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The Standard Model Precision Parameters at 200 GeV

Zamiul Alam
zamiul.alam@gmail.com

ABSTRACT

THE STANDARD MODEL PRECISION PARAMETERS AT 200 GEV

Zamiul Alam, M.S.
Department of Physics
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Stephen P. Martin, PhD, Director

The Standard Model can be defined quantitatively by running parameters in a mass-independent renormalization scheme at a fixed reference scale. We provide a set of simple interpolation formulas that give the fundamental Lagrangian parameters in the $\overline{\text{MS}}$ scheme at a renormalization scale of 200 GeV, safely above the top-quark mass and suitable for matching to candidate new physics models at very high mass scales using renormalization group equations. These interpolation formulas take as inputs the on-shell experimental quantities and use the best available calculations in the pure $\overline{\text{MS}}$ scheme. They also serve as an accounting of the parametric uncertainties for the short-distance Standard Model Lagrangian. We also include an interpolating formula for the W boson mass. A step-by-step calculation of the one-loop contributions to the mass of the Higgs boson is included.

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ZAMIUL ALAM

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DEDICATION

To Frenemy, who has been by my side through most things Kafkaesque.

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CHAPTER 1

INTRODUCTION

The Standard Model of fundamental particle physics has reached a high level of experimental maturity with the 2012 discovery of the Higgs boson and the increasingly accurate measurements of its mass, production modes, and decays. Meanwhile, the endeavors of the Large Hadron Collider (LHC) at the high-energy frontier have not revealed any substantial and lasting deviations that would compel extensions of the Standard Model, despite many motivations for such new physics.

Given this state of affairs, it is useful to summarize as accurately as possible our quantitative knowledge of the Standard Model in terms of the Lagrangian parameters that define the theory. These defining parameters can then be matched to a larger set of parameters in candidate new physics theories characterized by large mass scales. It is convenient to use the $\overline{\text{MS}}$ renormalization scheme [1, 2] based on dimensional regularization [3, 4, 5, 6, 7] for this purpose.

We have used the public computer code library, `SMDR` (Standard Model in Dimensional Regularization) which relates the renormalizable Lagrangian parameters that define the theory to the observables and on-shell quantities [8]. `SMDR` performs state-of-the-art calculations with amazing accuracy but some of the computations take some time even on well-equipped machines because of their complexity. We are presenting a calculation of the Higgs squared pole mass M_h at one-loop order by following simple Feynman rules to show the analytical approach to these calculations and to give an idea of how long just the one-loop calculations can get. Although `SMDR` incorporates a numerical approach instead of an analytical one, it does calculations at two-loops and beyond and one can easily comprehend the extensivity of such tasks. So, we have used `SMDR` to create multiple sets of data (which took long computational times) and then acquired interpolation formulas by taking fits of the data. These formulas enable almost instantaneous reproduction of some of the results from `SMDR`. The results of this work were recently published in the form of a paper [9].

We are also presenting very short reviews of some of the fundamental theoretical concepts involved in this work, such as renormalization, dimensional regularization and the Passarino-Veltman integrals.

CHAPTER 2

REVIEW OF THE STANDARD MODEL

The Standard Model of particle physics came into existence out of a a desire for the unification of the known four forces and a classification of the known elementary particles. Towards the beginning of the 20th century, with the gradual discovery of new fundamental particles, it became increasingly difficult for physicists to categorize them and explain their existence. As the world learned to come to terms with the bizarreness of quantum mechanics [10], a newer quantum theory of fields was developed which changed our approach to looking at particles and interactions. And finally, through the works of many great physicists, the current Standard Model of particle physics took form [11], [12].

2.1 Particles and Interactions

As physicists realized that 1-particle quantum mechanics starts to break down in the relativistic limit [13] [14], it became necessary to formulate the quantum theory of fields where particles are treated as fluctuations of all-pervading fields. In other words, particles are nothing but 'quantizations' of the quantum fields. By utilizing the tools already developed in classical field theory, the quantum fields can be described by Lagrangians. And the interactions between particles can be described by interaction terms in the Lagrangian.

