipated noise sources for the MAGIS-100 experiment and some of the strategies used in the detector design to minimize their impact. This noise analysis translates into experimental requirements, as summarized in table 3.1. An important aspect of the MAGIS-100 research program is experimentally studying these noise sources and the associated mitigation strategies in a long-baseline atom interferometry configuration. Modeling and analysis of many of the error contributions has been carried out in the context of atomic gravitational wave detector configurations similar to MAGIS-100 and are directly applicable [7, 10, 15, 17]. The MAGIS-100 experiment is designed to be sensitive to time-varying gravitational wave or dark matter signals in the frequency band $\sim 0.1$ Hz–10 Hz. The relevant backgrounds are those that temporally vary in this frequency band. This frequency selectivity eases a number of requirements. For instance, any sources of error that lead to constant phase offsets or long-term phase drifts would not affect the potential science signal. MAGIS-100 aims for $100\hbar k$ beam splitters and a phase resolution of $10^{-3}\text{rad}/\sqrt{\text{Hz}}$ for a differential measurement between interferometers separated by baselines of up to 100 m, which appears achievable in the near term. Research and development efforts aim to boost instrument sensitivity via increased momentum splitting and improved phase resolution (resulting from higher atom flux and/or squeezed atom sources) and to correspondingly further reduce the influence of the technical noise sources discussed here.

3.1.1 Laser frequency noise

The atom-laser interactions are dependent of the phase of the laser at each of the interaction points, so laser technical noise can cause noise in the atom interferometer signal.
Table 3.1: Summary of key experimental parameters and requirements. Spectral densities are taken to be in the $\sim 0.1 - 10$ Hz frequency band of interest. A more detailed discussion of error modeling and requirements can be found in the main text.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target Value</th>
<th>Primary Driving Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMT atom optics</td>
<td>$n = 100$</td>
<td>Increase sensitivity to science signals</td>
</tr>
<tr>
<td>Phase Resolution</td>
<td>$10^{-3}$ rad$/\sqrt{\text{Hz}}$</td>
<td>Increase sensitivity to science signals</td>
</tr>
<tr>
<td>Frequency noise/drift</td>
<td>$&lt; 10$ Hz</td>
<td>Increase pulse transfer efficiency (sec. 2.3.2)</td>
</tr>
<tr>
<td>Per shot measurement or stabilization of atom cloud kinematics</td>
<td>$10 \mu$m$/\sqrt{\text{Hz}}$, $10 \mu$m$/s$/\sqrt{\text{Hz}}$</td>
<td>Coupling of kinematic jitter to wavefront aberrations (sec. 3.3)</td>
</tr>
<tr>
<td>Laser wavefront transverse spatial phase variation</td>
<td>$5 \text{ mrad at length scales } \lesssim 3 \text{ mm}$</td>
<td>Coupling of atom kinematic jitter and laser pointing jitter to wavefront aberrations (sec. 3.3 and sec. 3.1.2)</td>
</tr>
<tr>
<td>Laser intensity stabilization</td>
<td>$0.1 %/\sqrt{\text{Hz}}$</td>
<td>AC Stark shifts (sec. 3.1.3)</td>
</tr>
<tr>
<td>Laser pointing stability</td>
<td>$30 \text{ nrad}/\sqrt{\text{Hz}}$</td>
<td>Coupling of laser pointing jitter to wavefront aberrations (sec. 3.1.2)</td>
</tr>
</tbody>
</table>

The multiple atom ensembles in the gradiometer are subject to the same laser pulses, so this noise effect is expected to be common-mode suppressed to a significant degree. Single photon atom optics on the clock transition [173] will be employed to realize the necessary level of laser noise rejection, as detailed in Ref. [7]. Specifically, the residual noise $\delta \phi_{\text{freq}}$ in the interferometer phase arising from laser frequency noise has the leading contribution [7]

$$
\delta \phi_{\text{freq}} \sim (10^{-13} \text{ rad}/\sqrt{\text{Hz}}) \left( \frac{n}{100} \right) \left( \frac{\Delta v}{100 \mu\text{m/s}} \right) \left( \frac{\delta f}{10 \text{ Hz}/\sqrt{\text{Hz}}} \right) \left( \frac{\Delta \tau}{100 \mu\text{s}} \right)
$$

related to the finite duration $\Delta \tau$ of each laser pulse, the velocity difference $\Delta v$ between the two atom clouds in the gradiometer, the beam splitter momentum $n \hbar k$, and the amplitude spectral density $\delta f$ of laser frequency noise. For all conceivable experimental parameters, $\delta \phi_{\text{freq}}$ is negligibly small.
3.1.2 Laser pointing jitter

Uncontrolled pointing jitter of the laser causes phase shifts in the interferometer since it changes the position of the atom with respect to the laser wavefronts. The behavior of these phase shifts is well-understood [1, 3, 10]. Pointing jitter in the target frequency band for the science signal (\(\sim 0.1-10\) Hz) can act as a noise background. Let \(\delta\Phi\) denote the amplitude of the power spectral density for pointing jitter in this band. If the pointing jitter originates from optics near one of the atom ensembles, the laser wavefronts are tilted and also transversely displaced by a distance \(L\delta\Phi\) for the far interferometer and much less for the near interferometer, where \(L \approx 100\) m is the baseline length. For MAGIS-100, the dominant source of noise in this situation arises from the coupling of this displacement to laser wavefront aberrations. With the wavefront aberrations parameterized as above, the level of background noise from laser pointing jitter is

\[
\delta\phi_{\text{pointing}} \sim (6 \times 10^{-4}\ \text{rad/}\sqrt{\text{Hz}}) \left( \frac{n}{100} \right) \left( \frac{\delta}{0.005} \right) \left( \frac{\delta\Phi}{30 \ \text{nrad/}\sqrt{\text{Hz}}} \right) \left( \frac{k_t}{(3 \ \text{mm})^{-1}} \right).
\]

Laser pointing will be monitored using split photodetectors (see Ref. [17] for a more detailed discussion), and feedback to control the laser direction can be used if needed. Also, spatially resolved detection of the atomic interference pattern can be leveraged to provide measurements of the pointing jitter on each shot [3, 116].

3.1.3 AC Stark shifts

Off-resonant light causes an AC Stark shift of the atomic line. For example, even for resonant excitation of the clock transition in Sr (\(\lambda = 698\) nm), there is an AC Stark shift due to off-resonant coupling to the other atomic levels. In particular, off-resonant coupling to the \(^1S_0 \rightarrow ^1P_1\) (461 nm) transition shifts the energy of the ground \(^1S_0\) state, and off-resonant
coupling to the $^3P_0 \rightarrow ^3S_1$ (679 nm) transition shifts the energy of the excited clock ($^3P_0$) state [174]. This energy shift can cause a phase shift in an interferometer that mimics the target signal to the extent that it varies spatially and fluctuates in time in the target frequency band. Intensity fluctuations of the laser are a dominant source of this, so laser intensity control can reduce the effect. For interferometers with $n\hbar k$ beam splitters, phase backgrounds from AC Stark shifts are at the level of $\delta\phi_{AC} \sim \left(6 \times 10^{-4} \text{ rad}/\sqrt{\text{Hz}}\right)\left(\frac{n}{100}\right)\left(\frac{\delta(\Delta I)/I}{10^{-5}/\sqrt{\text{Hz}}}\right)$, where $\delta(\Delta I)/I$ is the amplitude of the power spectral density in the target frequency band for the fractional fluctuation of the differential laser intensity (averaged over the atom ensembles) between the two interferometers. $\delta(\Delta I)/I \sim 10^{-5}/\sqrt{\text{Hz}}$ can be realized, for example, with a 1% spatial intensity variation between the two interferometers and laser intensity stabilization at the level of 0.1%, which is readily achievable [175]. Transverse-position-dependent AC Stark shifts can also couple to initial atom kinematic jitter in a manner analogous to wavefront aberrations (discussed above). For a given intensity/wavefront perturbation amplitude, phase errors from AC Stark couplings of this type will generally be smaller than the corresponding wavefront-induced phase errors. For such AC Stark couplings, analogous mitigation strategies can be applied as in the wavefront case. In situ measurements of intensity perturbations can be performed by implementing spatially resolved detection combined with short duration interferometers in a superposition of different internal states and leaving on a long, Doppler-detuned laser pulse.

### 3.1.4 Rotations and gravity gradients

Shot-to-shot fluctuations in the atom trajectory can couple to rotations and gravity gradients, and leading to time dependent phase errors [176–178]. Multi-loop interferometers provide a way to cancel phase shifts from the coupling of initial kinematics to gravity
gradients or rotations while preserving the time-varying dark matter or gravitational wave signal [10, 179]. As an illustrative example, a three-loop interferometer configured to cancel out leading order phase shift contributions from rotations and gravity gradients is considered [179]. In such an interferometer, the coupling of higher-order rotation/gravity gradient phase shifts to initial kinematic jitter can still be a source of noise. The dominant such noise term arises from a cross-coupling between rotations, gravity gradients, and initial atom velocity and has the form \(\delta \phi_{\text{RGGV}} = \left(\frac{17}{3} + 4\sqrt{2}\right) nk \Delta v_x \Omega y T_{zz} T^4\). Here, the \(x\), \(y\), and \(z\) axes are defined so that \(z\) is normal to Earth’s surface and Earth’s rotation vector lies in the \(yz\) plane. \(\Delta v_x\) denotes the shot-to-shot jitter in the atom cloud velocity along the \(x\) axis (or the accuracy to which this jitter can be measured on each experimental shot if post-processing corrections are implemented), \(\Omega y\) is the \(y\) component of Earth’s rotation vector, \(T_{zz}\) is the vertical gravity gradient, \(T\) is the duration between the first beam splitter and mirror interactions, and all other notation is as defined above. The associated noise has the magnitude \(\delta \phi_{\text{RGGV}} \sim \left(2 \times 10^{-5}\right) \text{ rad/}\sqrt{\text{Hz}} \left(\frac{n}{100}\right) \left(\frac{\Delta v_x}{10 \ \text{nm/s/}\sqrt{\text{Hz}}}\right) \left(\frac{T}{1\ \text{s}}\right)\). Additional loops would further suppress phase shifts from the cross-coupling of rotations, gravity gradients, and initial atom velocity.

### 3.1.5 Mean field shifts

Atom-atom interactions cause an energy shift of the clock transition proportional to the atomic density. This can cause a systematic error in an interferometer if the two arms occupy different atomic states (with different mean field shifts), or if the density is asymmetric. The detector is sensitive to any time-varying mean field shifts, which may arise if the density fluctuates shot-to-shot. This background can be suppressed by using ensembles with low density (after matter wave lensing), by employing sequences that use symmetric internal states for
the two arms, and by using symmetric beam splitting sequences [5, 180]. For \(^{87}\text{Sr}\), the mean field shift has been measured to be \(\sim (1 \ \text{Hz}) \left(10^{11} \text{ atoms/cm}^3\right)\) [18, 181], where \(n_{\text{atom}}\) is the atom number density. The mean field shift arises when atoms are inhomogeneously excited on the clock transition, as otherwise atom-atom interactions are suppressed by Pauli blocking [18]. Therefore, the times at which mean field shifts will affect phase evolution in MAGIS-100 are during the atom optics pulses. MAGIS-100 will have \(\sim 10^6\) atoms per interferometer in a volume of \(\sim 1\ \text{mm}^3\). Using these numbers and assuming a \(\pi\)-pulse duration of \(\Delta \tau\), phase backgrounds from mean field shifts are \(\delta \phi_{\text{MF}} \sim \left(5 \times 10^{-5} \ \text{rad/} \sqrt{\text{Hz}}\right) \left(\frac{n}{10^6}\right) \left(\frac{\Delta \tau}{200 \ \mu\text{s}}\right) \left(\frac{\delta(N_A)/N_A}{0.01}\right)\), where \(\delta(N_A)/N_A\) is the shot-to-shot fluctuation of the fractional atom population asymmetry between the two interferometer arms (the relevant fluctuations are those in the target frequency band). Additionally, closely matching the densities of the two separated interferometers will allow the mean field phase shift to be further suppressed as a common mode, and highly robust beam splitters and mirrors using composite pulses along with advanced quantum control [166–170, 182, 183] can dramatically reduce fluctuations in the population asymmetry.

### 3.1.6 Magnetic fields

The clock energy levels of Sr shift in response to magnetic fields. Time varying magnetic fields can cause systematic frequency shifts that behave like a gravitational wave or ultralight dark matter signal. As discussed in Section 2.3.2, magnetic shielding will be employed to reduce the influence of stray fields in the interferometer region, and the field will be monitored with magnetometers. Using multiple sequential transitions, the atom interferometry sequence can be designed so that both arms of the interferometer spend most of the time in the ground state, reducing the differential phase shifts. For \(^{87}\text{Sr}\), a co-magnetometer can be
realized by simultaneously operating two interferometers using states with opposite magnetic field response, suppressing the linear response to magnetic fields and allowing the magnetic field dependent phase shift to be measured and subtracted. The residual quadratic Zeeman shift coefficient is $-0.23 \text{ Hz/G}^2$ [19], implying that control or measurement of magnetic field variations at the level of $\sim 1 \text{ mG}/\sqrt{\text{Hz}}$ in the relevant band should be sufficient for the target sensitivity goal.

### 3.1.7 Blackbody radiation shifts.

Blackbody radiation causes an energy shift of the atomic energy levels. This can result in phase noise in the interferometer if the temperature of the vacuum tube varies in time in the target frequency band. For the strontium clock transition, the blackbody shift has a temperature coefficient of $-2.3 \text{ Hz } (\frac{T_{\text{system}}}{300 \text{ K}})^4$, where $T_{\text{system}}$ is the temperature of the apparatus [19]. It is important to note that apparatus temperature drifts will naturally occur at frequencies much lower than the target frequency band. For a three-loop interferometer with $T$ as defined above and a temperature oscillation at frequency $\omega_{\text{Temp}}$, the interferometer phase response to temperature variations at low frequency $\omega_{\text{Temp}}$ is suppressed by a factor of $(\omega_{\text{Temp}} T)^2$. For a temperature oscillation of amplitude 1 K and period 1 hour, the associated interferometer noise is at the level of $\sim 1 \times 10^{-6} \text{ rad}/\sqrt{\text{Hz}}$ for $T = 1 \text{ s}$. 
3.1.8 Timing jitter

A timing-jitter induced asymmetry $\delta T$ in the duration of different free propagation zones of the interferometer, in combination with a velocity mismatch $\Delta v$ between the two clouds in the gradiometer, leads to interferometer phase noise of magnitude [7]

$$\delta \phi_{\text{timing}} \sim \left( 10^{-4} \text{ rad}/\sqrt{\text{Hz}} \right) \left( \frac{n}{100} \right) \left( \frac{\Delta v}{100 \mu \text{m/s}} \right) \left( \frac{\delta T}{1 \text{ ns}/\sqrt{\text{Hz}}} \right).$$

The necessary timing stability can be achieved with stable pulse generators.

3.1.9 Background gas index of refraction

The index of refraction from background gas in the pipe modifies the optical path length associated with the baseline. Noise $\delta \eta$ in the index of refraction $\eta$ therefore leads to a spurious strain signal $\delta h_{\text{index}} = \delta \eta$ [15]. The index of refraction of air is $\eta \sim 1 + 3 \times 10^{-4} \left( \frac{P}{760 \text{ Torr}} \right) \left( \frac{300 \text{ K}}{T_{\text{system}}} \right)$, where $P$ is the pressure and $T_{\text{system}}$ is the temperature of the system. The spurious strain signal associated with index of refraction variation due to temperature fluctuation $\delta T_{\text{system}}$ with a period of 1 hour (for $T = 1$ s) is $\delta h_{\text{index}} = \delta \eta \sim \left( 4 \times 10^{-26}/\sqrt{\text{Hz}} \right) \left( \frac{P}{10^{-11} \text{ Torr}} \right) \left( \frac{300 \text{ K}}{T_{\text{system}}} \right) \left( \frac{\delta T_{\text{system}}}{1 \text{ K}} \right)$. The spurious strain signal associated with index of refraction variation due to fractional pressure fluctuation $\delta P/P$ in the frequency band of interest is $\delta h_{\text{index}} = \delta \eta \sim \left( 4 \times 10^{-21}/\sqrt{\text{Hz}} \right) \left( \frac{P}{10^{-11} \text{ Torr}} \right) \left( \frac{300 \text{ K}}{T_{\text{system}}} \right) \left( \frac{\delta P/P}{0.001/\sqrt{\text{Hz}}} \right)$. For reference, it is noted that the same index of refraction effects apply to LIGO, which maintains ultra-high vacuum of $10^{-8}$ to $10^{-9}$ Torr [184]. Additionally, the index of refraction of the
ultracold atom clouds will induce a phase shift on the laser beam when it passes through a cloud. Interferometer phase noise from this effect has the magnitude

\[
\delta \phi_{\text{cloud}} \sim \left(10^{-7} \text{rad}/\sqrt{\text{Hz}}\right) \left(\frac{n}{100}\right) \left(\frac{N_a}{10^6}\right) \left(\frac{r_c}{1 \text{ mm}}\right)^2 \left[\left(\frac{\delta N_a/N_a}{0.01/\sqrt{\text{Hz}}}\right)^2 + 4 \left(\frac{\delta r_c/r_c}{0.01/\sqrt{\text{Hz}}}\right)^2\right]^{1/2}
\]

where \(N_a\) is the atom number in a given experimental shot, \(r_c\) is the atom cloud radius, \(\delta N_a/N_a\) is the fractional shot-to-shot fluctuation in the atom number, and \(\delta r_c/r_c\) is the fractional shot-to-shot fluctuation in the cloud radius. If needed, \(\delta N_a/N_a\) and \(\delta r_c/r_c\) can be measured on each shot and associated phase shifts can be subtracted in post-processing.