2.2 Gauge Symmetries

The most successful of the field theories is perhaps Quantum Electrodynamics. If we have a fermionic field Ψ (the complex conjugate would be $\overline{\Psi}$) for a particle with mass m, then the Lagrangian (actually Lagrangian density) [15] is expressed as

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\overline{\Psi}\not{D}\Psi - m\overline{\Psi}\Psi \tag{2.1}$$

where we have the covariant derivative

$$D_\mu = \partial_\mu + iQeA_\mu \tag{2.2}$$

for a fermion with charge Qe, and e turns out to be the charge of the electron. The field strength in terms of the electromagnetic four-potential A^μ is,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.3}$$

This Lagrangian is invariant under both the global gauge transformation,

$$\Psi \to e^{i\theta}\Psi \tag{2.4}$$

and, the local gauge transformation,

$$\Psi \to e^{i\theta(x)}\Psi \tag{2.5}$$

which requires,

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \theta \tag{2.6}$$

where $\theta(x)$ is any function of spacetime, called a gauge parameter. This transformation may be thought of as a multiplication of Ψ by a unitary 1×1 matrix:

$$\Psi \rightarrow U\Psi \tag{2.7}$$

where

$$U^\dagger U = 1 \tag{2.8}$$

The group of all such matrices is called $U(1)$ and hence the underlying symmetry is called a '$U(1)$ gauge symmetry'. This describes scalar fields and electromagnetic interactions where photons are the mediators. If the determinant of $U(1)$ is 1, then it can be called a special unitary matrix $SU(1)$.

Similarly, the Lagrangian for a theory of Dirac fermions and gauge bosons transforming under a non-Abelian gauge group can be constructed which is called Yang-Mills theory. The Lagrangian [16] is,

$$\mathcal{L}_{Yang-Mills} = \mathcal{L}_{gauge} + \mathcal{L}_{fermions} \tag{2.9}$$

where,

$$\begin{aligned} \mathcal{L}_{gauge} &= -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} \\ \mathcal{L}_{fermions} &= i\overline{\Psi}^i\gamma^\mu D_\mu \Psi_i - m\overline{\Psi}^i\Psi_i \end{aligned} \tag{2.10}$$

The covariant derivative is,

$$D_\mu \Psi_i = \partial_\mu \Psi_i + igA_\mu^a T_i^{aj} \Psi_j, \tag{2.11}$$

and the field transforms as,

$$D_\mu \Psi_i \rightarrow (1 + i\epsilon^a T^a)_i^{\ j} D_\mu \Psi_j, \tag{2.12}$$

where ϵ^a is an infinitesimal gauge transformation and T^a are Hermitian matrices. The gauge transformations are again in the form of eq. (2.7) where U is $n \times n$ unitary matrix with determinant 1. And the underlying symmetry is called a '$SU(n)$ gauge symmetry'. Although the original idea behind the Yang-Mills theory was not useful at first, it later helped in the formulation of the theory of weak interactions in the form of an isospin-hypercharge $SU(2)_L \times U(1)_Y$ symmetry and the theory of strong interactions in the form of a color $SU(3)_c$ symmetry, where the subscripts L, Y, and c stand for weak isospin, weak hypercharge and color respectively.

2.3 Particles in the Standard Model

The Standard Model Lagrangian is invariant under global and local transformations from the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group and different gauge bosons act as mediators for each of the individual gauge symmetries [17], [18], [19], [20]. Just as photons act as mediators in the theory of electrodynamics, the W^+, W^- and Z bosons act as mediators in the theory of weak interactions and the gluon acts as the mediator in the theory of strong interactions. We also have the Higgs boson, the theory of which explains why some particles are massive and others are not. The Higgs boson has zero spin and the rest of the bosons have spin 1.

The elementary particles described by the Standard Model include three generations of quarks and three generations of leptons, all having spin 1/2. The quarks come in three colors- red, blue, and green which is just a convention and does not refer to physical colors. All the fermions (which include the quarks and leptons) have their anti-particles with opposite charges. For instance, the up and down quarks have their counterparts, anti-up and anti-down; the electron and muon have their counterparts, the positron and anti-muon and so on. All the bosons except the W^+ and W^- act as their own antiparticles. Table 2.1 shows all the particles (the anti-particles are not being shown for conciseness) currently part of the Standard Model and their respective spins. We are also including the massless Goldstone bosons in the table since they play a part in our calculations in Chapter 3.

Table 2.1: **Particles in the Standard Model with Their Symbols, Charge and Spin**

	Particles	Symbol	Charge	Spin
Quarks	up	u	2/3	$\frac{1}{2}$
	charm	c		
	top	t		
	down	d	$-1/3$	
	strange	s		
	bottom	b		
Leptons	electron	e	-1	$\frac{1}{2}$
	muon	μ		
	tau	τ		
	electron neutrino	ν_e	0	
	muon neutrino	ν_μ		
	tau neutrino	ν_τ		
Bosons	gluon	g	0	1
	photon	γ		
	Z boson	Z		
	W boson	$W^\pm$	±1	
Scalar Bosons	Higgs	h	0	0
	Neutral Goldstone	G^0		
	Charged Goldstone	$G^\pm$	±1	

2.4 Spontaneous Symmetry Breaking and the Higgs Mechanism

The situation in which the laws of physics are invariant under some symmetry transformations, but the vacuum state is not, is called spontaneous symmetry breaking [16]. We again have two versions of this phenomenon, global and local. For instance, if we have a complex scalar field $\varphi(x)$ with a Lagrangian density,

$$\mathcal{L} = \partial^\mu \varphi^* \partial_\mu \varphi - V(\varphi, \varphi^*) \tag{2.13}$$

and the potential has a form,

$$V(\varphi, \varphi^*) = m^2 \varphi^* \varphi + \lambda(\varphi^* \varphi)^2, \tag{2.14}$$

where m^2 and λ are parameters of the theory, then the Lagrangian is invariant under a global $U(1)$ transformation like the one in eq. 2.4. If $m^2 < 0$ and $\lambda > 0$ (other variations do not lead to viable theories), then the potential $V(\varphi, \varphi^*)$ has a "Mexican hat" shape as shown in Fig. 2.1, with a local maximum at $\varphi = 0$, and a degenerate set of minima with,

$$|\varphi_{min}|^2 = \frac{v^2}{2}, \tag{2.15}$$

where, $v = \sqrt{-m^2/\lambda}$ is a measurable property of the vacuum state, known as the vacuum expectation value, or VEV of $\varphi(x)$. The vacuum state is not invariant under the global transformation and so the symmetry is spontaneously broken.

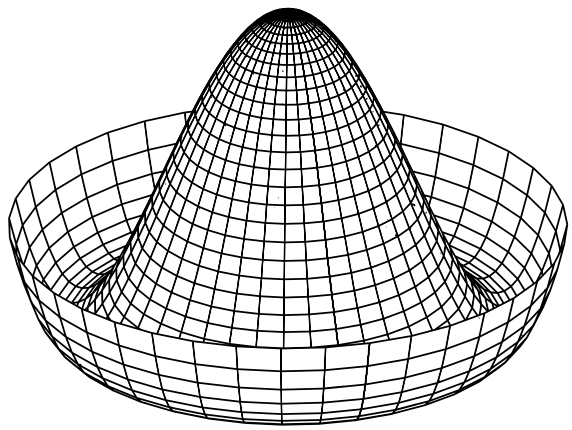


Figure 2.1: Plot of a potential, $V = -5|\varphi^2| + |\varphi^4|$ showing the "Mexican hat" shape [21]

The problem is solved by writing the phase of φ as a massless scalar field $G(x)$ which is associated with an unphysical particle called the Nambu-Goldstone boson [22], [23]. The Standard Model Higgs mechanism is an example of local symmetry breaking where we have a Lagrangian remaining invariant under a transformation like that in eq. (2.5). It can be said that the vector bosons attain mass by "eating" up the Nambu-Goldstone boson and this is known as the Higgs mechanism [24], [25].