3.2 Coriolis

When making measurements of any massive object in an inertial reference frame on or near the surface of the Earth the Coriolis force will need to be accounted for. The effective acceleration that arises as a consequence of this effect comes from the fact that we are working in a rotating reference frame, the Earth, while the atoms are in a stationary reference frame with respect to the vacuum pipe they are dropped in. While normally Coriolis acceleration is small for such light atoms we are working with a sensitive detector utilizing laser interactions with the atoms over large launch distances and times. Because of this the atoms under free fall can either fall out of the laser beam path leading to a lower number of atoms being acted on or having non-uniform phase shears imprinted on the atoms directly affecting the atom interference phase measurement.

The leading systematic for atom interferometry phase measurements is the Coriolis effect; caused by the transverse deflection of the atoms while under vertical free fall in the rotating reference frame of the Earth. In MAGIS-100 we plan on using two types of initial launch
configurations. The first launch configuration called an atom fountain will use optical lattice launches to propel the atom cloud vertically upwards and back down to the initial position for imaging. The second launch will be a vertical drop in free-fall. It is necessary to specify the different types of launches because the Coriolis force has differing magnitudes and directions of effect on the transverse deflection.

We first approach the Coriolis effect analytically on a single particle under various launching and dropping schemes followed by the effect on a cloud of atoms where there may be transverse velocity fluctuations. The run configurations we consider using in MAGIS-100 are

- Dual 10 m drops from 2 atom sources
- Dual 10 m fountains from 2 atom sources
- Dual 50 m drops from 2 atom sources
- Dual 50 m fountains from 2 atom sources
- 100 m vertical drop
- 100 m launch fountain (the most ambitious)

### 3.2.1 Analytical Calculation

**Single Particle**

The Coriolis acceleration is found from classical mechanics using the equation

\[ a_c = -2\Omega \times v. \] (3.1)
We take $\Omega$ to be the rotation vector of the Earth and $\mathbf{v}$ the velocity vector of the atom. For our coordinate system we choose, $(x = \text{East}, y = \text{North}, z = \text{Perpendicular to surface})$, with respect to a reference frame embedded in the Earth.

\[
\Omega = \begin{pmatrix} 0 \\ \omega \cos(\phi) \\ \omega \sin(\phi) \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix},
\]

so we have

\[
a_c = 2\omega \begin{pmatrix} v_y \sin(\phi) - v_z \cos(\phi) \\ -v_x \sin(\phi) \\ v_x \cos(\phi) \end{pmatrix}.
\]

Deflections caused by this acceleration are found by integrating over time $t$ twice to arrive at the displacement vector. For our particle moving along the vertical vacuum pipe in the $\pm z$ direction, depending on a drop or launch configuration, we replace $v_z = v_{0,z} - gt$ and arrive at the displacement vector

\[
\Delta \mathbf{r} = \omega \begin{pmatrix} v_y \sin(\phi) t^2 - \left[ v_{0,z} t^2 - \frac{gt^3}{3} \right] \cos(\phi) \\ -v_x \sin(\phi) t^2 \\ v_x \cos(\phi) t^2 \end{pmatrix}.
\]

Given the latitude, $\phi$, at the Fermilab MINOS shaft where MAGIS-100 is being built, and the initial velocities we can calculate how much the particle will deflect. From a simple kinematics approach we need to find the flight times of our different scenarios. We use

\[
T = \sqrt{\frac{2h}{g}}.
\]
Table 3.2: Table of flight times for various configurations of atom launch schemes.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( T )</th>
<th>( 2T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 m Drop</td>
<td>2.26 s</td>
<td>4.52 s</td>
</tr>
<tr>
<td>100 m Fountain</td>
<td>4.52 s</td>
<td>9.03 s</td>
</tr>
<tr>
<td>50 m Drop</td>
<td>1.595 s</td>
<td>3.19 s</td>
</tr>
<tr>
<td>50 m Fountain</td>
<td>3.19 s</td>
<td>6.38 s</td>
</tr>
<tr>
<td>10 m Drop</td>
<td>0.72 s</td>
<td>1.43 s</td>
</tr>
<tr>
<td>10 m Fountain</td>
<td>1.43 s</td>
<td>2.86 s</td>
</tr>
</tbody>
</table>

Using these derived times we now calculate the transverse deflection, \( \Delta x \), and the vertical deflection, \( \Delta z \).

Table 3.3: The calculation was performed with \( \omega = 7.27 \times 10^{-5} \) rad/s, \( v_x = v_y = 1 \) mm/s, and initial launch velocities for the fountain configurations of \( v_0 = 44.29 \) m/s, 31.32 m/s and 13.03 m/s for the 100 m, 50 m, and 10 m fountains respectively.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( \Delta x )</th>
<th>( \Delta z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 m Drop</td>
<td>1.64 cm</td>
<td>( 1.1 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>100 m Fountain</td>
<td>-6.52 cm</td>
<td>( 4.4 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>50 m Drop</td>
<td>5.75 mm</td>
<td>( 5.5 \times 10^{-7} ) m</td>
</tr>
<tr>
<td>50 m Fountain</td>
<td>-2.31 cm</td>
<td>( 2.2 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>10 m Drop</td>
<td>0.518 mm</td>
<td>( 1.1 \times 10^{-7} ) m</td>
</tr>
<tr>
<td>10 m Fountain</td>
<td>-1.63 mm</td>
<td>( 4.4 \times 10^{-7} ) m</td>
</tr>
</tbody>
</table>

**Atom Cloud**

For a more realistic analysis of the effect of Coriolis in MAGIS-100 a distribution of Strontium atoms were simulated with fluctuating velocities on the order of micrometers. The Coriolis effect alters the measurement in two ways: it becomes imprinted in the phase shift between the two ports of the interferometer by shifting the interference location and not allowing the interferometer to close, and it alters the position of the atom cloud in the laser beam during free-fall. Starting with an initial atom cloud distribution and propagating it
through a 3-pulse interferometry sequence we find a phase shift of \(2k_{\text{eff}}\delta v_x \Omega_y T^2\) where \(k_{\text{eff}}\) is the laser wavenumber, \(\delta v_x\) is the initial velocity fluctuation of the atoms in the distribution, \(\Omega_y\) is the component of the Earth’s rotation in the East direction on the surface. Suppressing this phase shift then allows for sensitive measurements of gravity and rotations. Without compensation Coriolis acts as velocity selector and spreads out the initial atom distribution according to the initial velocities. We depict this by starting with a general Gaussian spread of atoms with random velocity jitters on the order of 1 mm/s in the transverse \(x\) and \(y\) directions of the interferometer and calculate the final distribution displacement shifts for the 50 m and 10 m configurations. Figure 3.1 shows a single port comparison of the atom distribution positions, with a phase fringe purposely imprinted at the final beam-splitter pulse for better resolution of the interference pattern, with and without Coriolis deflection. As expected from classical mechanics the drop towards the Earth ends with a deflection to the positive \(x\) direction while a launch up and down ends up in the opposite direction with a much larger magnitude of deflection.

### 3.2.2 Mitigation Strategies

Some of the topics discussed for eliminating or canceling out the effect of Coriolis deflection as a systematic and a noise source include a retro-reflective mirror rotating counter to the Earth’s rotation. This method will be discussed further in the next section. Another possibility would be a multi mirror pulse scheme for oscillating the upper and lower arms in order to close the path after the full trajectory. More generally, there are also the possibilities of altering the initial transverse velocity profile, a controlled magnetic field tuned for the Coriolis force, or having a tilted optical lattice launch.
Figure 3.1: Effect of Coriolis deflection on single port atom cloud position after final beam splitter, with imprinted phase fringes. Blue distribution represent the position without Coriolis deflection. Green distribution represents the position with Coriolis. Top row is, left to right, 10 m drop and 10 m fountain. Second row, left to right, is 50 m drop and 50 m fountain.

**Actuated Mirror**

In the EP tower experiment at Stanford with a 10 m fountain the best method for eliminating the Coriolis effect was by having a retro reflector mirror that could rotate on piezo electric motors counter to the rotation of the Earth. This compensated for the atom’s deflection away from the interferometry laser beam [116, 156]. For the larger case of a 100 m drop tower this is useful for shorter time and distance runs however for the 50 m and 100 m measurements this is no longer adequate to be the only method of mitigation. As an example
for the longer flight times of the schemes above the laser’s transverse displacement at the
top of the vacuum pipe goes as

\[ \Delta x \propto z \omega t. \] (3.6)

The following table shows the values for this laser displacement at the top of the tower
for our scenarios.

Table 3.4: Laser arc deflection values for a counter rotating mirror for the corresponding
flight times of different atom interferometric scenarios

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( \Delta x ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 m Drop</td>
<td>3.29</td>
</tr>
<tr>
<td>100 m Fountain</td>
<td>6.56</td>
</tr>
<tr>
<td>50 m Drop</td>
<td>1.16</td>
</tr>
<tr>
<td>50 m Fountain</td>
<td>2.32</td>
</tr>
<tr>
<td>10 m Drop</td>
<td>0.10</td>
</tr>
<tr>
<td>10 m Fountain</td>
<td>0.21</td>
</tr>
</tbody>
</table>

As we see from these values, the scaling up of this method rapidly increases the laser’s
position at the top of the vacuum tube. This can be physically avoided by using a larger di-
ameter tube, however there are other methods that will be more appropriate for the kilometer
scale and beyond detectors.

**Multi-pulse Sequences**

Another method of countering the displacement and phase shift caused by Coriolis is by
implementing a staggered series of interferometry mirror pules causing the atom trajectories
to cross over each other multiple times creating a looping structure that suppresses the shift
between the arms of the atom interferometer [179]. This method has been investigated and
will be implemented and tested in the MAGIS-100 detector for use in future kilometer scale
and space experiments.
**Velocity Kicks and Magnetic Fields**

An active way to make the two paths of the atom close is by adding an extra longitudinal momentum kick to the atoms at the top of the fountain trajectory by an amount that brings the second half of the trajectory back to the center axis of the laser beam in the accelerating reference frame. This will introduce a known phase shift between the upper and lower ports that can be accounted for while keeping the atom clouds centered. Further analysis and other effects of this method should be investigated.

For magnetic isotopes of Strontium yet another possibility that can be explored is generating a magnetic field that bends the atom trajectories opposite to the Coriolis force. This might be implemented using bias coils that are installed around the vacuum tube to create a uniform magnetic environment.
3.3 Laser Wavefront Abberation

3.3.1 Background

We consider here a treatment of higher order effects of wavefront perturbations in atom interferometry, which will become increasingly important as atom interferometers become more sensitive. The influence of wavefront perturbations in atom interferometry has been explored in, references [8–13]. The perturbed laser wavefront is imprinted on the atom’s wavefunction whenever momentum is transferred to the atom, leading to errors in the atom interferometer phase shift proportional to the amplitude of the wavefront perturbation. Laser wavefront perturbations also modify the momentum transferred to the atom by each laser pulse [8, 9, 14]. The influence of modifications to the longitudinal momentum transfer have recently been studied and measured [14]. Here, we extend this work to perform a full atom interferometer phase shift calculation including the influence of wavefront-perturbation-induced transverse momentum kicks, and also including wavefront-perturbation-induced longitudinal momentum kicks in a more general context in which the longitudinal laser phase gradient experienced by the atoms varies from pulse-to-pulse. Including these effects leads to contributions to the atom interferometer that scale quadratically with the wavefront perturbation amplitude and with the number $n$ of beam splitter momentum kicks. These higher order effects may ultimately be important to consider for MAGIS-100.

The following discusses the interferometer phase response to a laser beam perturbation. Consider a Fourier component with amplitude $\delta$ and transverse spatial frequency $k_x$. The beam is treated paraxially, which means that the wavevector, $\vec{k}$, is nearly parallel to the longitudinal axis down the pipe. The coordinate system is defined so that the $xy$-plane is transverse to the laser propagation and the $z$-axis is in the direction of propagation, which
is vertical. The $x$ and $y$ axes are defined so that Earth’s rotation vector lies in the $yz$ plane. As an illustrative example, a perturbation Fourier component $k_x$ along the $x$-axis is considered. An analogous treatment applies for a Fourier component along a general axis in the $xy$-plane. For the phase calculation, the equations of motion of the atoms are solved by expanding in a power series and keeping terms up to the third order in time. We also only consider Earth rotations $\Omega_y$ as these couple with the $x$ and $z$ positions of the atoms and the launch velocity. Whereas the $\Omega_z$ terms couple with the $y$ position of the atom and the transverse launch velocities which are much smaller. This allows us to reduce the problem to a two dimensional phase calculation in the $xz$-plane.

Field perturbations of a Gaussian laser being scattered off a mirror take the form

$$E(x, z) = u(x, z)e^{ikz}. \quad (3.7)$$

The initial amplitude and the amplitude some farther distance is

$$u(x, 0) = 1 + \delta \cos(k_xx), \quad (3.8a)$$

$$u(x, z) = 1 + \delta \cos(k_xx)e^{-\frac{k_z^2}{2k}z}. \quad (3.8b)$$

In these equations $k_x = 2\pi/\lambda_x$, where $\lambda_x$ is the characteristic spatial scale of the perturbations from reflecting off of the mirror. The paraxial approximation allows us to replace the wavevector in the $z$ direction as follows

$$k_z \approx \sqrt{k^2 - k_x^2} \approx k \left(1 - \frac{k_x^2}{2k}\right). \quad (3.9)$$
This leads to the amplitude and phase perturbations

\begin{align}
\text{Amplitude: } & \quad \delta \cos(k_x x) \cos\left(\frac{k_x^2 z}{2k}\right), \\
\text{Phase: } & \quad \phi_w = \delta \cos(k_x x) \sin\left(\frac{k_x^2 z}{2k}\right). 
\end{align}  

(3.10a)

(3.10b)

Using equation (3.10b) we can also write down the effective momentum kicks

\begin{align}
\delta k_x &= \frac{\partial \phi_w}{\partial x} = -k_x \delta \sin(k_x x) \sin\left(\frac{k_x^2 z}{2k}\right), \\
\delta k_z &= \frac{\partial \phi_w}{\partial z} = \frac{k_x^2}{2k} \delta \cos(k_x x) \cos\left(\frac{k_x^2 z}{2k}\right). 
\end{align}  

(3.11a)

(3.11b)

These are treated as momentum kicks because during the laser interaction atoms in the cloud will interact with the gradient of perturbed wavefront phase at different points along the wavefront surface and experience a force, see figure 3.2.

Figure 3.2: Conceptual diagram of the wavefront aberration phase (black curved lines) traveling upwards after reflecting of the tip-tilt mirror. Blue dots represent atoms in the atom cloud and the effective transverse $\delta k_x$ and longitudinal $\delta k_z$ momentum kicks are depicted with arrows.
We will first examine the perturbation's effect on the laser phase that is imprinted on the atoms as \( \phi_{\text{laser}} = n(kz + \phi_w) \), where \( n \) is the size of the large momentum transfer to the atom, at each laser-atom interaction point, then move on to the propagation and separation phases. We will use Figure 3.3 to denote the laser interaction points.