2.5 Renormalization

One of the discrepancies that initially clouded the success of quantum field theory or quantum electrodynamics in particular was the appearance of divergences while calculating contributions beyond the tree-level diagrams. One particular example is the calculation of the electron self-energy [26] that gave an infinite value, which was absurd. The development of quantum electrodynamics was stalled for almost two decades until the combined efforts of many great physicists developed the tools of renormalization [27]. Renormalization is a collection of techniques that can be used to deal with the divergences or infinities that arise in the calculations. One general approach is to introduce a very high cut-off mass scale, M.

Other general approaches include the use of Feynman parameters and dimensional regularization which will be discussed in the following section. Gauge theories with dimensionless couplings are often more useful than theories with dimensionful couplings. If the mass dimension of $[g]$ is negative, then the perturbative expansion breaks down and this is related to the non-renormalizability of the theories.

2.6 Dimensional Regularization and the $\overline{\mathrm{MS}}$ Scheme

To discuss briefly about dimensional regularization and the $\overline{\mathrm{MS}}$ scheme, we follow the conventions of [28] and consider a φ^3 theory with a lagrangian of the form,

$$\mathcal{L} = -\frac{1}{2}Z_\varphi \partial^\mu \varphi \partial_\mu \varphi - \frac{1}{2}Z_m m^2 \varphi^2 + \frac{1}{6}Z_g g \varphi^3 + Y\varphi \tag{2.16}$$

Here, the parameter m is the mass of the particle and g is the coupling. The remaining parameters can be determined from the values of m, g and the normalization conditions. In fact, the Lagrangian in 2.16 can be re-written as,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \tag{2.17}$$

where,

$$\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{2}\partial^\mu \varphi \partial_\mu \varphi - \frac{1}{2}m^2\varphi^2 \\
\mathcal{L}_1 &= \frac{1}{6}Z_g g \varphi^3 + \mathcal{L}_{ct} \\
\mathcal{L}_{ct} &= -\frac{1}{2}(Z_\varphi - 1)\partial^\mu \varphi \partial_\mu \varphi - \frac{1}{2}(Z_m - 1)m^2\varphi^2 + Y\varphi.
\end{aligned} \tag{2.18}$$

Here, $\mathcal{L}_{ct}$ is called the counterterm Lagrangian and all the tadpole diagrams that arise are canceled by the Y counterterm. A tadpole is a one-loop diagram with only one leg. Then the momentum-space propagator can be expanded using a geometric series, the sum of which can be expressed as,

$$\tilde{\mathbf{\Delta}}(k^2) = \frac{1}{k^2 + m^2 - i\epsilon - \Pi(k^2)} \tag{2.19}$$

where ϵ is a positive infinitesimal and $\Pi(k^2)$ is the self-energy which is expressed in terms of integrals. The diagrams which contribute corrections to the propagator at one-loop order and two-loop order are as follows,

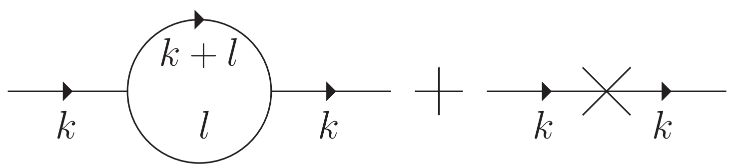


Figure 2.2: Corrections to the propagator at one-loop order

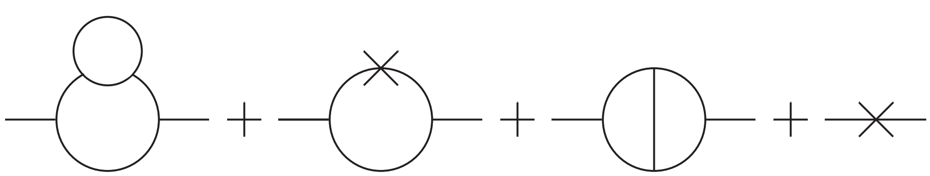


Figure 2.3: Corrections to the propagator at two-loop order

Here, the " × " in the diagrams represents the counterterms. The integrals diverge at large l for $d \geq 4$. Different tricks can be applied so that the integrals do not diverge and can be evaluated as $d = 6$, since the coupling constant is dimensionless for that value of d. One such trick is Dimensional Regularization. By using Feynman's formula, we can re-write the terms in the 1-loop divergent integral as,

$$\begin{aligned}\tilde{\mathbf{\Delta}}((l+k)^2)\tilde{\mathbf{\Delta}}(l^2) &= \frac{1}{(l^2+m^2)((l+k)^2+m^2)} \\ &= \int_0^1 dx[q^2+D^2]^{-2} \end{aligned} \tag{2.20}$$

where the $i\epsilon$s have been suppressed for notational convenience and,

$$\begin{aligned} q &= l + xk, \\ D &= x(1-x)k^2 + m^2. \end{aligned} \tag{2.21}$$

The integration variable is changed from l to q and a Wick rotation is performed to avoid integration over any poles. Then a d dimensional vector $\bar{q}$ is defined such that $d^dq = id^d\bar{q}$, and we can write,

$$\Pi(k^2) = \frac{1}{2}g^2 I(k^2) - Ak^2 - Bm^2 + O(g^4) \tag{2.22}$$

where,

$$\begin{aligned} I(k^2) &= \int_0^1 dx \int \frac{d^d\bar{q}}{(2\pi)^d}\frac{1}{(\bar{q}^2+D)^2}, \\ A &= Z_\varphi - 1, \\ B &= Z_m - 1. \end{aligned} \tag{2.23}$$

This d-dimensional integral can be evaluated over $\bar{q}$ in spherical coordinates. The integral over $\bar{q}$ yields the area Ω_d of the unit sphere in d dimensions, which is,

$$\Omega_d = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{1}{2}d)}. \tag{2.24}$$

Here, $\Gamma(x)$ is the Euler gamma function. And the partial integral takes the form,

$$\int \frac{d^d\bar{q}}{(2\pi)^d} \frac{1}{(\bar{q}^2 + D)^2} = \frac{\Gamma(2 - \frac{d}{2})\Gamma(\frac{d}{2})}{(4\pi)^{\frac{d}{2}}\Gamma(2)\Gamma(\frac{d}{2})} D^{-(2-\frac{d}{2})}. \tag{2.25}$$

This approach of evaluating the self-energy $\Pi(k^2)$ as a function of d for $d < 4$ and then analytically continuing the result to arbitrary d is called dimensional regularization [28].

When dimensional regularization is used, the divergences appear as poles at isolated values of the space-time dimension d. The renormalization prescription, where the counterterms are pure poles at the physical value of d, is called minimal subtraction, MS scheme [29]. Here, g has mass dimension $\epsilon/2$, where $\epsilon = 6 - d$. To make g dimensionless, we can introduce a new parameter $\tilde{\mu}$ and make the replacement $g \rightarrow g\tilde{\mu}^{\frac{\epsilon}{2}}$.

One disadvantage that appears in the Minimal Subtraction scheme is that it tends to produce large coefficients in perturbative expansions. So, the modified minimal subtraction scheme, $\overline{\text{MS}}$ (pronounced MS-bar) was introduced. In this scheme, a new parameter μ can be defined as,

$$\mu = \sqrt{4\pi}\, e^{-\frac{\gamma}{2}} \tilde{\mu} \tag{2.26}$$

where $\gamma = 0.5772...$ is the Euler-Mascheroni constant. The Lagrangian parameter m will not correspond to the physical mass of the particle anymore. In both schemes, all the tadpole diagrams are canceled out by the Y counterterm in the Lagrangian.

2.7 Passarino-Veltman Integrals

After following the steps of Dimensional Regularization and while doing calculations involving one-loop integrals, one comes across some integral functions which share the same form. Techniques were developed to conduct all such calculations in terms of certain basic scalar integrals [30], [31]. These integrals are called the Passarino-Veltman integrals and we use them in our one-loop calculations for the mass of the Higgs M_h.