The total laser phase built up after a three pulse sequence \((\pi/2, \pi, \pi/2)\), is defined as

\[
\Delta \phi_{\text{laser}} = \phi_A - \phi_B - \phi_C + \phi_D. \tag{3.12}
\]

For simplicity we will start by ignoring the \( \delta k \) kicks as well as rotations and show that the perturbative method employed reduces to the expected result to zeroth order in \( \delta \). The laser phase perturbed by the wavefront at the four interaction points is

\[
\phi_A = n(kz(t) + \phi_w)
= n \left( \delta \cos(k_x x_i) \sin \left( \frac{k_z^2 z_i}{2k} \right) + kz_i \right), \tag{3.13}
\]

\[
\phi_B = n \left( \delta \cos(k_x(Tv_x + x_i)) \sin \left( \frac{k_x^2}{2k} \left( -\frac{gT^2}{2} + T \left( \frac{\hbar n}{m} + v_z \right) + z_i \right) \right) \right)
+ k \left( -\frac{gT^2}{2} + T \left( \frac{\hbar n}{m} + v_z \right) + z_i \right), \tag{3.14}
\]
\[
\phi_C = n \left( \delta \cos(k_x(T v_x + x_i)) \sin \left( \frac{k_x^2 \left( -\frac{gT^2}{2} + T v_z + z_i \right)}{2k} \right) + k \left( -\frac{gT^2}{2} + T v_z + z_i \right) \right),
\]

(3.15)

\[
\phi_D = n \left( \delta \cos(k_x(2T v_x + x_i)) \sin \left( \frac{k_x^2 (-2g T^2 + kn T h + 2m T v_z + m z_i)}{2km} \right) + k \left( -2gT + \frac{kn h}{m} + 2v_z \right) \right) \sin \left( \frac{g k^2 T^2}{4k} - \frac{k_x^2 z_i}{2k} - \frac{k_x^2 n T h}{2m} \right)
\]

(3.16)

Adding the four of these equations together as in equation (3.12) we find the total laser phase shift to first order reduces to

\[
\Delta \phi_{\text{laser}} = -g k n T^2 + \left[ -n \cos(2k_x T v_x + k_x x_i) \sin \left( \frac{g k_x^2 T^2}{4k} - \frac{k_x^2 z_i}{2k} - \frac{k_x^2 n T h}{2m} \right) \right]
\]

Propagation and separation phases are calculated in the same manner as [113]. For a full perturbative calculation of the phase shift the total phase difference \( \Delta \phi_{\text{total}} = \Delta \phi_{\text{prop}} + \Delta \phi_{\text{laser}} + \Delta \phi_{\text{sep}} \). In order to keep the terms to a manageable number we made physical cuts based on the size of product terms. Large numbers were multiplied by a \( 1/\varepsilon \) term and small numbers by \( \varepsilon \). Then we set \( \varepsilon \) to zero to cut terms that were negligible. The total phase difference was then expanded to second order in \( \delta \). A list of the zeroth, first, and second order terms can be found in Tables 3.5–3.8. This calculation builds from the previous calculations [10] by implementing a full Fourier analysis including the effective transverse and longitudinal momentum kicks caused by gradients in the wavefront aberration phase. To second order in the full calculation terms that go as \( \delta^2 \) arise. This makes sense as the kicks
Table 3.5: First order in $\delta$ phase shift terms without Earth rotation

<table>
<thead>
<tr>
<th>Term</th>
<th>Phase shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$n \cos(k_x x_i) \sin \left( \frac{k_z^2}{2k} \right)$</td>
</tr>
<tr>
<td>2</td>
<td>$n \cos(k_x T v_x + k_x x_i) \sin \left( \frac{g k_x^2 T^2}{4k} - \frac{k_x^2 v_x}{2k} - \frac{k_x^2 z_i}{2k} - \frac{k_x^2 n T h}{2m} \right)$</td>
</tr>
<tr>
<td>3</td>
<td>$-n \cos(2k_x T v_x + k_x x_i) \sin \left( \frac{g k_x^2 T^2}{k} - \frac{k_x^2 v_x}{k} - \frac{k_x^2 z_i}{2k} - \frac{k_x^2 n T h}{2m} \right)$</td>
</tr>
<tr>
<td>4</td>
<td>$n \cos(k_x T v_x + k_x x_i) \sin \left( \frac{g k_x^2 T^2}{4k} - \frac{k_x^2 v_x}{2k} - \frac{k_x^2 z_i}{2k} - \frac{k_x^2 n T h}{2m} \right)$</td>
</tr>
</tbody>
</table>

go as a perturbation $\sim \delta$ and these couple with the imprinted wavefront perturbations that also go as $\delta$. These terms are expected physically and some of the terms are listed below.

### 3.3.2 Leading Order Terms

We have listed here a selection of the leading order terms for two versions of the calculation. The first set assume that the Earth’s rotation will be succesfully compensated using either multi-loop atom interferometry schemes or by the use of a tip-tilt mirror as discussed in section 3.2. The next set of tables show an example selection of the terms arising with rotation couplings.

Here are listed some of the leading terms of the calculation. In Table 3.6 the terms physically represent the recoil phase shift from the added kinetic energy from the momentum kicks imparted on the atoms in the $x$ and $z$ direction by the wavefronts. This shift goes as $\hbar^2 \delta k_i^2 / 2m$ where $i = x, z$ for the $x$ and $z$ momentum kicks respectively. Terms 1 through 5 are associated with the $\delta k_z$ kick as they scale with $(k_x^2 / 2k)^2$ and terms 6 through 10 are associated with the $\delta k_x$ kick as they scale with $k_x^2$. The second order terms from the transverse ($x$) kicks will therefore generally be larger than those from the longitudinal ($z$) kicks by a factor of $\sim (k_x / k)^2$. 
Table 3.6: Second order in $\delta$ phase shift terms without Earth rotation.

<table>
<thead>
<tr>
<th>Term</th>
<th>Phase shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$k_s n^2 T h \cos(k_s x) \cos\left(\frac{k_s z}{2\pi}\right) \cos(k_s T v_x + k_s x) \cos\left(\frac{g k^2 T^2}{4\pi} - \frac{k^2 T v_z}{2\pi} - \frac{k^2 z}{2\pi}\right)$</td>
</tr>
<tr>
<td>2</td>
<td>$k_s n^2 T h \cos(k_s x) \cos\left(\frac{k_s z}{2\pi}\right) \cos(2k_s T v_x + k_s x) \cos\left(\frac{g k^2 T^2}{4\pi} - \frac{k^2 T v_z}{2\pi} - \frac{k^2 z}{2\pi}\right)$</td>
</tr>
<tr>
<td>3</td>
<td>$k_s n^2 T h \cos(k_s x) \cos\left(\frac{k_s z}{2\pi}\right) \cos(k_s T v_x + k_s x) \cos\left(\frac{g k^2 T^2}{4\pi} - \frac{k^2 T v_z}{2\pi} - \frac{k^2 z}{2\pi}\right)$</td>
</tr>
<tr>
<td>4</td>
<td>$k_s n^2 T h \cos(k_s x) \cos\left(\frac{k_s z}{2\pi}\right) \cos(k_s T v_x + k_s x) \cos\left(\frac{g k^2 T^2}{4\pi} - \frac{k^2 T v_z}{2\pi} - \frac{k^2 z}{2\pi}\right)$</td>
</tr>
<tr>
<td>5</td>
<td>$-\frac{g k n T^2}{2\pi}$</td>
</tr>
<tr>
<td>6</td>
<td>$2 k n T^2 v_x \Omega_y$</td>
</tr>
<tr>
<td>7</td>
<td>$-3 k n R T^2 \Omega_y^2$</td>
</tr>
<tr>
<td>8</td>
<td>$-\frac{3 k^2 n^2 T^3 \Omega_y^2 h}{2\pi}$</td>
</tr>
</tbody>
</table>

Table 3.7: Zeroth order in $\delta$ phase shift terms with Earth rotation.

<table>
<thead>
<tr>
<th>Term</th>
<th>Phase shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-\frac{g k n T^2}{2\pi}$</td>
</tr>
<tr>
<td>2</td>
<td>$2 k n T^2 v_x \Omega_y$</td>
</tr>
<tr>
<td>3</td>
<td>$-3 k n R T^2 \Omega_y^2$</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{3 k^2 n^2 T^3 \Omega_y^2 h}{2\pi}$</td>
</tr>
</tbody>
</table>

Table 3.8: Selection of first order in $\delta$ phase shift terms with Earth rotation.

<table>
<thead>
<tr>
<th>Term</th>
<th>Phase shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-\frac{3 k^2 n^2 T^3 \Omega_y^2 h \cos(k_s x) \cos\left(\frac{k_s z}{2\pi}\right)}{2\pi}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{k_s n^2 T^2 \Omega_y h \sin\left(\frac{1}{2} k_s T^2 \Omega_y - \frac{1}{2} k_s R T x \Omega_y + k_s T v_x \Omega_y + k_s x \right) \sin\left(\frac{g k^2 T^2}{4\pi} - \frac{k^2 T v_z}{2\pi} - \frac{k^2 z}{2\pi}\right)}{2\pi}$</td>
</tr>
</tbody>
</table>
Table 3.9: Selection of example values for wavefront phase shifts at the first and second order in $\delta$.

<table>
<thead>
<tr>
<th>Term</th>
<th>Phase shift</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$n\delta \cos(k_t x_i) \sin\left(\frac{k^2}{2k} H\right)$</td>
<td>First Order in $\delta$</td>
</tr>
<tr>
<td>1</td>
<td>$(0.325 , \text{rad}) \left(\frac{n}{100}\right) \left(\frac{\delta}{0.005}\right) \left(\frac{\sin(k^2 H/2k)}{0.65}\right)$</td>
<td>Longitudinal Kicks</td>
</tr>
<tr>
<td>2</td>
<td>$-\kappa_t^2 n^2 \delta^2 T H \cos(k_t x_i) \cos\left(\frac{k^2}{2k} H\right) \cos\left(k_t T v_t + k_t x_i\right) \cos\left(\frac{g k^2 y^2}{4k} - \frac{k^2 T v_x}{2k} - \frac{k^2}{2k} H\right)$</td>
<td>Transverse Kicks</td>
</tr>
<tr>
<td>3</td>
<td>$\left((-4.33 \times 10^{-9} , \text{rad}) \left(\frac{n}{100}\right) \left(\frac{\delta}{0.005}\right) \left(\frac{\sin(k^2 H/2k)}{-0.2}\right)\right)$</td>
<td></td>
</tr>
</tbody>
</table>

For MAGIS-100 example values of such wavefront perturbations are shown in table 3.9. The values chosen assume an LMT $n = 100$, a transverse spatial length scale of 3 mm for the perturbation, and a 698 nm laser pulse.

### 3.3.3 Wave-front Imaging

To mitigate wavefront-perturbation-induced phase shifts, it will be important to characterize the laser wavefront *in situ* using the MAGIS-100 atomic interferometer. Several methods have been thought of so far and can be used in conjunction with one another. First, a three-pulse laser sequence can be implemented where for the third and final beam splitting pulse one of the tip-tilt mirrors is tilted slightly to add an angle $\theta$ to laser beam. This shifts the wavefront by a distance $d_{\text{atom}} \theta$ before the final pulse ($d_{\text{atom}}$ is the distance between the mirror and the location of the atoms), allowing a measurement of the first derivative of the wavefront across the entire atom cloud to be made using spatially resolved detection,
see figure 3.4. Having both the down-shooting and retro-reflection mirrors mounted on piezo tip-tilt stages will allow wavefront perturbations on the upward and downward propagating beams to be independently characterized. Another method is to purposefully apply an initial transverse velocity kick to the atom cloud, which would provide information about spatial variation of the wavefront along the kick direction. Point source atom interferometry can provide similar information [1]. For the laser tilting method, care must be used with the range of angles that can be applied. If too much tilt is added, the frequencies of the atom fringes can grow so large as to be unresolvable by the imaging system. In addition, methods of 3D image reconstruction are actively being pursued which would allow fitting of the wavefront perturbation based on images of the atom cloud from multiple angles.

Figure 3.4: Plots of laser wavefront perturbation phase shifts. Left plot shows the phase shift arising from a calibration interferometry sequence with no beam tilt after the final beam splitter pulse. Right shows the laser wavefront perturbation phase shift with the beam tilt \( d_{\text{atom}} \theta \) allowing for the characterization of the wavefront.
3.4 Temperature and Humidity

For temperature, humidity, and dew point measurements we used an OM-DVTH logger. They are compact and battery run which allowed for placing them in multiple locations around the MAGIS-100 site. One logger was placed at the top of the shaft and another was placed underground near the bottom of the shaft. Data was then sampled every minute for a period from January 2019 to June 2019. The temperature accuracy of the logger is ±0.5°C with a resolution of 0.005°C. Relative humidity accuracy and resolution are ±2% RH and 0.01% RH respectively.

Data was then cleaned to account for the temperature change when moving the logger to connect to the PC. The logger at the bottom of the shaft was found to have a single bad bit which was adjusted during processing.

For the period between late January and early June analysis of the temperature readings show a tightly constrained distribution underground at the bottom of the shaft and a slightly larger spread of temperatures at the surface. As can be seen in figure 3.5, above ground there exist two peaks around 15.5°C and 17°C. This variation implies a need to have an extra layer of temperature control at the surface. The MINOS service building that houses the shaft has a large roll-up door that provides large surface area for temperature change when opened. Underground the environment is controlled through air-handling units and the only connections to the surface environment are through the shaft and a smaller pipe near the end of the underground tunnel.

Humidity and dew point follow the same pattern having a narrow distribution underground and a broader distribution above ground show in figures 3.6 and 3.7. This is again caused by the control of the underground air and the variability of the temperature caused by the opening and closing of the entry way to the MINOS building. The humidity at
Figure 3.5: Histograms of temperature recorded at the top of the shaft in the MINOS service building and at the bottom of the shaft.

The surface level is between 25 %RH–35 %RH. Below ground the humidity remains between 18 %RH–20 %RH. Above ground the humidity fluctuates with the seasons between a large range of 20 %RH–80 %RH. The dew point underground falls between $-8^\circ C$–$-3^\circ C$ while at the surface has two main peaks that range between $-3^\circ C$–$0^\circ C$ and $10^\circ C$–$11^\circ C$.

Figure 3.6: Histograms of humidity recorded at the top of the shaft in the MINOS service building and at the bottom of the shaft.

Figure 3.8 shows at the surface, median temperature readings remain bound between $13^\circ C$ and $19^\circ C$ for the full 5 month period of temperature data, but fluctuate with the seasons. Underground median temperatures are stable between $18^\circ C$–$18.5^\circ C$ owing to the HVAC controlled underground environment.
Figure 3.7: Histograms of dew point recorded at the top of the shaft in the MINOS service building and at the bottom of the shaft.

Figure 3.8: Minimum, Maximum, and Median values of weekly temperature at the top of the MAGIS shaft and the bottom of the shaft over a 5 month period and a 3 month period respectively.

We also note that for average days in January and May the hourly fluctuations at the surface of the shaft can show fluctuations caused by opening and closing of the main building roll-up door, however they remain steady outside of these events. This is observed in figure 3.9.

Temperature fluctuations directly impact the atom interferometer phase through blackbody radiation shifts and background gas index of refraction changes. Blackbody radiation can shift the atomic energy levels of strontium resulting in phase noise in the interferometer if the temperature of the vacuum tube fluctuates with a rate inside the target frequency band. The strontium clock transition has a blackbody temperature coefficient of $-2.3 \text{Hz}(\frac{T_S}{300\text{K}})^4$, 


where $T_S$ is the system temperature [172]. For a three-loop interferometer where $T$ is the time from the initial beamsplitter to the mirror pulses and with a temperature frequency of $\omega_{\text{Temp}}$ the atom interferometer response is suppressed by a factor of $(\omega_{\text{Temp}}T)^2$ for low frequencies. With a temperature fluctuation of 1°C over a period of 1 hour the noise magnitude is $1 \times 10^{-6} \text{rad}/\sqrt{\text{Hz}}$. The effect coming from background index of refraction fluctuations alters the optical path length associated with the baseline of the interferometer. Index of refraction noise leads to false strain signals [15]. Fluctuations in the system temperature $T_S$ with a fluctuation of $\delta T_s$ over a period of 1 hour leads to a strain signal associated with the index of refraction change of $$\delta h = \delta \eta \sim \left(4 \times 10^{-25} \text{ rad}/\sqrt{\text{Hz}}\right)\left(\frac{P}{10^{-10} \text{ Torr}}\right)\left(\frac{300 \text{ K}}{T_S}\right)\left(\frac{\delta T_S}{1 \text{ K}}\right).$$

From the results above it is clear that the temperature fluctuations underground are not of concern as we are worried about fluctuations on the order of 1°C per hour. At the surface however, there are daily temperature fluctuations that are aligned with the weather outside of the MINOS service building and the large roll-up door provides a wall of external air to enter all at once. We intend to gate out the opening of the large door during runs as this will allow control of the largest deviations. The atom sources to be installed down the shaft wall will also be sealed in temperature controlled enclosures. Finally temperature and humidity controls will be incorporated into the laser room design as well as the vacuum tube transport system for the interferometry laser beam from the laser room to the vertical vacuum pipe in
the shaft. Long term diagnostics of the temperature, humidity, dew point, and pressure will also be actively measured and analyzed by an environmental DAQ system. These mitigation strategies will reduce the systematic noise in the phase measurements related to blackbody radiation shifts, and background gas index of refraction fluctuations in the vacuum tube [1, 3, 5].
3.5 Seismic Vibrations

To measure ground motion a Trimble RefTek 151B-120 Observer [185] seismometer was installed in two locations. Our primary focus was to measure the vertical displacement of the ground motion which is driven by Rayleigh waves along the surface. In order to measure the ambient seismic noise we needed a low self-noise seismometer. As a preliminary step we collected data from the surface of the shaft and underground near the lower exit of the shaft. The environment underground is much quieter so the two locations allow us to look for various vibrational noise sources that could be contaminating the signal at the surface. The seismometer has a self-noise lower than the “New Low Noise Model” from 0.0167 Hz–10Hz. This makes it ideal for the modeling we performed. Data was then collected for two months above ground and nine months underground. Data collection was taken at a sampling rate of 50 samples per second above ground and 40 samples per second underground. The time traces were sampled into hour long segments.

Power spectra of the data were then constructed. Since we plan to model GGN, which is a low amplitude effect, we followed the methods used to construct the new low and high noise model (NHNM,NLNM) [186, 187]. These hourly power spectra could then be averaged and binned in a 2D histogram for the period of observation and the mean values analyzed.