We use the $(-+++)$ metric where the external momentum invariant is $s = -p^2$. And, the conventions are taken from [32], by defining a loop factor,

$$C = (16\pi^2)\frac{\mu^{2\epsilon}}{(2\pi)^d}. \tag{2.27}$$

And, the one-loop integrals are defined as,

$$A(x) = C\int d^dq \frac{1}{[p^2+x]} \tag{2.28}$$

$$B(x,y) = C\int d^dq \frac{1}{[p^2+x][(p-q)^2+y]}. \tag{2.29}$$

Computer programs like `SMDR` [33] make use of this approach and perform the numerical calculations of these 1-loop integrals and analogous 2-loop integrals.

CHAPTER 3

ONE LOOP CALCULATION OF M_H

We present a detailed step-by-step calculation of the mass of the Higgs boson in terms of the underlying Lagrangian parameters at 1-loop order. In Landau gauge and the $\overline{\text{MS}}$ renormalization scheme based on dimensional regularization, the 1-loop order contribution to the Standard Model effective potential is,

$$V_1 = 3f(G) + f(H) - 12f(t) + 6f(W) + W^2 + 3f(Z) + \frac{1}{2}Z^2 \tag{3.1}$$

where,

$$f(x) \equiv \frac{x^2}{4}\left[\overline{ln}(x) - \frac{3}{2}\right]. \tag{3.2}$$

And, the field-dependent running squared masses are,

$$\begin{aligned}
G &= m^2_{G^0} = m^2_{G^\pm} = m^2 + \lambda\varphi^2 \\
H &= m^2_H = m^2 + 3\lambda\varphi^2 \\
t &= m^2_t = \frac{y_t^2\varphi^2}{2} \\
W &= m^2_W = \frac{g^2\varphi^2}{4} \\
Z &= m^2_Z = \frac{(g^2 + g'^2)\varphi^2}{4},
\end{aligned} \tag{3.3}$$

where, we have defined φ to be the real background field, about which are expanded the real Higgs quantum field H, and the real neutral and complex charged Goldstone boson fields G^0 and $G^+ = G^{-*}$.

As shown in [34], we can take the vacuum expectation value to be $v = \varphi_{min}$ and the effective potential $\tilde{V}_{eff}$ can be minimized using the condition,

$$G = m^2 + \lambda v^2 = -\frac{1}{16\pi^2}\hat{\Delta}_1 - \frac{1}{(16\pi^2)^2}\hat{\Delta}_2 - ... \tag{3.4}$$

where,

$$\hat{\Delta}_1 = -6{y_t}^2 A(t) + 3\lambda A(H) + \frac{g^2}{2}[3A(W) + 2W] + \frac{g^2 + g'^2}{4}[3A(Z) + 2Z]. \tag{3.5}$$

Here, $\hat{\Delta}_1$ is the derivative of the 1-loop part of $\tilde{V}_{eff}$ with respect to the vacuum expectation value v. To calculate the mass of the Higgs at 1-loop order, we need to consider contributions from the self-interactions of the Higgs itself, the Goldstone boson, the top quark, and the W and Z bosons. As shown in [35], the Higgs squared pole mass M_H can be obtained from the following expression,

$$M_h^2 - i\Gamma_h M_h = 2\lambda v^2 + \frac{1}{16\pi^2}\Delta^{(1)}_{M_h^2} + ... \tag{3.6}$$

We include all the possible Feynman diagrams at one-loop order and sum up the individual contributions to obtain the explicit result for the 1-loop part,

$$\begin{aligned}\Delta^{(1)}_{M_h^2} \;=\;& 3{y_t}^2(4t - s)B(t,t) - 18\lambda^2 v^2 B(h,h) \\ &+\frac{1}{2}(g^2 + g'^2)[(s - 3Z - \frac{s^2}{4Z})B(Z,Z) - \frac{s}{2Z}A(Z) + 2Z] \\ &+g^2[(s - 3W - \frac{s^2}{4W})B(W,W) - \frac{s}{2W}A(W) + 2W].\end{aligned} \tag{3.7}$$

Using the Feynman rules given in the appendix, we compute the contributions from the following Feynman diagrams.