3.5.1 Seismometer Specs

A state of the art seismometer was purchased to reach the sensitivity required for a preliminary model of gravity gradient noise (GGN). The specs of the seismometer are as follows:
### Table 3.10: Seismometer specification list

<table>
<thead>
<tr>
<th>Sensor Type</th>
<th>3 component orthogonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback</td>
<td>Force balance with capacitive displacement transducer</td>
</tr>
<tr>
<td>Frequency Response</td>
<td>151B-120: 0.0083 Hz – 50 Hz</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>2000 V/m/s (Differential Output)</td>
</tr>
<tr>
<td>Output Signal</td>
<td>±20 V</td>
</tr>
<tr>
<td>Dynamic Range</td>
<td>Greater than 140 dB (5 Hz)</td>
</tr>
<tr>
<td>Self Noise</td>
<td>Lower than NLNM from 0.0167 Hz – 10 Hz</td>
</tr>
</tbody>
</table>

#### 3.5.2 Analysis

For environmental characterization near the proposed detector site we look at the power spectral density of the ambient seismic waves using a broadband low-frequency seismometer. This data is also required in making a model of the location’s gravity gradient noise spectrum. The analysis of the seismic data is split into three main steps. First, the binary data is imported from the seismometer’s digitizer. Second, the data is then cleaned and formatted in Mathematica according to the methods in [186]. Finally the data is moved and plotted using Python.

A thorough consideration of the seismic environment requires measurements at the surface of the 100 m shaft as well as underground. The seismometer was positioned in the MINOS Service Building near the opening to the shaft for 58 days, from December 18, 2018 to February 13, 2019, then moved underground. The seismometer remained underground from February 25, 2019 to May 2019. Data was taken at a sampling rate of 100 samples/sec for the surface measurements and 40 samples/sec for the underground measurements. This was done to allow for a high enough Nyquist frequency cut-off. The choices were somewhat arbitrary and set by required storage size for the recorded data. The digitizer was set to save data in one hour segments.
Mathematica is used primarily for formatting the data files and converting the voltage counts into units of m/s as well as finding the power spectrum of each one hour file. The data is scaled to the proper units using the sensitivity of the seismometer from the calibration data. Any linear trend is subtracted from the measurements and the data is zero meaned. This allows for the analysis on just the ambient noise. Events such as earthquakes or the shaft elevator running are not removed from the calculation of the power spectrum allowing us to see the full characteristics of the site noise.

A power spectrum is calculated using a discrete Fourier transform for a total time $T = N\Delta t$ and total number of measurements $N$

$$PSD\{x(t)\} = \frac{2(\Delta t)^2}{T}|x(\omega)|^2$$

(3.18)

where,

$$x(\omega) = DFT\{x(t)\} = \sum_{t=0}^{N-1} x(t)e^{2\pi it/N}.$$  

(3.19)

For large datasets Mathematica is on the slower side of plotting so 2-D histograms of the multiple days worth of data were made using Python.

Seismic data is shown in histograms where the color corresponds to the counts per bin of the amplitudes associated with the frequency bins figure 3.10. Frequency was binned for each recorded frequency. The yellow region represents the 95th percentile. The vertical velocity seismic spectra above and below ground are consistent with expected ground motion of northern Illinois and fall between the NLNM/NHNM (depicted on the histograms as the lower and upper dashed black lines respectively). Time traces were taken at a sampling rate of 50 Hz above ground and 40 Hz below ground. Data was taken from December 18, 2018 to February 25, 2019 for surface measurements and from February 25, 2019 to May 30, 2019 underground measurements. Below 1 Hz the microseism peaks from the ocean movement can
be seen. Higher frequency content in the amplitude spectrum can be attributed to mechanical vibrations of nearby air-handling units, air compressors, and a ratchet-clank elevator used for accessing the underground facilities.

![Seismic amplitude spectra](image)

Figure 3.10: Seismic amplitude spectra for the surface of the shaft (left) and underground near the exit of the shaft (right). The color bar represents the histogram counts for the amplitude for each frequency bin. Also plotted are the NLNM (lower black line) and the NHNM (upper black line).

Comparing surface spectra to underground spectra show differences in the amplitude of the higher frequency content which is associated with anthropic generated Newtonian Noise. We also observe a decrease in human generated noise at night figure 3.11. Even for the full spectrum the acceleration vibration amplitudes are within acceptable limits for our current mitigation strategies.

In addition to the low frequency seismic acceleration spectrum measurements we took higher frequency samples in the planned location of the laser system to ensure design specs would meet the required vibration isolation. Figure 3.12 shows a large increase of anthropogenic Newtonian noise above 10 Hz which will be isolated through design choices of the laser tables and laser transport and support system.
Figure 3.11: Seismic amplitude spectra for the surface of the shaft (left) and underground near the exit of the shaft (right) between 6 pm and 6 am. The color bar represents the histogram counts for the amplitude for each frequency bin. Also plotted are the NLNM (lower black line) and the NHNM (upper black line).

Figure 3.12: Seismic acceleration amplitude spectral density plot at the planned laser table site sampled at 500 Hz.
3.5.3 Seismic Systematics

Seismic vibrations source two different systematic effects on the interferometer. Direct vibration of the laser and optics system can lead to phase noise in the same way that inherent laser noise affects the phase [7, 188]. The secondary effect of ground motion is to generate a perturbation in the gravitational potential field around the atom clouds causing time dependent gravity gradients. For a gradiometer configuration of the atom interferometer the velocity difference $\Delta v$ between the two atom clouds leads to a phase shift of

$$\delta \phi_{\text{vibration}} \sim \left( 10^{-9} \text{rad}/\sqrt{\text{Hz}} \right) \left( \frac{n}{100} \right) \left( \frac{\Delta v}{10 \mu \text{m/s}} \right) \left( \frac{T}{1 \text{s}} \right) \left( \frac{\delta a}{10^{-4} \text{m/s}^2/\sqrt{\text{Hz}}} \right),$$

where $n$ and $T$ are defined as above, caused by the vibration of the beam optics with acceleration amplitude spectral density $\delta a$. An example of the expected noise levels associated with initial cloud kinematics we consider a three loop interferometry sequence which suppresses leading order phase shifts derived from rotation and gravity gradients [179]. The primary noise term for such a sequence has the form

$$\delta \phi_{RGGV} = \left( \frac{17}{3} + 4\sqrt{2} \right) nk \Delta v_x \Omega_y T_{zz} T^4.$$  

Here the coordinate axes are such that $z$ is normal to Earth’s surface and the Earth’s rotation is in the $y$ direction. $nk$ is the number of large momentum transfer velocity kicks imparted on the atom clouds times the laser wavenumber. $\Delta v_x$ is the shot-to-shot velocity jitter of the atom cloud in the $x$ direction, $\Omega_y$ is the Earth’s rotation speed in the $y$ direction, $T_{zz}$ is the vertical gravity gradient, and $T$ is the time between the first beam splitter laser pulse and mirror pulses. The expected noise magnitude is

$$\delta \phi_{RGGV} \sim \left( 2 \times 10^{-6} \text{rad}/\sqrt{\text{Hz}} \right) \left( \frac{n}{100} \right) \left( \frac{\Delta v_y}{1 \mu \text{m/s}/\sqrt{\text{Hz}}} \right) \left( \frac{T}{1 \text{s}} \right).$$

Seismic vibrations cause an initial fluctuation in the atom source leading to velocity jitters of the atom cloud. The last systematic is an effective strain that is generated in the interferometer
response by the GGN generated by Rayleigh surface waves. From current analysis the phase shift of the atom interferometer leads to a spurious strain noise magnitude of

$$\delta h_{GGN} = \frac{\pi G \gamma_\nu \rho_0}{2LT \omega_{ggn}^3} \langle \delta \xi_z \rangle \left( e^{-\sqrt{2}L \omega_{ggn}/c_R} - 1 \right) \sin(\omega_{ggn} T) \cos(\phi_{ggn}).$$

Here $\delta h_{ggn} \sim \left(10^{-19} / \sqrt{\text{Hz}}\right) \left(\frac{100 \text{ m}}{L}\right) \left(\frac{1.5 \text{ s}}{T}\right) \left(\frac{1 \text{ Hz}}{\omega_{ggn}}\right)^3 \left(\frac{\delta \xi_z}{1 \mu\text{m}}\right) \left(\frac{300 \text{ m/s}}{c_R}\right)$, where $\omega_{ggn}$ is the frequency of the generating Rayleigh wave, $L$ is the baseline length, $c_R$ is the Rayleigh wave speed, $\delta \xi_z$ is the vertical surface displacement, and $\gamma_\nu$ and $\rho_0$ are ground dependent quantities. This level of strain noise will mask signals of gravitational waves (GWs) that we wish to look for in our frequency band of 0.3 Hz–10 Hz with future long baseline atom interferometers. A benefit of atom interferometers is that we can use many interferometers down the baseline to distinguish GGN from GWs which cannot be done in laser based interferometers [189].
3.6 Gravity Gradient Noise

For the experimental sensitivities that we hope to achieve with MAGIS-100, understanding and being able to map out so-called “gravity gradient noise” (GGN) is important [190]. First, it is an important background that will limit our sensitivity in the mid-band especially when we move to frequencies lower than 1 Hz. Second, it is a theorized effect that has not yet been measured in this frequency range.

GGN arises as a secondary effect caused by ground motion. From seismology we know that mass density perturbations – these include earthquakes, slip faults, or other irregular density fluctuations – cause waves to propagate through the surface of the Earth. We classify these waves as primary and secondary waves. The former is made up of longitudinal waves while the latter is a transverse wave. The third relevant type of wave is the Rayleigh wave which contains components of both primary and secondary waves. A common place where these appear is as waves along the surface of lakes and rivers. These waves are also important because they are surface waves that can exist along interfaces between different media. Included in these interfaces is the surface of the Earth and the atmosphere. The direct effect of these seismic waves is mechanical vibration of the apparatus and lasers. We also see a perturbation in the gravity field’s potential. Since our masses are measured in a differential configuration [7] the vibration associated with the lasers used to split the atom clouds at the top and bottom of the baseline is subtracted out. The leftover systematics are that of gravity gradients, magnetic field gradients, and GGN.
3.6.1 Seismic and Gravitational Interactions

The following is a review of the necessary backgrounds for understanding the couplings of seismic fluctuations and gravity [191]. There are four main response mechanisms to consider in terms of gravitational perturbations. These are test mass accelerations and tidal forces, the Shapiro time delay, gravity induced ground motion, and static gravity field couplings.

**Accelerations and Tidal Forces.** For test mass accelerations we consider the two cases where the test mass is supported by either vertical suspension (strings), or a horizontal cantilever. Both of these scenarios have an acceleration response caused by gravity perturbations as well as vibrations of the support. The representative equations are

\[ \delta a(\omega) = \frac{\omega^2}{\omega^2 - \omega_0^2 + i\gamma\omega} \delta g(\omega) \equiv R(\omega; \omega_0, \gamma)\delta g(\omega) \]  
\[ \delta a(\omega) = \frac{\omega_0^2 - i\gamma\omega}{\omega_0^2 - \omega^2 - i\gamma\omega} \delta \alpha(\omega) \equiv S(\omega; \omega_0, \gamma)\delta \alpha(\omega). \]

In the case of freely falling test masses there is also a tidal force caused by relative accelerations given by

\[ \delta g_{12}(\omega) = -\nabla \psi(\mathbf{r}_2, \omega) + \nabla \psi(\mathbf{r}_1, \omega) \]
\[ \approx -(\nabla \otimes \nabla \psi(\mathbf{r}_1, \omega)) \cdot \mathbf{r}_{12}. \]

The first term in parenthesis is called the gravity gradient tensor. The second time integral of this corresponds to the gravity strain \( \mathbf{h}(\mathbf{r}, t) \). This approximation holds so long as the distance, \( \mathbf{r}_{12} \) is sufficiently small between the test masses.

**Shapiro Time Delay.** This effect becomes relevant whenever the sensor in question uses the readout of lasers and their corresponding phases. Since we know that the curvature
of space-time influences light as well as matter there should be some phase shift caused by the perturbation of the light’s path. From the following metric it is possible to calculate the first order effect of this time delay.

\[ ds^2 = -(1 + 2\psi(\mathbf{r}, t)/c^2)(c\,dt)^2 + (1 - 2\psi(\mathbf{r}, t)/c^2)|d\mathbf{r}|^2. \]  (3.24)

\[ \frac{d^2t}{d\lambda^2} = -\frac{2}{c^2} \frac{dt}{d\lambda} \frac{d\mathbf{r}}{d\lambda} \cdot \nabla \psi(\mathbf{r}, t), \]  (3.25)

\[ \frac{d^2\mathbf{r}}{d\lambda^2} = \frac{2}{c^2} \frac{d\mathbf{r}}{d\lambda} \times \left( \frac{d\mathbf{r}}{d\lambda} \times \nabla \psi(\mathbf{r}, t) \right). \]  (3.26)

With these equations at hand one need only do a perturbation expansion to first order to find that the deviation in the spatial path is orthogonal to the unperturbed path. With this we simply calculate the time delay as an integral over the straight line trajectory and arrive at the final phase shift of

\[ \Delta \phi(\mathbf{r}_0, t_0) = \frac{\omega_0}{c} \int_0^{L/c} d\lambda \frac{dt}{d\lambda} = \frac{\omega_0 L}{c} - \frac{2\omega_0}{c^2} \int_0^{L/c} d\lambda \psi(\mathbf{r}^{(0)}(\lambda), t^{(0)}(\lambda)). \]  (3.27)

**Gravity induced ground motion.** Another important mechanism to consider is seismic field and gravity perturbation interactions. A seismic field will create a gravity perturbation that will in turn react back on the seismic field. This self interaction is present at low frequencies. The best plan of attack is to look at the creation of a gravity perturbation by a seismic perturbation then to look at the interaction of the gravity perturbation on the seismic field.

We will first investigate the equations for small seismic displacements

\[ \rho \partial_t^2 \xi(\mathbf{r}, t) = \nabla \cdot \mathbf{\sigma}(\mathbf{r}, t). \]  (3.28)
Here $\xi$ is the displacement field, and $\sigma$ is the stress tensor. The stress tensor can also be written in terms of the strain tensor for homogeneous and isotropic medium

$$
\sigma_{\epsilon}(\mathbf{r}, t) = \lambda \text{Tr}(\epsilon(\mathbf{r}, t)) \mathbf{1} + 2\mu \epsilon(\mathbf{r}, t),
$$

(3.29)

where,

$$
\epsilon_{ij}(\mathbf{r}, t) = \frac{1}{2}(\partial_i \xi_j + \partial_j \xi_i).
$$

(3.30)

In our tidal field description we can mimic these equations by replacing the strain with the gravity strain $h(\mathbf{r}, t)$. This is because the distance between two freely falling masses experiences a strain $\delta \mathbf{L}(\mathbf{r}, t) = h(\mathbf{r}, t) \cdot \mathbf{L}$. So we can recast the above equations as

$$
\sigma_h(\mathbf{r}, t) = \lambda \text{Tr}(h(\mathbf{r}, t)) \mathbf{1} - 2\mu h(\mathbf{r}, t),
$$

(3.31)

$$
\partial_t^2 \sigma_h(\mathbf{r}, t) = \lambda(\nabla^2 \psi(\mathbf{r}, t)) \mathbf{1} + 2\mu \nabla \otimes \nabla \psi(\mathbf{r}, t).
$$

(3.32)

calculating the divergence of our stress tensor now gives us

$$
\partial_t^2 (\nabla \cdot \sigma_h(\mathbf{r}, t)) = 2(\nabla \mu) \cdot (\nabla \otimes \nabla \psi(\mathbf{r}, t)) = -2(\nabla \mu) \cdot \partial_t^2 h(\mathbf{r}, t).
$$

(3.33)

This equation reveals that tidal fields produce forces on elastic mediums through the gradient of the shear modulus $\mu$. The gradient can be represented as a Dirac delta function $\nabla \mu = -\mu \delta(z) \hat{e}_n$, for a surface at $z = 0$. So the response to gravity strain fields is

$$
\lambda \text{Tr}(\epsilon(\mathbf{r}, t)) \hat{e}_n + 2\mu \hat{e}_n \cdot (\epsilon(\mathbf{r}, t) - h(\mathbf{r}, t)) = 0.
$$

(3.34)

**Atom-interferometric strainmeters.** When considering atom-interferometers as a strain detector we must consider their sensitivities and limiting factors. Treating an AI as a gradiometer we must note that the acceleration measurement sensitivities are limited
by seismic perturbations of the optics and laser source that provide the phase measurement of the free falling atom clouds. This is

\[ \delta a_{12}(\omega) = \frac{\omega L}{c} \delta \alpha(\omega). \]  

(3.35)

Here \( L \) is the separation of the two atom clouds.

Atom interferometers with long enough baselines (which is what we are considering for MAGIS-100 and beyond) can also be considered gravity strainmeters. They are capable of reading out strains caused by gravitational waves. The effect of a gravity strain field on two test masses is represented as

\[ \delta a_{12}(\omega) = \frac{1}{2} L \hat{e}_{12}^\top \cdot \hat{h}(r, t) \cdot \hat{e}_{12}. \]  

(3.36)

For a displacement noise with amplitude \( \xi(\omega) \) the resulting strain noise for the atom interferometer is

\[ \frac{\omega \xi(\omega)}{c}. \]  

(3.37)

This is on the order of \( 10^{-17}/\sqrt{\text{Hz}} \) for a frequency of 0.1 Hz.

In this section we will discuss the three types of seismic waves and the fields that generate them. From there we will look at the basic formalism that allows us to define the gravitational perturbation based on the fields. The potential formalism will also be presented.

From seismology the two main waves are P-waves and S-waves. “P” stands for primary and “S” stands for secondary. These waves correspond to compression waves, which displace along their direction of travel, and shear waves which act transversely to compression waves. The third type of wave is known as a Rayleigh wave. These are surface waves that travel along the boundary while the other two travel within the body of the medium. Our defining
characteristics are the usual $\omega = k_P \alpha$, with $\omega$ the frequency of the wave, $k_P$ the wave vector, and $\alpha$ the wave speed.

For the P-waves and S-waves their corresponding displacement fields are given by

$$
\xi^P(\vec{r}, t) = \xi_0^P(\vec{k}_P, \omega) \exp \left( i \left( \vec{k}_P \cdot \vec{r} - \omega t \right) \right) \vec{e}_k,
$$

$$
\xi^S(\vec{r}, t) = \xi_0^S(\vec{k}_S, \omega) \exp \left( i \left( \vec{k}_S \cdot \vec{r} - \omega t \right) \right) \vec{e}_p.
$$

In these equations $\vec{e}_k = k_P/k_P$, and $\vec{e}_p \cdot \vec{k}_S = 0$. The speeds of these waves are defined using the Lamé constants, $\lambda$, and $\mu$.

$$
\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad \beta = \sqrt{\frac{\mu}{\rho}}.
$$

The relation between these two speeds is

$$
\beta = \alpha \sqrt{\frac{1 - 2\nu}{2 - 2\nu}},
$$

where $\nu$ is the Poisson ratio of the given medium.

A better representation of the displacement field is by the potentials for the two types of field.

$$
\vec{\xi}(\vec{r}, t) = \nabla \phi_s(\vec{r}, t) + \nabla \times \vec{\psi}_s(\vec{r}, t).
$$

The scalar and vector potentials $\phi_s$ and $\vec{\psi}_s$, represent the P-wave potential and the S-wave potential respectively. We note that the P-wave potential is irrotational, and the S-wave potential is divergence free. These are the proper characteristics for our waves. It is also possible to write the vector potential $\vec{\psi}_s$ as scalar quantities by

$$
\vec{\psi}_s = \nabla \times (0, 0, \psi_s(x, y, t)) + (0, 0, \chi_s(x, y, t)).
$$
Rayleigh Surface Waves. Rayleigh waves propagate along the horizontal direction $\vec{e}_k$ along the surface. They can be split into their vertical and horizontal directions as

$$\vec{k} = \vec{k}_z + \vec{k}_\varrho.$$  \hspace{1cm} (3.44)

The vertical wave numbers correspond to

$$k_z^P = \sqrt{(k^P)^2 - k_\varrho^2}, \quad k_z^S = \sqrt{(k^S)^2 - k_\varrho^2}. \hspace{1cm} (3.45)$$

Rayleigh waves also exist as an evanescent wave that falls off exponentially under the surface. Thus we also need the wave vectors defined for imaginary components

$$q_z^P = \sqrt{k_\varrho^2 - (k^P)^2}, \quad q_z^S = \sqrt{k_\varrho^2 - (k^S)^2}. \hspace{1cm} (3.46)$$

From these definitions the horizontal and vertical field amplitudes are

$$\xi_k(\vec{r}, t) = A \left( k_\varrho e^{q_z^P z} - \zeta q_z^S e^{q_z^S z} \right) \sin \left( \vec{k}_\varrho \cdot \vec{\varrho} - \omega t \right), \hspace{1cm} (3.47)$$

$$\xi_z(\vec{r}, t) = A \left( q_z^P e^{q_z^P z} - \zeta k_\varrho e^{q_z^S z} \right) \cos \left( \vec{k}_\varrho \cdot \vec{\varrho} - \omega t \right). \hspace{1cm} (3.48)$$

The function $\zeta(k_\varrho) \equiv \sqrt{q_z^P/q_z^S}$, and the speed of the Rayleigh wave is $c_R = k_\varrho/\omega$, and obeys the following

$$R((c_R/\beta)^2) = 0, \hspace{1cm} (3.49)$$

$$R(x) = x^3 - 8x^2 + 8x \left( \frac{2 - \nu}{1 - \nu} \right) - \frac{8}{1 - \nu}. \hspace{1cm} (3.50)$$
This finally leads to the displacement field

\[ \vec{\xi}(\vec{r}, t) = \xi_k(\vec{r}, t)\vec{e}_k + \xi_z(\vec{r}, t)\vec{e}_z. \]  

(3.51)

**Basics of Seismic Gravity Perturbations.** These are the basic techniques for doing calculations with plane wave seismic field displacements.

To grasp a deeper understanding of the cause of GGN we will imagine a train of mass moving just under the surface. This moving mass density causes a variation in the gravitational potential around masses underground and above ground. If we imagine the effects as radiation coming off the front of the mass train we can see that the radiation from the front will act as a high frequency effect which is highly dependent on the baseline length between two vertically aligned test masses. This is because the time difference it takes for the effect to be realized at the two locations is large. Once the mass train is directly above the test masses the effect becomes independent of the distance between them. This same scenario is realized with Rayleigh waves moving along the surface and their interaction with the gravitational potentials near the test masses of an atomic interferometer.

### 3.6.2 Derivation for MAGIS-100

This is still only a first order calculation. We are able to expand it a little further to realize a two regime solution to the expected GGN with respect to the frequency of the Rayleigh wave.

Our beginning assumptions are that of our configuration starting at the top of the shaft and descending to the bottom with a baseline \( L \) of 100 m. The baseline will be along the \( z \) direction and the incoming wave will be represented by its displacement vector field. We
can decompose the displacement field into a vertical component and horizontal component as,

\[ \vec{\xi}(\vec{r}, t) = \xi_k(\vec{r}, t)\vec{e}_k + \xi_z(\vec{r}, t)\vec{e}_z. \]  

(3.52)

Where, \( \vec{e}_k \) and \( \vec{e}_z \) are polarization vectors and

\[
\begin{align*}
\xi_k(\vec{r}, t) &\propto A \cdot \sin(\vec{k}_\ell \cdot \vec{\ell} - \omega t), \\
\xi_z(\vec{r}, t) &\propto A \cdot \cos(\vec{k}_\ell \cdot \vec{\ell} - \omega t).
\end{align*}
\]

In these expressions for the amplitudes in the horizontal direction and the vertical direction, respectively, the parameter \( A \) is a function of the wave vectors for the Rayleigh waves, \( \vec{k}_\ell \), and \( \vec{\ell} \) is the projection of \( \vec{r} \) onto the surface.

With this seismic displacement field we begin the derivation of the gravity potential perturbation using a continuity equation

\[ \delta\rho(\vec{r}, t) = -\nabla \cdot \left( \rho(\vec{r}, t)\vec{\xi}(\vec{r}, t) \right). \]

(3.53)

\[ \delta\phi(\vec{r}_0, t) = -G \int \frac{\delta\rho(\vec{r}, t)}{|\vec{r} - \vec{r}_0|} dV \]

\[ = G \int \rho(\vec{r})\vec{\xi}(\vec{r}, t) \cdot \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} dV. \]

(3.54)

In the case of Rayleigh wave perturbations it is found in Harms Equation (93) that the sum of the surface and bulk contributions to the gravity perturbation takes the form

\[\delta\phi(\vec{r}_0, t) = -2\pi G\rho_0 A e^{-z k_\ell} e^{i(\vec{k}_\ell \cdot \vec{r}_0 - \omega t)} (1 - \zeta(k_\ell)). \]

(3.55)
After substituting the surface displacement in for \( A = \frac{\xi_z}{(q_z - k_t \zeta(k_t))} \) we take the gradient and arrive at

\[
\delta a_z(\vec{r}_0, t) = -2\pi G \rho_0 e^{-zk_t} e^{i(\vec{k}_t \cdot \vec{\ell}_0 - \omega t)} \gamma(\nu) \xi_z. \tag{3.56}
\]

Here \( \gamma(\nu) \) is a term dependent only on the properties of the medium. \( \nu \) is the Poisson ratio for the ground in this case and to first order approximations can be taken to be 1. All that is left for us to do here is to take a differential measurement of this test mass perturbation along the baseline, \( \delta a_z(L\hat{e}_z) - \delta a_z(0) \). Following that we convert from an acceleration to a displacement strain of the test masses and convert from a seismic velocity spectrum to a seismic displacement spectrum.

\[
\delta a_z(h_2) - \delta a_z(h_1) = 2\pi G \rho_0 \gamma(\nu) \xi_z \left( e^{-k_t h_1} - e^{-k_t h_2} \right). \tag{3.57}
\]

We chose a coordinate system such that the origin is at the bottom of the atom interferometer leading to the term \( \vec{k}_t \cdot \vec{\ell} \to 0 \).

From this equation we derive two expressions. One for the low frequency regime, where the mass train we discussed earlier is directly above the two test masses, and one for the high frequency regime. The determining factor in this separation is the speed of the Rayleigh wave. For large baselines \( L \) we have the form

\[
\langle \delta x_z \rangle = 2\pi G \rho_0 e^{-k_t L} \frac{\langle \xi_z \rangle}{(2\pi f)^2}.
\]

\[
= \frac{G \rho_0 \gamma(\nu) e^{-k_t L}}{2\pi f^2} \langle \xi_z \rangle. \tag{3.58}
\]

If \( k_t L \ll 1 \) this reduces to

\[
\langle \delta x_z \rangle = \frac{G \rho_0}{2\pi f^2} \langle \xi_z \rangle. \tag{3.59}
\]
And in terms of strain noise this is

\[ h_{GGN} = \frac{G\rho_0}{2\pi f^2 L} \langle \xi_z \rangle. \]  \hfill (3.60)

In the low frequency regime we take a Taylor expansion of (3.57) and see that

\[
\begin{align*}
\delta a_z(h_2) - \delta a_z(h_1) &= 2\pi G\rho_0 \gamma(\nu) \xi_z \left( e^{-k_\ell h_1} - e^{-k_\ell h_2} \right) \\
&= 2\pi G\rho_0 \gamma(\nu) \xi_z (k_\ell L).
\end{align*}
\]  \hfill (3.61)

So,

\[
\begin{align*}
h_{GGN} &= \frac{\langle \delta x_z \rangle}{L} = \frac{\langle \delta a_z (L) - \delta a_z (0) \rangle}{L} \\
&= \frac{G\rho_0 k_\ell}{2\pi f^2 L} \langle \xi_z \rangle \\
&= \frac{G\rho_0}{f c_R} \langle \xi_z \rangle,
\end{align*}
\]  \hfill (3.62)

where \( k_\ell = 2\pi f / c_R \) and \( c_R \) is the velocity of the incoming Rayleigh wave. This velocity also determines the inflection point of our frequency dependence as

\[ f \lesssim \frac{c_R}{2\pi L}. \]  \hfill (3.63)

**Beyond MAGIS-100**

Throughout this calculation we have taken the assumption that the baseline does not go far enough underground for other bulk waves to have an effect. This allows for the approximation that only the surface waves (Rayleigh waves) contribute. There are other surface waves (Love waves) that may have an effect but have not been explored yet. The full form calculation including bulk waves after a certain distance underground would only
be needed for future kilometer scale detectors where the extra contributions would alter the
strain response along the baseline. The calculation implementing the phase response of the
atom interferometer by GGN at the MAGIS-100 site can be found in section 3.6.4.

3.6.3 Gravity Gradient Noise Model

Modeled GGN shows levels that will not affect MAGIS-100 until the late stages of the
experiment. The histograms of the modeled GGN figure 3.13 show little variation between
the surface measurements and the underground measurements as was expected from the
analytical model since the amplitude is exponential with a characteristic length scale of
order $k_L \sim 1$. The modeled GGN also has a knee frequency of 0.48 Hz where the analytical
model changes from the high frequency regime to the low frequency regime.

Figure 3.13: Modeled GGN strain amplitude spectra for the surface of the shaft (left) and
underground (right). The dashed black line denotes the expected strain sensitivity after
detector advancements.
3.6.4 GGN Analysis

For future suppression and mitigation of GGN we investigated the use of a string-of-pearls method where multiple simultaneous interferometers are generated along the baseline and run simultaneously to probe the gravitational environment similar to the method planned for MIGA and ELGAR [144, 147] except in our scenario the baseline is vertical and the atom trajectories are perpendicular to the surface of the Earth. We then measure the phase shift in a 10 m fountain gradiometer configuration. A differential gradiometer phase measurement along the baseline of each interferometer removes common mode noise sources such as laser beam jitter and wavefront aberrations. In this simulation we generated five atom clouds per atom source, 15 atom clouds in total, for both a 100 m baseline (MAGIS-100) and a 1 km baseline and 1000 LMT. With some technical advancement this scheme can be realized in the MAGIS-100 detector as a proof of concept. A plane Rayleigh surface wave was assumed with velocity of 300 m/s and a vertical displacement amplitude of $10^{-6}$ m in agreement with seismometer measurements around the shaft. The phase shift of the detector was calculated using a perturbative semi classical method [116]. The resulting GGN phase shift in the atom interferometer to leading order has the form

$$\delta \phi_{\text{GGN}} = \frac{\pi G \gamma \nu \rho_0}{\omega_{\text{ggn}}} \langle \delta \xi_z \rangle \, n k_{\text{eff}} e^{-\sqrt{2}k_l z} \sin(\omega_{\text{ggn}} T) \cos(\phi_{\text{ggn}}).$$

(3.64)

Here $G$ is Newton’s gravitational constant, $\gamma_\nu$ is a material dependent factor in our case $\sim 0.27$, $\rho_0$ is the average ground density at the site and $k_l$ is the Rayleigh wavenumber. All other parameters are the same as above.

By randomizing the phase of the Rayleigh wave we stochastically simulate 1000 random measurements of the GGN phase shift and then take the root-mean-square (RMS) value of
each set of samples allowing us to extract the exponential character directly associated with terrestrial ground motion dependent on the depth of the atom cloud. Figure 3.14 shows the results of fitting the phase shift to an exponential model and a linear model for three specific frequencies of the source Rayleigh wave.

\[
\text{Exponential Model: } \phi(z) = A \exp(-\alpha z) + B, \\
\text{Linear Model: } \phi(z) = Cz, \\
\text{Mixed Model: } \phi(z) = Cz + A \exp(-\alpha z). \tag{3.65}
\]

In practice a Fourier decomposition of the phase signal will be used to separate out the frequencies which will then be fit. This discriminatory power between a linear fit and an exponential fit is important because the response of the interferometer to GWs is linear with the baseline \( L \) \[133\]

\[
\delta \phi_{gw} = 2k_{eff} h L \sin^2 \left( \frac{\omega_{gw} T}{2} \right) \sin(\phi_0), \tag{3.66}
\]

while the GGN phase shift is exponential.

In addition we examined a mixed model fit and the dependence of the fit on the total number of atom interferometers down the baseline. In this study we assumed a perfect ability to extract the phase shift of the GGN alone when in reality there would be other noise sources present that need to be accounted for \[10\]. For this analysis we equally spaced the atom interferometers along the baseline. The simulated input data was also standardized in order to increase the numerical stability of the fit and to allow comparisons for different baseline lengths. Our confidence in the estimated linear component of the fitted model for a single measurement increased with higher resolution of atom interferometers, because the estimated standard error of the fit decreased. Using the exponential term from the fit we took a simulated phase difference across the entire baseline (100 m/1000 m) and subtracted the estimated exponential phase difference calculated from the fit. We computed the ratio
of this residual linear component with the initial simulated phase difference. We chose this to be our suppression factor, see figure 3.15. This metric then provides a heuristic of our ability to remove the exponential components of the GGN signal where the linear component serves as noise in any GW measurements. The suppression factor of the GGN gets better with increasing sample size of phase shift measurements and approaches an asymptote with number of atom interferometers along the baseline. For a phase shift containing only GGN components there should be no linear term present, however the uncertainty in distinguishing between a linear or exponential dependence leads to a spurious linear signal that looks like a GW signal. Finally, we examined the amplitude transfer function which takes us from a measured vertical displacement amplitude spectral density to the atom interferometer’s phase response as shown in figure 3.16. We plotted the transfer function at 100 m and at 1 km. Combining active seismometer arrays [191], to provide priors, with in-situ calibration measurements using the atom interferometer will allow us to map out the GGN and suppress it from the atom interferometer signal.