3.1 Higgs Self-interactions

First, we calculate the one-loop corrections from the self-interactions of the Higgs. We look at the 1-vertex diagram,

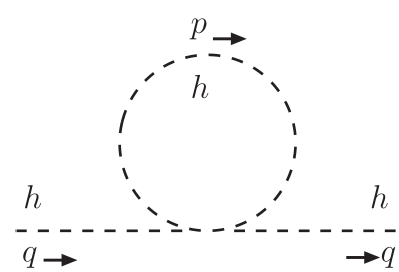


Figure 3.1: Higgs Self-interaction with 1 vertex

Here, we get a symmetry factor of $\frac{1}{2}$ since the Higgs is its own antiparticle. And the vertex gives a factor of $(-6i\lambda)$. The other term is from the propagator. The contribution is calculated to be,

$$\begin{aligned} & -\frac{1}{2}(-6i\lambda)(16\pi^2)\mu^{2\epsilon}\int\frac{d^dp}{(2\pi)^d}\frac{-i}{p^2+{m_h}^2} \\ = \;& 3\lambda A(h). \end{aligned} \tag{3.8}$$

For future calculations, we will be absorbing $(16\pi^2)\mu^{2\epsilon}$ into the integral $\int d^dp$ for brevity. So,

$$\int \frac{d^d p}{(2\pi)^d} \to (16\pi^2)\mu^{2\epsilon} \int \frac{d^d p}{(2\pi)^d}. \tag{3.9}$$

Now, we look at the 2-vertex diagram,

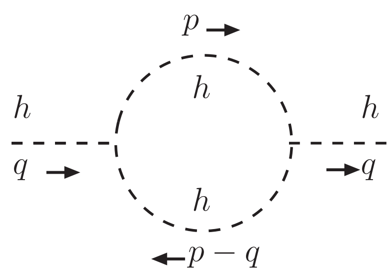


Figure 3.2: Higgs Self-interaction with 2 vertices

Here, we get a symmetry factor of $\frac{1}{2}$ since we can interchange the top and bottom Higgs particles. We get a factor of $(-6i\lambda)$ from each of the vertices. And the other two terms come from the internal propagators. The contribution is calculated to be,

$$\begin{aligned} & -\frac{1}{2}(-6i\lambda)^2 \int \frac{d^d p}{(2\pi)^d} \frac{-i}{p^2 + m_h{}^2} \frac{-i}{(p-q)^2 + m_h{}^2} \\ = \;& -18\lambda^2 v^2 B(h,h). \end{aligned} \tag{3.10}$$

3.2 Higgs One Loop Contribution from the Top Quark

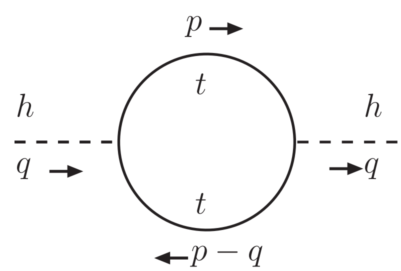


Figure 3.3: Top quark loop with 2 vertices

Here, we get an extra factor of (-1) since there is an integral over a fermion loop and it follows from the usual Feynman rules for electrodynamics. We also get a factor of 3 since we need to consider the three colors of the top quark. Each vertex contributes a factor of $\left(-iy_t/\sqrt{2}\right)^2$. We consider the integral after taking the trace of the propagator terms and the 4-momentums follow the Feynman slash notation, where $\not{p} = \gamma^\mu p_\mu$ and $\not{q} = \gamma^\mu q_\mu$. The contribution is calculated to be,

$$\begin{aligned} & (-1)(-1)3\left(\frac{-iy_t}{\sqrt{2}}\right)^2 \int \frac{d^dp}{(2\pi)^d} Tr\left[\frac{-i(\not{p}-m_t)}{p^2+{m_t}^2}\frac{-i[(\not{q}-\not{p})-m_t]}{(p-q)^2+{m_t}^2}\right] \\ = \quad & \frac{3}{2}{y_t}^2 \int \frac{d^dp}{(2\pi)^d}\frac{Tr[(\not{p}-m_t)((\not{q}-\not{p})-m_t)]}{(p^2+{m_t}^2)[(p-q)^2+{m_t}^2]} \\ = \quad & 3{y_t}^2 \int \frac{d^dp}{(2\pi)^d}\frac{2(-p^2+p\cdot q+{m_t}^2)}{(p^2+{m_t}^2)((p-q)^2+{m_t}^2)}. \end{aligned} \tag{3.11}$$