The secondary seismic vibration effect of GGN as shown in figure 3.13 is not an early systematic for the MAGIS-100 detector as we will not reach the strain sensitivity to detect it until technical advancements have been made. Active monitoring will be used for seismic vibration mitigation as well as early diagnostic measurements. Once the MAGIS-100 detector has reached an advanced stage we hope to be sensitive enough after R&D to reach strains where GGN becomes detectable. We would be interested to both measure as well as suppress the effect of GGN on our measurements as it interferes with GW detection. In addition to GGN generated by surface seismic waves the atmospheric pressure fluctuations also act as a source for gravity gradients detectable by atom interferometers [192]. Further analysis will collect data on the pressure fluctuations near the site to determine what level this systematic will have.
Figure 3.14: Plots of RMS phase shift amplitude from stochastically simulated measurements of the atom interferometer phase shift generated by Rayleigh waves with velocity 300 m/s and frequencies of 0.1 Hz (first row), 1 Hz (second row), and 5 Hz (third row). Left column plots are run with a baseline of 100 m and 15 simultaneous 10 m fountain launches with 5 clouds generated per atom source equally spaced down the baseline. The plots on the right column are for a baseline of 1000 m adding a fourth atom source generating 5 more atom interferometers at 1000 m. The shaded region of the curves is the 95% confidence region for the fitted phase shift amplitude.
Figure 3.15: Suppression factor of GGN noise as a function of number of atom interferometers down the baseline of 100 m and 1000 m. Plots show the effect for a Rayleigh wave of 0.5 Hz (top), 1 Hz (middle), and 5 Hz (bottom).
Figure 3.16: Transfer function of the atom interferometer response to a vertical displacement as measured by a seismometer at a depth of 100 m, left, and 1 km, right.

**100 m vs 1 km**

Future studies of GGN in longer baselines will consider the effect of higher velocity Rayleigh waves as you get deeper underground where the dense rock supports faster traveling waves. Underground there will also be contributions from body waves that can become surface waves between differing mediums after scattering that will also contribute to the seismic perturbation spectrum. In addition the longer baseline will contribute directly to reducing the impact of the GGN on strain measurements. Investigations are also ongoing for more advanced methods of seismic mitigation by modifying the physical environment around the shaft to passively scatter seismic waves.

### 3.6.5 Open Questions

Beyond the effects of seismic gravity gradient noise in the frequency range of interest we must also consider atmospheric gravity gradient noise. Arising from a similar coupling of the acceleration perturbation caused by seismic waves we can construct a model of the effects of pressure waves (sound waves) as well as shock waves from the atmosphere that then perturb the gravitational potential field around the atoms. Preliminary estimations have been done
in this direction but none for our specific low frequency atomic interferometer. This is an area that will require modeling efforts as well as more environmental data at the MINOS shaft.

Further insights are also needed in order to construct methods to reduce the expected seismic gravity gradient noise. These include an array of seismometers to subtract the signal of seismic waves positioned at the surface and underground. Combining these active measurements with \textit{in situ} characterization may allow the creation of filters that can be used to clean the phase signal after extraction enhancing the low frequency sensitivity of MAGIS-100 and beyond.

\section{MAGIS-100 Simulation}

During the research and development phase of MAGIS-100 simulating the design components and specifications that are necessary to attain our goal sensitivity is extremely useful. Design choices are guided by a goal sensitivity in the phase shift between the upper and lower arms of the MAGIS atom interferometer. Estimations for various parameters from the laser system to the atom sources have been made to meet the requirements laid out in section 3.1, however decisions such as the optics and imaging system required a deeper investigation into the impact on the phase extraction from obtained measurement images. This guided the case study of imaging parameters and further led to understanding effects of diffusion, cooling, linear and quadratic phase shears, and other systematics for the MAGIS system.

Building the software is also a first step at creating an automated system for analyzing the data that comes out of MAGIS-100. From the case study a full suite of fitting methods have been developed in conjunction with previous work done at Stanford. This new package of
Mathematica functions allows for simulating realistic images and fitting various parameters from the simulations and is under active development by the MAGIS-100 collaboration.

### 3.7.1 Case Study

The MAGIS-100 case study serves as a prototyping tool for the eventual data analysis workflow as well as a means of testing the experimental parameters against our fitting accuracy. In order to test out the effects of altering different design parameters we first need to generate an example distribution of atoms after a typical atom interferometer sequence. From this distribution we simulate photons through the proposed optics system and project them onto a pixelated plane representative of the camera used for imaging. Appropriate noise is added to the image based on the specifications of various test cameras. With this image we then study the accuracy and precision of different methods of fitting the phase of the phase difference between the two ports.

To construct the atom distribution we assume a three dimensional Gaussian wavefunction. This is possible because we are considering an ultra cold atom cloud, however a density matrix formulation would be more appropriate for unknown mixed states. For our purposes this approximation is valid for the noise and systematics we are investigating. We then take the initial wave function and propagate it through momentum space using a Fourier transform where we apply the momentum kicks that would be acquired by the atom cloud from the laser pulses during interferometry. After the last beam splitter pulse we apply a fringe shear pattern across the atom cloud, which is a modulated sine wave, in the transverse direction. In practice this added phase shear increases the accuracy of phase extraction [1].

We start with the initial three dimensional distribution properly normalized and a coordinate system where the $xy$-plane is transverse to the vertical direction of the vacuum
pipe and the z-axis is longitudinal along the vertical shaft. We then integrate out the y direction as there is cylindrical symmetry for the simplified simulation leading to the initial 2 dimensional Gaussian

$$\psi_{\text{initial}}(x, z) = \frac{1}{\sqrt{2\pi w_0^2}} e^{-\frac{(x^2 + z^2)}{4w_0^2}}. \quad (3.67)$$

with a width of $w_0$. This wavefunction is then Fourier transformed where we define the forward and inverse transform as

$$f(k) = \mathcal{F}[f(x)] \equiv \frac{1}{2\pi^{d/2}} \int_{-\infty}^{\infty} f(x) e^{-i\cdot k \cdot x} \, dx,$$

$$f(x) = \mathcal{F}^{-1}[f(k)] \equiv \frac{1}{2\pi^{d/2}} \int_{-\infty}^{\infty} f(k) e^{i\cdot k \cdot x} \, dk. \quad (3.68)$$

where $d$ is the dimension of the vector function being transformed. Equipped with these definitions we transform the wavefunction to momentum space and apply a propagation in time\(^1\) to simulate the interferometry trajectory of the atoms.

$$\psi_{\text{final}}(x, z) = \mathcal{F}^{-1}\left[\mathcal{F}[\psi_{\text{initial}}(x, z)] \exp\left(-i\frac{\hbar}{2m} \left(k_x^2 + k_z^2 \right) T\right)\right],$$

$$= \frac{mw_0 \sqrt{2/\pi}}{2mw_0^2 + i\hbar T} \exp\left(-\frac{m(x^2 + z^2)}{4mw_0^2 + 2i\hbar T}\right). \quad (3.69)$$

Here $m$ is the mass of the atom and $T$ is the elapsed time from the initial beamsplitter light pulse to the final beamsplitter pulse. After the final beamsplitter the two ports of the atom interferometer, port A and port B, separate out over some time $T_{\text{extra}}$ because of the different velocities imparted on the arms of the interferometer. A fringe pattern is also applied

$$\psi_A(x, z) = \psi_{\text{final}}(x - x_0, z - z_0) \frac{1}{\sqrt{2}} \left(1 + e^{i(k_{\text{fringe}}x + ak_{\text{quad}}x^2) + i\phi}\right),$$

$$\psi_B(x, z) = \psi_{\text{final}}(x - x_0, z - z_0) \frac{1}{\sqrt{2}} \left(1 + e^{-i(k_{\text{fringe}}x + ak_{\text{quad}}x^2) - i\phi}\right) e^{i(k_{z, \text{spread}}z)}. \quad (3.70)$$

\(^1\)Following the methods prescribed in [193] to move a quantum wavepacket in momentum space.
The phase shear fringe applied is specified by \( k_{\text{fringe}} \) which is the spatial frequency, \( a \) is the amplitude of the quadratic phase shear allowing for more general systematic effects to be incorporated into the simulated wavefunction, and \( \phi \) is the generic phase of the wavefunction to be fit. Port B also has an added velocity term, \( e^{i(k_{z,\text{spread}}z)} \), applied to allow for the separation of the two ports after the final beamsplitter. The final analytical form of the two port wavefunction is then found by applying the final propagation in Fourier space until imaging

\[
\psi_{A,\text{final}}(x, z) = \mathcal{F}^{-1} \left[ \mathcal{F}[\psi_A(x, z)] * e^{-i \frac{a}{2m} (k_x^2 + k_z^2) T_{\text{extra}}} \right], \\
\psi_{B,\text{final}}(x, z) = \mathcal{F}^{-1} \left[ \mathcal{F}[\psi_B(x, z)] * e^{-i \frac{a}{2m} (k_x^2 + k_z^2) T_{\text{extra}}} \right].
\]

(3.71)

We then plot the form of these very complex analytical functions in figure 3.17 for reasonable parameters chosen for the MAGIS-100 detector.

![Figure 3.17: Plot of analytical two port wavefunction. Parameters used for strontium atoms with interferometry time \( T = 3 \) s, extra drift time \( T_{\text{extra}} = 0.15 \) s, cloud radius \( w_0 = 0.002 \) m, port separation velocity \( \Delta v_z = \hbar k_{z,\text{spread}}/m = 0.10 \) m/s, and a fringe shear \( k_{\text{fringe}} = 1/0.0003 \) m\(^{-1}\) with a relative phase \( \phi = \pi/2 \) between the ports.](image)

For a chosen number of atoms and atom cloud radius we sample this distribution and simulate a number of photons being emitted during resonant scattering and traveling to the camera. Using a Poisson distribution for each photon and assuming a Gaussian noise of the camera as provided in the specs for the simulated camera we build up an image. During
this imaging time work has been done on investigating the effects of diffusion and Doppler cooling on the atoms during the fluorescence to determine the optimal imaging time for MAGIS-100 [194]$^2$.

Current analysis methods are built for daily runs on the 10 m scale and require manual analysis of raw data. We plan to build out a robust analysis framework on a shot by shot basis. Using the case study we also built up various fitting protocols for estimating the precision we can measure the phase difference with. The current simulation shows that we can fit the phase difference to a precision of 1 mrad [195]$^2$.

The case study has led to a tool that allows us to simulate realistic atom wavefunctions with specified imaging parameters. This will continue to help us develop a mock data challenge for testing the analysis tools as well as allow us to investigate the effect of other systematics and noise sources appearing during the interferometer measurement. I plan to further develop this simulation for the cases of thermal atom cloud distributions as well as adding in the possibility of having mixed states by the density matrix formulation.

### 3.7.2 MAGISsims Package

What started as a small case study into the effects of different camera parameters on our phase extraction abilities quickly developed into a system that allows for simulating atomic cloud distributions and fitting the phase difference of the simulated images. The initial case study code evolved into having two primary features: a set of functions for generating either randomized initial atom positions on the CCD plane or creating pixel signals, and a set of fitting and statistics functions for fitting various models against the generated images.

$^2$Accessible through internal document database. Can produce source if requested
The package is split into two libraries with functions for simulating atom cloud images with various systematics embedded and functions for doing various fitting procedures. Many collaborators on MAGIS-100 will be developing and adding functionalities to the package. I have contributed by converting the existing code base into a packaged format and adding the functions for simulating the atom interferometry sequence and generating the atom distributions as well as generating initial atom positions.

As was shown in the initial analytical distribution above one can generate a random pixel signal following a Poisson distribution of counts at each pixel in the simulated image plane by projecting a sampled number of atoms from the distribution and binning into the pixel grid. Figure 3.18 shows three example simulated images for projections in the $xz$-plane, $yz$-plane, and $xy$-plane.
Figure 3.18: Simulated atom cloud interference pattern pixelated images. Images, going from the top to the bottom, are of the $xz$ projection where $z$ runs horizontally, $yz$ projection with $z$ running horizontally, and $xy$ projection from top down view. Pixel plane is a $1340 \times 400$ grid with a pixel size of 2 $\mu m$ and an atom cloud radius of 2 mm.
CHAPTER 4
THEORETICAL MODELS AND INVESTIGATION

Primary science goals laid out in chapter 2 for MAGIS-100 and beyond are dark matter tests, prototyping for gravitational wave measurements, advanced tests of quantum mechanics and searches for new physics. In this chapter I will explore other possible science that can be done with the MAGIS-100 detector and how we might test these theories.

4.1 Gravitational Waves and Early Universe Models

Gravitational waves have been since the theory of General Relativity first came out and have finally been measured radiating from some of the largest events in our universe; a binary black hole merger, and neutron star mergers. There are many sources of gravitational waves that are theorized, such as signals from cosmic strings and a background of white dwarf binaries, and have yet to be measured as they occupy different frequency ranges and strain amplitudes than what is currently accessible.

The leading model of the evolution of the universe is still the inflationary model which predicts and era of rapid expansion and cooling. This period could have generated a primordial background of gravitational radiation that still permeates space-time. Other models such as the cyclic and Ekpyrotic model do not predict a background of gravitational radiation which would serve as a method of discriminating between the two theories. If a direct measurement were made of a primordial background the standard model of cosmology would be able to narrow down the possible models of the early universe.
Following the derivation from the summary in reference [91] we will derive the form of gravitational radiation sourced by slow-roll inflation as an example. Inflation results in a spectrum of gravitational waves theorized to be generated from the stretching of quantum vacuum fluctuations during the inflationary period. This stretching makes the perturbations larger than the Hubble horizon and then freezes out specific wavelengths during the end of inflation leading to a background of gravitational radiation when the Hubble horizon expands to contain the smaller fluctuations. We begin with the tensor perturbations of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric in the transverse traceless gauge

\[ g_{ij} = a^2(\eta) \left[ 1 - 2 \sum_{r=1}^{\infty} \frac{1}{r!} \phi^{(r)}(r) \right] \delta_{ij} + \sum_{r=1}^{\infty} \frac{1}{r!} h^{(r)}_{ij} \right), \]

(4.1)

Here \( a(\eta) \) is the scale factor in terms of the conformal time \( \eta = \int dt/a(t) \), \( \phi^{(r)} \) is the \( r \)th order perturbation trace imprinted portion of the tensor component and \( h^{(r)}_{ij} \) is the transverse traceless tensor. The equations of motion of the tensor \( h_{ij} \) to first order can be found using the gravitational wave action found in [91] and the references therein

\[ S_T = \frac{M^2_{pl}}{8} \int d^4x a^2(t) \left[ h_{ij} \dot{h}_{ij} - \frac{1}{a^2} (\nabla h_{ij})^2 \right]. \]

(4.2)

with the \( M_{pl} \) the Planck mass, varying with respect to the tensor \( h_{ij} \) the resulting equations are

\[ \nabla^2 h_{ij} - a^2 \ddot{h}_{ij} - 3a \dot{a} \dot{h}_{ij} = 0, \]

(4.3)

with a solution

\[ h_{ij}(\vec{x}, t) = \sum_{i=(+, \times)} h^{(i)}(t) e^{(i)}_{ij}(\vec{x}), \]

(4.4)
where (+, ×) are the gravitational wave polarizations. The resulting power spectrum for the fluctuations in Fourier components $h_{k}^{(i)}$ are

$$P_T(k) = \frac{k^3}{2\pi^2} \sum_i |h_{k}^{(i)}|^2$$

$$= \frac{8}{M_{\text{pl}}^2} \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{aH} \right)^{-2\epsilon}$$

(4.5)

where the sum is over the two polarization states, $H$ is the Hubble constant, $\epsilon$ is a slow-roll inflation parameter. This power spectrum is nearly flat and thus would require a correlated measurement to be directly detected. Future investigations will explore the sensitivity of a network of atom interferometers on the ability to measure stochastic cosmological signals.

### 4.1.1 Model Parameter Estimation

For various models of gravitational waves that we hope to measure with MAGIS-100 or more realistically MAGIS-km, a method of estimating our maximum ability to measure model dependent parameters is provided by Fisher analysis. Given the noise sources and geometric layout of the atom interferometer and given an assumed signal we can model the error on specific model parameters and a priori understand the fundamental limits on our measurement capabilities.

Fisher analysis is a maximum likelihood method for examining the information about an unknown parameter held in a random variable observable. For instance, we take a model for the gravitational waveform which is dependent on some parameters. Given some prior information about the relevant parameters we can form a Fisher information matrix. This matrix encodes the information we know about our given model. The inversion of the Fisher matrix leads to a covariance matrix which contains lower bounds on the accuracy of
measurement, known as the Cramér-Rao bound, for each of the parameters as well as their correlations. In the preliminaries I will layout the mathematics behind the construction and follow that up with its application to MAGIS and how it is used in setting a lower bound on our expected errors in parameter measurements.

The current work focuses on gravitational waves measured by satellite and terrestrial detectors but can be expanded to signals from different sources or generalized to unknown signals. The method is purely for parameter estimation. It follows similar analysis from the LISA space mission and previous studies on error accuracy of post Newtonian gravitational wave-forms by LIGO and others.

We will first examine the statistical formalism only briefly and explore the more physical intuition of this method. An important assumption that needs to be made for the fisher analysis method to hold is that the probability distributions for the parameters must by Gaussian. If not then one would need to fall back on other familiar methods such as Monte-Carlo statistics. The Fisher information is formally denoted

$$ F(\theta) = \int \left( \frac{\partial}{\partial \theta} \log f(x; \theta) \right)^2 f(x; \theta) \, dx. $$  \hspace{1cm} (4.6)\

The function $f(x; \theta)$ is the probability density function. It depends on $x$, the random variable, and $\theta$, the parameter. As can be seen this is nothing more than the second moment or variance of the function in parenthesis.

For a less statistical approach we take the fisher matrix to be defined as an inner product on the vector space of waveform signals.

$$ \Gamma_{ij} = \left( \frac{\partial h^*}{\partial \theta^i} \bigg| \frac{\partial h}{\partial \theta^j} \right). $$  \hspace{1cm} (4.7)
Using this definition we can form the covariance matrix by

$$
\Sigma_{ij} = (\Gamma_{ij})^{-1}.
$$

The MAGIS configuration will measure a signal

$$
h(t) = D_{ij}h_{ij}(t) = h_\times(t)D_{ij}e_{ij}^\times + h_+(t)D_{ij}e_{ij}^+ \\
= h_\times(t)F^\times(t) + h_+(t)F^+(t).
$$

This equation represents the strain that is measured at the detector. To break down the pieces that make up this expression, $h_{\times,+}$ are the gravitational wave polarization amplitudes for the two modes. The $e_{ij}$’s are the gravitational wave polarization tensors for the respective modes. Finally the tensor $D_{ij}$ represents the detector, and is made up of the tensor product of the detector baseline vector $\vec{a}_i(t)$. The resulting functions $F^{\times,+}(t)$ are the antenna functions and are representative of what is observed by the atomic interferometer. In our case we take the signal, $s(t)$, to be $h(t) + n(t)$. This allows us to write our fisher matrix finally as

$$
F_{ij} = 4 \text{Re} \int_{f_n}^{f_o} \frac{\partial \tilde{h}^* \partial \tilde{h}}{S_n(f)} \, df.
$$

This expression is in the Fourier space where tildes represent the Fourier transform of the signal $h(t)$.

Once we have constructed the Fisher information matrix we can invert it and find our covariance matrix. This gives us the ability to make an order of magnitude statement about the expected accuracy of our measurement of parameters such as: sky localization, binary inclination angle, binary polarization angle, coalescence time and phase, and spin.
4.2 Quantum Mechanics Tests

The MAGIS-100 detector will push the limits on atomic superposition length and time scales. Using large momentum transfers to the atoms the separation between the ground and excited state of each atom can be pushed to their physical limits. If these states can be held for a long enough time during either launch configurations or drop configurations we may be able to go beyond testing the classical quantum mechanics superpositions and begin probing other theories such as quantum equivalence principles, gravitational decoherence of quantum superpositions to collapsed states, and quantum time dilation. These tests will be made possible by the extended operational modes utilizing the strontium atoms atomic clock modes and by using dual species launches.

4.2.1 Quantum Equivalence Principle

Atom interferometers make an excellent test bed for measuring the Einstein Equivalence Principle (EEP) as they use extremely sensitive phase shift measurements to sense gravitational accelerations. The EEP can be separated into three essential constituents: the Weak Equivalence Principle (WEP), local Lorentz invariance (LLI), and local position invariance (LPI). Physically the WEP states that all masses in a uniform gravitational field with the same initial conditions will follow the same trajectory i.e. \( m_i = m_g \), where \( m_i \) is the inertial mass and \( m_g \) is the gravitational mass. Local Lorentz invariance comes from special relativity and the statement that a systems dynamics are independent of the velocity of the frame that they are being observed in i.e. rest mass is equivalent to inertial mass \( m_r = m_i \). The final component is the LPI which states results of an experiment are invariant under translations in space and time leading to the statement \( m_r = m_g \). We note from the equation
$E = mc^2$, where $m$ is the rest mass leads to the effective mass of a particle being given by $m = m^{\text{ext}} + E^{\text{int}}/c^2$, where $m^{\text{ext}}$ is the mass when the particle is at it’s minimum internal energy. Any of difference found in the equality of these masses would lead to a violation of EEP.

Atom interferometers also have the ability to hold massive atoms in a superposition which naturally leads to testing extensions to the EEP. These extensions fall under the category of Quantum Einstein Equivalence Principles (QEP) and have been structured and phrased in many different ways. As was discussed in section 1.3.2, the classical equivalence principle should be naturally violated in quantum mechanics. This is because the Schrödinger equation explicitly involves the mass of the quantum particle when determining the trajectory through space-time. Where as in classical mechanics the motion is independent of the mass. In some versions of the QEP one would show that the initial and final states of an experiment with a quantum system remains the same regardless of the internal parameters such as mass, charge, and spin [112]. For this formulation no violation has been found in current atom interferometry experiments [5, 108, 129, 176, 196]. Others have started from the bottom up by creating quantum coordinate systems on which paths through space-time can be properly tracked for non-local wavepackets [197]. A more explicit formalism was developed in [110] where the Hamiltonian of the quantum system separates out the inertial, gravitational, and rest mass. In addition they are promoted to quantum operators. This leads to dynamics of the system where superpositions contain coupled mass terms that may violate the classical EP and can possibly be searched for in experiments comparing atoms with differing masses or differing internal states in free-fall.

From the Zych and Brukner formulation [110] of the QEP the Hamiltonian describing the atom in the weak gravitational field limit with low energy internal states is given by

$$
\hat{H} = mc^2 + \frac{\hat{p}^2}{2m} + m\phi(\hat{q}) - \hat{H}_{\text{int}} \frac{\hat{p}^2}{2m^2c^2} + \frac{\hat{H}_{\text{int}} c^2}{2} \phi(\hat{q}) + \hat{H}_{\text{int}}. 
$$ (4.11)
This Hamiltonian represents the external contributions to the energy, the first three terms, and the contributions of the internal quantized energies labeled by $\hat{H}_{\text{int}}$, and $\hat{p}$ and $\hat{q}$ represent the center of mass momentum and position operators respectively. For testing violations of the QEP they demonstrate comparing between two atoms of different masses and verifying each of the EEP equivalences by using

$$\hat{H}_{\text{exp}} = m_r c^2 + \frac{\hat{p}^2}{2m_i} + m_g \phi(\hat{q}) - \hat{H}_{\text{int},i} \frac{\hat{p}^2}{2m_i^2 c^2} + \frac{\hat{H}_{\text{int},g}}{c^2} \phi(\hat{q}) + \hat{H}_{\text{int},r}. \quad (4.12)$$

In order to measure any difference in the masses a parameterization as matrices is made of the mass operator ratios for testing the WEP, LLI, and LPI. In this form the tests of classical EEP have only examined the diagonal terms where the off diagonals are representatives of the superposition of energy states. These parameter matrices have the form

$$\text{WEP} : \hat{\eta} = \hat{I}_{\text{int}} - \hat{M}_g \hat{M}_i^{-1}$$
$$\text{LLI} : \hat{\alpha} = \hat{I}_{\text{int}} - \hat{H}_{\text{int},i} \hat{H}_{\text{int},r}^{-1}$$
$$\text{LPI} : \hat{\beta} = \hat{I}_{\text{int}} - \hat{H}_{\text{int},g} \hat{H}_{\text{int},r}^{-1}. \quad (4.13)$$

Violations of the QEP can be searched for in atomic interferometers in a series of ways. To look for WEP differences the overall spatial center-of-mass of a quantum system would evolve under free-fall to an entangled state with internal degrees of freedom coupled to external degrees of freedom. This separation would become distinguishable when the spatial difference of the center-of-mass position of the entangled states becomes larger than the coherence length of the atom cloud. For a coherence length larger than this separation the effect would resemble an anomalous spread in the spatial profile of the distribution. Precisely measuring such a spread would be a direct detection of a quantum violation of WEP, however it would be very difficult to achieve and is bounded by $\Delta \hat{\eta} < 8 \times 10^{-5}$ from Bose-Einstein
condensate drop experiments. Testing the LLI and LPI can be done by looking for variations in the visibility or contrast of the interference pattern. This effect would not arise from a classical analog. This modulation of the contrast of the interference pattern is dependent on fluctuations of the parameter $\Delta \hat{\alpha}$.

A first test of this theory was done using an atom interferometer in a gradiometer mode [198] akin to MAGIS-100. In their test they used a gradiometer with baseline $L = 30 \text{ cm}$, $n = 3$ momentum transfer to the atoms and two quantum hyperfine states of rubidium. Their measurement was sensitive to measuring accelerations $a = g \langle s'|\hat{M}_g \hat{M}^{-1}_g|s\rangle$ for different combinations of eigenstates of the internal Hamiltonian $(s', s) = (1, 1), (2, 2), (1, 2)$, the last being a superposition of the ground and excited states. By comparing the differential phase between two atom interferometers in different combinations of these states they established an upper-limit on quantum WEP of $5 \times 10^{-8}$ where this was inferred as added phase noise on the differential acceleration measurement between the reference and superposition configuration.

With the dynamic and configurable setup for MAGIS-100, and a future MAGIS-km, each of these tests for violations of QEP can be explored. For the method of testing for internal and external entangled degrees of freedom a coherent state of strontium can be created at the top most atom source and dropped to either the middle detection region or with enough stability the lowest detection region. The first would provide a 50 m drop and the second a 100 m drop. With the extended fall times alone the estimated upper bound on the atomic cloud broadening can be pushed to $\Delta \hat{\eta} < 8 \times 10^{-6}$ for 50 m and $\Delta \hat{\eta} < 4 \times 10^{-6}$ for 100 m as long as no spurious spreading outside of known effects on atom wavepackets occurs. Following the methodology of the first test of the QEP through LPI (LLI) violations above MAGIS-100 can also operate in a gradiometer mode to make comparisons between different combinations of states. A main difference is that the energy states for strontium would be optically separated where in the Tino experiment the states
were using different hyperfine state splittings. With the use of LMT, advanced atom optics including single photon Bragg transitions, and multiloop sequences that can increase the sensitivity to acceleration measurements that go as $2n k_{eff} g T^2$ many of the noise sources are reduced thus enhancing sensitivity of the phase shift measurement. The excess noise that could test QEP would also be more tightly constrained by up to an order of magnitude.

### 4.2.2 Gravitational Decoherence

Arising from the great interest in understanding how the discrete and probabilistic nature of quantum mechanics coalesces with the continuous and deterministic formalism of gravity many theories have been presented as possible routes towards quantum gravity. Among these are the Karolyhazy model [199, 200], the Diósi model [201, 202], and the Penrose model [203, 204]. They each approach the problem from different starting points. The K-model begins with examining stochastic corrections to quantum states which take into account the uncertainty relation even for microscopic and macroscopic objects. Similar to the K-model, Diósi introduces a stochastic nature into the Schrödinger equation and the gravitational potential as measured by a quantum sensor. Finally, Penrose introduces a starting point of the Shrödinger equation by adding in gravitation. This make the equation inherently non-linear with a self gravitation force of the quantum particle dependent on the uncertainty of its center of mass position in space-time\(^1\).

The primary assumptions of the K-model are that microscopic massive objects also obey the uncertainty relation $\delta x \delta v \geq \hbar / 2m$. Because of the stochastic correction to the quantum states space-time fuzziness from this uncertainty relation constrains the spread of center of mass wavefunctions. For some characteristic length scale coherent components of the

\(^1\)A full review of these theories can be found in reference [205]
wavefunction are destroyed leading to the observed collapsed wavefunction. To derive the minimum uncertainty length one starts with the uncertainty in the velocity of a wavepacket moving a long a worldline $s = cT$. The corresponding velocity spread is $\Delta v = \hbar/2m\Delta x_0$ for a mass $m$. After a travel time $T$ the uncertainty in the position is then $\Delta x = \Delta vT = \frac{\hbar}{2mc\Delta x_0}cT$. Bounding the maximum position uncertainty by the gravitational radius $\Delta x \approx \frac{Gm}{c^2}$ giving the minimum length scale

$$(\Delta s)^2 = \left(\frac{G\hbar}{2c^3}\right)^{2/3} s^{2/3}. \quad (4.14)$$

Equipped with this uncertainty of spacetime segments he goes on to assign a different metric for each chunk and the proper path through the overall spacetime – which is made up of the set of all sub metric spaces instantaneously – is the average over the length $s = \bar{s}$. For a family of wavefunctions $\Psi = \{\psi_\beta\}$ corresponding to each metric $g^{\beta}_{\mu\nu}$ propagating through this space the fuzziness of spacetime acts as a perturbation on the dynamics of the wavefunction. A small separation $a = |\vec{x}_1 - \vec{x}_2|$ between the wavefunctions $\psi_\beta$ leads to a phase difference between the wavefunctions. For a critical value $a_c$ the wavefunctions are completely out of phase and separate leading to coherence cells of this size. These cells are characterized by

$$a_c = \frac{\hbar^2}{Gm^3} \approx \left(\frac{L}{L_p}\right)^2 L; \quad L = \frac{\hbar}{mc}. \quad (4.15)$$

and a critical reduction time of

$$\tau_c \approx \frac{ma_c^2}{\hbar}. \quad (4.16)$$

As an example these values for a strontium atom in MAGIS-100 would be

$$a_c \approx 10^{16} \text{ m}, \quad \tau_c \approx 10^{12} \text{ s}. \quad (4.17)$$

This is clearly a very large distance the wavefunction can travel before decoherence occurs and a long period of time. A rough estimate for the center of mass critical length for
stochastic reduction can be given assuming an atom cloud of strontium with \( n = 10^7 \) atoms and a cloud radius of \( r = 2 \times 10^{-6} \text{ m} \) using the equation for a cubic volume

\[
a_c \approx \left( \frac{\hbar^2}{Gm^3} \right)^{1/3} (r)^{2/3},
\]

This leads to a critical length of \( a_c \approx 60 \text{ m} \) and a critical time \( \tau_c \approx 10^{12} \text{ s} \). The K-model also establishes the values of interest for investigating the transition region between microscopic and macroscopic behavior as

\[
a^{\text{tr}} \approx 10^{-7} \text{ m}, \quad \tau^{\text{tr}} \approx 10^3 \text{ s}, \quad m^{\text{tr}} \approx 10^{-17} \text{ kg}.
\]

This range is interesting because the terrestrial MAGIS detectors would make possible testing grounds for gravitational decoherence by separating superposition wavepackets to the maximum baseline length.

In the model formulated by Diósi the formulation begins from assuming a stochastic variation entering through the gravitational potential leading to a slightly different uncertainty relation [201]

\[
(\delta \tilde{g}(\vec{r}, t))^2 \geq \frac{\hbar G}{VT}.
\]

Where the stochastic gravitation is \( \delta \tilde{g} \), \( V \) is the volume averaged over by a quantum measurement, and \( T \) is the time of the measurement. The main equation he derived for the time dependence of the density operator \( \hat{\rho} = \langle \psi(t)\psi^\dagger(t) \rangle \), where \( \psi \) is a stochastic variable is

\[
\langle x|\hat{\rho}|x' \rangle = -\frac{i}{\hbar} \langle x|\left[ \hat{H}_0, \hat{\rho}(t) \right]|x' \rangle - (\tau_d(x, x'))^{-1} \langle x|\hat{\rho}(t)|x' \rangle.
\]

\[
(4.21)
\]
In this equation $\hat{H}_0$ is the Hamiltonian of the system and $\tau_d$ is the damping time dependent on the mass density of the system. From this main equation a critical length can be derived where the second term destroys coherence between the two positions.

$$l_{\text{crit}} \sim \begin{cases} \left( \frac{\hbar^2}{Gm^3} \right)^{1/4} r^{3/4} & \text{if } rm^3 \gg \frac{\hbar^2}{G}, \\ \left( \frac{\hbar^2}{Gm^3} \right)^{1/2} r^{1/2} & \text{if } rm^3 \ll \frac{\hbar^2}{G} \end{cases}$$ (4.22)

Instead of beginning with an uncertainty principle as the K-model and the Diósi model do, Penrose begins by adding in self gravitation as the bounding mechanism of the particle’s wavefunction. The gravitational field can then be introduced into the Schrödinger equation through a potential coupling to the wavefunction while taking the wavefunction to physically represent the mass density of the particle as

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + m\Phi \Psi; \quad \nabla^2 \Phi = 4\pi Gm|\Psi|^2.$$ (4.23)

This is the nonlinear equation known as the Schrödinger-Newton equation [203, 206]. Numerical studies have shown that the ground state solution to this equation goes as $E \sim -\frac{1}{8} \frac{G^2 m^5}{\hbar^2}$ and the width of the mass distribution is $a_0 = \frac{2\hbar^2}{Gm^3}$. This matches up with the results found in the other two models examined above. With a dimensionless coupling constant

$$K = \frac{2Gm^3}{\hbar^2} l = 2 \left( \frac{l}{L_p} \right) \left( \frac{m}{M_p} \right)^3.$$ (4.24)

From this coupling constant the quantum to classical transition is derived by setting the constant to unity. Observing a quantum to classical transition in this framework would require a mass $m = 10^{-20}$ kg over a coherence cell of $a = 100$ m. This is just outside of the
mass range for strontium but may be possible with either heavier elements in the future or larger baseline distances.

4.2.3 Quantum Time Dilation

Atom interferometers have been shown to have extreme sensitivities towards testing general and special relativistic effects including time dilation. The effect of time dilation on interference patterns in atom interferometers has been studied but not yet experimentally verified [207]. In addition to these effects on the overall contrast and phase shift of the atom interferometer new theories have been in development for testing if an atomic clock prepared in a superposition state experiences time in the same way an atomic clock in a coherent state does [208]. The strontium atoms we will be using in MAGIS have a long lived clock mode that may be able to put limits on these theories.

The primary effect of time dilation in an atom interferometer is to entangle atom’s internal degrees of freedom with external degrees of freedom such as center of mass position. In this way the effects that arise such as loss of coherence are purely quantum mechanical. Previous studies found the visibility or contrast to depend on the time dilation for an initial superposition to be

\[ C = \left| \cos \left( \frac{\Delta E}{2\hbar} \Delta \tau \right) \right|, \]

(4.25)

where \( \Delta E \) is the energy difference between the two states of the system and \( \Delta \tau \) is the proper time difference between the two trajectories. In the case of a vertical drop experiment such as MAGIS-100 the proper time difference can be expanded dependent on the position of the atoms in the Earth’s gravitational potential as \( \Delta \tau = (\phi(x_2) - \phi(x_1))t/c^2 \). This would then be measurable as an overall modulation to the contrast of the atom interferometer.
Using the clock modes of the atom interferometer may prove to be interesting as well. As proposed in [208] two atomic clocks moving relativistically with one in a superposition of momentum states measuring a proper time $\tau_1$ and the other in a coherent state measuring $\tau_2$ can be compared. The output leads to a Doppler shift of the required resonance frequency for the clock transition. The proper time as measured by the superposition clock with respect to the single state clock will then be shifted. This shift can be quantified by $\tau_1 = \gamma_{\text{eff}}^{-1}\tau_2$, with $\gamma_{\text{eff}}^{-1} = (1 - K_c - K_q)$ and from reference [208]

$$K_q = \frac{\sin 2\theta \cos \phi}{8m^2c^2N} e^{-(\bar{p}' - \bar{p})^2/4\Delta^2} \left[ 2(\bar{p}'^2 - \bar{p}^2) \cos 2\theta - (\bar{p}' - \bar{p})^2 \right]. \quad (4.26)$$

This time dilation is affected by the classical contribution $K_c$ and the quantum contribution $K_q$. The quantum contribution is dependent on the average momentum difference, $\bar{p}' - \bar{p}$, between the two superposition states of the atomic clock, $\Delta$ is the wavepacket width, and $N$ is the normalization constant. The size of this effect for MAGIS-100 can be estimated to the order of $K_q \sim 10^{-16}$ given a velocity difference 10 m/s.
CHAPTER 5
EXPERIMENTAL WORK ON EXPERIMENT PROTOTYPES

5.1 Test Laser System

A test laser system was built at Northwestern for research and development purposes for the MAGIS-100 laser setup. The setup utilizes 689 nm light which is used for the two photon Bragg transitions on strontium and for various configurations of atom interferometry. Before testing coherent combination of the light from two lasers for more precise control the first point of instability comes from the laser drift stabilization over long periods of time. This is controlled by locking the laser to a cavity with a precise stability. In the actual experiment a highly tuned frequency comb will be used to manage the locking of multiple wavelengths of light and provide confident stability over long times and with frequency fluctuations less than 10 Hz.

The primary method that is used to achieve this is the Pound-Drever-Hall locking method [209, 210]. This relies upon measuring the power of the light reflected off the cavity. We cannot use the reflected light directly because it disappears at resonance so we need to generate an additional signal that can indirectly give us information about which side of the resonance the laser beam frequency is on and inform how we should feedback the modulation of the laser to keep it locked to the resonant mode of the cavity. The reflection coefficient of the laser beam is given by

\[ F(\omega) = \frac{E_{\text{ref}}}{E_{\text{inc}}} = \frac{r(\exp(i\omega/\Delta \nu) - 1)}{1 - r^2 \exp(i\omega/\Delta \nu)}, \]  \hspace{1cm} (5.1)
where $r$ is the cavity reflection coefficient and $\Delta \nu = c/2L$ is the free spectral range of the cavity.

We generate an error signal by modulating the laser frequency, by $\Omega$, via physical modulation using a Piezo motor inside the laser beam controller or by varying the current of the laser. This generates side-bands on the reflected light of the laser beam that can be used to mix with a reference signal to read out the phase. This mixed signal is what we call the error signal. We then use the error signal to set a threshold for the servo feedback loop to modulate the laser to remain on resonance. The reflected power at the photo-detector is

$$P_{\text{ref}} = |E_{\text{ref}}|^2,$$

$$= P_c |F(\omega)|^2 + P_s \left(|F(\omega + \Omega)|^2 + |F(\omega - \Omega)|^2\right)$$

$$+ 2 \sqrt{P_c P_s} \text{Re}[F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)] \cos(\Omega t)$$

$$+ 2 \sqrt{P_c P_s} \text{Im}[F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)] \sin(\Omega t).$$

For large modulation frequencies, $\Omega$, after mixing the photo-detector signal with the reference oscillator signal the DC component, or error signal, is

$$\varepsilon = 2 \sqrt{P_c P_s} \text{Im}[F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)].$$

$$\varepsilon = D \delta f; \quad D = \frac{8 \sqrt{P_c P_s}}{\delta \nu}.$$ 

(5.4)

Here $\delta \nu$ is the linewidth of the cavity.

**Fabry-Pérot Ringdown Measurement**

Our laser beam stability is determined by the characteristics of the cavity. To properly model how well we can stabilize the laser it is important to measure the linewidth of the
cavity. Finding the linewidth requires us to measure the decay time, or ringdown, of the modes of the cavity when light entering the cavity is turned off. With the linewidth we can then determine other quantities such as the finesse $F_c = \Delta v_{fr} / \Delta v_c$, which is the parameter characterizing the quality of the cavity as the ratio of the free-spectral range of the cavity and the full-width-at-half-maximum bandwidth. This is an important characterization as the manufacturer only provides an expected range of values for the finesse.

First attempts at measuring the ringdown of the cavity were done with a locked laser beam and a photodetector imaging the light passing through the cavity. The laser was then unlocked and allowed to drift and the decay of the cavity modes was read out through the photodetector. In our setup we chose to use an acousto-optic modulator (AOM) in order to control the switching of the light on and off. This was done to keep the cavity from having random mode excitations caused by unlocking the laser beam which were apparent in the shape of the cavity decay signal. We then measured the decay time constant by fitting the curves to an exponential decay.

To make the analysis more robust we also measured the time decay of the amplified photodetector used for the ringdown measurements as a reference. This allowed us to deconvolve the measured signal through the filtering of the inherent decay of the detector’s circuitry. Using the fact that in Fourier space the convolution of two signals can be written as

$$\mathcal{F}[f(t) * g(t)] = \mathcal{F}[f(t)] \cdot \mathcal{F}[g(t)],$$  \hspace{1cm} (5.5)

where $*$ denotes the convolution operator. Here, $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the forward and inverse Fourier transforms respectively with appropriate normalizations. If we assume our measured signal is the convolution $f(t) * g(t)$ we can extract the signal of the cavity ringdown by computing

$$f(t) = \mathcal{F}^{-1} \left[ \frac{\mathcal{F}[f(t) * g(t)]}{\mathcal{F}[g(t)]} \right].$$  \hspace{1cm} (5.6)
In addition to the previous method we fit the measured signal to a convolution of two general exponential decay functions substituting fitting parameters obtained from the reference signal \( g(t) \).

\[
\begin{align*}
  f(t) &= A \exp\left( -\frac{(t - t_1)}{\tau_1} \right) - f_0, \\
  g(t) &= B \exp\left( -\frac{(t - t_2)}{\tau_2} \right) - g_0, \\
  f(t) \ast g(t) &= -\frac{AB\tau_1\tau_2 \exp\left( -\frac{t}{\tau_1} + \frac{t_1}{\tau_1} + \frac{t_2}{\tau_2} \right) \left( \exp\left( t \left( \frac{1}{\tau_1} - \frac{1}{\tau_2} \right) \right) - 1 \right)}{\tau_1 - \tau_2} \\
  &\quad + Ag_0\tau_1 \exp\left( \frac{t_1}{\tau_1} \right) \left( \exp\left( -\frac{t}{\tau_1} \right) - 1 \right) \\
  &\quad - Bf_0\tau_2 \exp\left( \frac{t_2 - t}{\tau_2} \right) \left( \exp\left( \frac{t}{\tau_2} \right) - 1 \right) + f_0g_0t. 
\end{align*}
\]  

The combination of these two methods of measuring the decay time constant give us a high level of confidence in the retrieved value of \( \tau_1 \).

For our measurements we used an amplified photodetector to measure the time traces of the cavity ringdown on an oscilloscope. The traces were taken using a gain of 20 dB on the photodetector and are shown in figures 5.1, 5.2, 5.4 and 5.7.
Figure 5.1: Plot of time trace of cavity mode decay signal. A photodetector at 20 dB gain was used. The best fit time decay constant was found to be $\tau_1 = 1.27 \mu$s.

Figure 5.2: Plot of time trace of photodetector decay signal. Used as a reference for the filtering of the cavity ringdown signal. A photodetector at 20 dB gain was used. The best fit time decay constant was found to be $\tau_2 = 0.17 \mu$s.
Figure 5.3: Plot of time trace of cavity mode decay signal fit with Eq [5.7]. A photodetector at 20 dB gain was used. The best fit time decay constant was found to be $\tau_1 = 1.23 \mu s$.

Figure 5.4: Plot of reconstructed time trace of cavity mode decay signal via deconvolution method shown in Eq [5.6]. A photodetector at 20 dB gain was used. The best fit time decay constant was found to be $\tau_1 = 1.26 \mu s$. 
**Fiber Couple 689 nm Laser Light**

In addition to helping with the characterization of the cavity used for locking the 689 nm laser I setup the fiber couple that will deliver the main laser light for the atom interferometry sequences in the coherent combination test system. The setup consists of an optical isolator, a $\lambda/2$ waveplate, a beamsplitter and a telescope to match the iris of the fiber couple for a power efficiency of 80%. Figure 5.5 shows the block diagram and an image of the optics on the table.

![Block diagram of fiber couple setup for 689 nm laser for main atom interferometry sequences](image)

Figure 5.5: Block diagram of fiber couple setup for 689 nm laser for main atom interferometry sequences

### 5.2 Atom Shuttling and Trapping

In the MAGIS-100 atom sources there is a need to be able to trap the ultra-cold atom clouds and then transition smoothly to shuttling them over to the connection node where the atom source and vacuum tube meet. Here they are transitioned to the optical lattice that will be used to launch the atoms. To simplify the design and system resources a spatial
light modulator will be used to perform the tasks of dipole trapping, evaporative cooling, matter-wave lensing, and shuttling to the vacuum tube.

To characterize the 100 W 1064 nm laser beam that will be used with the SLM measurements and fits where made on the Gaussian profile of the laser intensity at different wattages. We also tested the initial optical set-ups to be used with the SLM. Various phase images for the SLM were generated by the Gerchberg-Saxton algorithm (GSA) and tested for their accuracy and stability of recreating the reverse engineered intensity profile.

A pseudo-code of the GSA can be found below. Figure 5.6 shows the initial intensity profile and the target profile of the light that was generated in Mathematica and is fed into the algorithm. The resulting phase image to be displayed on the SLM screen to create the transition between these two profiles is shown in figure 5.7. The method works well however there are some side effects that required further study and understanding to optimize the phase images retrieved. For example in the process of building the required phase profile after each iteration there is the possibility of creating discontinuities in the phase between 0 and $2\pi$ which lead to divergence points in the phase profile seen as forks. These then act as sources to creating vorticies in the intensity profile that appear as speckles in the images rather than a smooth profile. Methods for mitigating this speckle includes adding an extra step into the iteration process by which these points are found during each pass and are pushed out of the phase image or swept away so the phase image can be built up cleanly.
Gerchberg-Saxton Pseudo-code:

Definitions:
FT, IFT – Fourier and Inverse Fourier Transforms

begin
  initialize source and target
  \( A = \text{IFT}(\text{target}) \)
  while error conditions not met/iterations not exceeded do
    \( B = \text{Amplitude}(\text{source}) \ast \text{Exp}(i \ast \text{Phase}(A)) \)
    \( C = \text{FT}(A) \)
    \( D = \text{Amplitude}(\text{target}) \ast \text{Exp}(i \ast \text{Phase}(B)) \)
    \( A = \text{IFT}(C) \)
  end
  return \text{Phase}(A)
end

Figure 5.6: Simulated initial Gaussian intensity profile of incoming laser light on the SLM (left). Target ring intensity profile after reflection from the SLM screen (right). The axes show the number of pixels of the SLM screen.
Figure 5.7: Phase image in bitmap format to display on the SLM screen. Imprints a phase on the incoming laser beam upon reflection by altering the LCD index of refraction.
CHAPTER 6
FUTURE PROSPECTS

The future of the MAGIS program will learn from MAGIS-100 leading to even longer terrestrial baselines and eventually leading to space. In order to achieve these goals many efforts and technical requirements remain to be tested and optimized while being experimentally implemented. This opens up a large amount of possible routes of exploration for researchers with interest in design and theory. As a path finder experiment MAGIS-100 aims to create connections on the international stage and inspire collaboration and networks across the planet in the same vain as the AION program which MAGIS-100 is complementary too and will work in collaboration with to develop atom interferometry as an international detector platform.

6.1 Advanced Atom Interferometry

To reach the next steps some effort and thought should be put into more methods of mitigation for the systematics that scale with the baseline such as those I explored in section 3.1. For larger experiments there exist some interesting opportunities for mitigation in addition to the methods outlined there.

Since laser wavefronts expand and are modified the farther they travel after reflecting off optical elements some possible methods around this include rotating laser beams and dithering. The wavefront spreads out from the center of the beamwaist dependent on the position \((x, y, z)\) of the beam along the vacuum pipe. If a rotation was applied to the laser
beam before entering the interferometry region then the phase wavefront imprinted could be averaged over allowing for a uniform wavefront applied at each light-atom interaction point thus compensating noise by common-mode subtraction. Another possibility that is commonly used in audio and video processing is dithering the beam. Dithering is the process of adding noise to the laser beam intensity. Adding noise at a level that would smear the wavefront at each pulse may lead to suppression by again making the interactions uniform with depth. A thorough investigation of the feasibility of these methods is of great interest.

As the strain sensitivity of extreme baseline atom interferometers gets lower GGN will inevitably being limiting sensitivity to small signals. The active strategies outlined in 3.6.4 can be applied however a passive method may also provide a layer of suppression. In particular using the fact that the surface acts as a medium that carries the seismic energy and implementing well established methods of seismic isolation in engineering [211–213] we could target an area, a mining shaft, where seismic waves cannot propagate or exist. Further analysis and numerical analysis of the amount of suppression in Rayleigh waves could be used to infer the effect this would have on GGN sourced primarily by seismic Rayleigh waves.

Increasing the scope of the science case for the MAGIS program includes taking the next steps towards MAGIS-km and MAGIS-space but also by incorporating correlated measurements with other atom interferometers across the planet.

Many of the science signals that MAGIS-100 and future MAGIS detectors are searching for act as cyclic signals that oscillate over some large time period. To build statistics over them the detectors located on the surface of the Earth must accumulate multiple cycles of the incoming wave signal – whether it be an ultra-light dark matter field or a gravitational wave – to be able to extract the phase shift with great precision. One way to enhance these measurements is by incorporating multiple atom interferometers at different locations on the Earth to give not only more time resolution but also spatial information about the signals.
Correlating these measurements can lead to an enhancement of sensitivity as well as allowing for detection of signals with a stochastic nature such as fluctuations from the early universe.

6.2 Open Questions

Much technical work and testing with the MAGIS-100 detector is still needed to know exactly how effective the proposed methods will be for MAGIS-100 and beyond. The primary questions for noise sources lie in the ability to control the laser beam’s wavefront and pointing stability at such large distances using \textit{in situ} calibrations as well as the combination of tip-tilt mirrors and multi-loop atom interferometry sequences for Coriolis effect mitigation. More numerical analysis on suppression and filtering of GGN is also needed to solidify our confidence in being able to measure and compensate at a level allowing for sensitive terrestrial strain measurements. In terms of the science goals there is great interest in exploring the mid-band because there is always the chance that a spurious signal may be detected which would further inspire networked searches. A full study of all possible signals in this frequency range would also be of great interest. This could be informed by studies of the sensitivity and reach of correlated measurements from MAGIS and AION.
CHAPTER 7
CONCLUSIONS

The MAGIS-100 detector and beyond is an exciting new avenue to test theories of the dark sector and the early universe using quantum mechanics. Searching the unexplored areas of frequency of the mid-band will hopefully lead to setting new bounds in the dark sector and show the capabilities of pushing quantum mechanics to its known limits and further. We also hope that new physics may make itself known through this channel of quantum sensing. With the full study of systematics we have worked on we have quantified the requirements on the different subsystems of the experiment and set the path for analyzing noise sources that will prove interesting in future large baseline atom interferometers. I have also investigated some more esoteric theories that the MAGIS program is capable of testing and presented my experimental efforts towards working on test systems guiding research and development of the MAGIS-100 laser system and the possible atom source shuttling and trapping techniques. The future prospects and results from MAGIS-100 are vast and will hopefully guide the international atom interferometry community to build the beginnings of a cohesive network and search platform.
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