Here, we can use the trick,

$$p \cdot q = -\frac{1}{2}\left[[(p-q)^2+m^2]-(p^2+m^2)-q^2\right] \tag{3.12}$$

So we have,

$$\begin{aligned}
& 3{y_t}^2 \int \frac{d^dp}{(2\pi)^d} \left[\frac{-2p^2 - [(p-q)^2 + {m_t}^2] + (p^2 + {m_t}^2) + q^2 + 2{m_t}^2}{(p^2+{m_t}^2)((p-q)^2+{m_t}^2)} \right] \\
= \ & 3{y_t}^2 \int \frac{d^dp}{(2\pi)^d} \left[-\frac{2}{(p-q)^2+{m_t}^2} + \frac{2{m_t}^2}{(p^2+{m_t}^2)[(p-q)^2+{m_t}^2]} - \frac{1}{p^2+{m_t}^2} \right. \\
& \left. + \frac{1}{(p-q)^2+{m_t}^2} + \frac{q^2}{(p^2+{m_t}^2)[(p-q)^2+{m_t}^2]} + \frac{2{m_t}^2}{(p^2+{m_t}^2)((p-q)^2+{m_t}^2)} \right] \\
= \ & 3{y_t}^2[-2A(t) + 2tB(t,t) - A(t) + A(t) - sB(t,t) + 2tB(t,t)] \\
= \ & 3{y_t}^2[(4t-s)B(t,t)] - 6{y_t}^2 A(t). \qquad (3.13)
\end{aligned}$$

3.3 Higgs One Loop Contribution from the Goldstone Boson

For the Goldstone Boson, we get two diagrams again like the Higgs. We first look at the 1-vertex diagram,

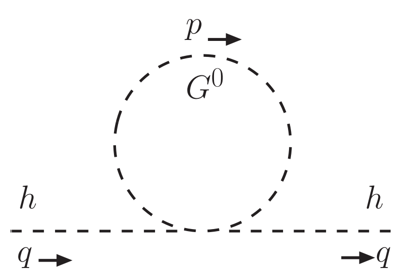


Figure 3.4: Goldstone loop with 1 vertex

Here, we have the vertex term $(-i2\lambda)$ and we consider the integral over a massless propagator term since the Goldstone bosons are massles. The contribution is, calculated to be,

$$(-1)(-i2\lambda) \int \frac{d^dp}{(2\pi)^d} \frac{-i}{p^2}$$

$$= \quad 2\lambda A(0)$$
$$= \quad 0. \qquad (3.14)$$

Now, we look at the 2-vertex case. We get two separate Feynman diagrams, one for the neutral Goldstone and the other for the charged Goldstone pair. For the charged Goldstone loop, if the Goldstone at the upper loop is G^+, then the Goldstone at the lower loop is G^- and vice-versa.

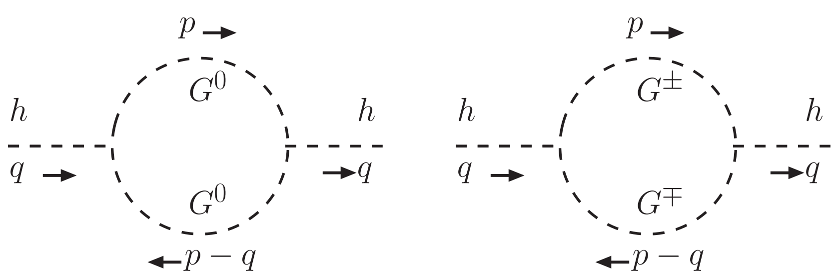


Figure 3.5: Goldstone loop with 2 vertices

The diagram with the neutral Goldstone has an additional symmetry factor of 1/2 since the neutral Goldstone is its own anti-particle. The other diagram does not have a symmetry factor. So we get a total factor of $1 + 1/2 = 3/2$. Here again, we have two vertex terms $(-i2\lambda)$ from the two vertices and we consider the integral over two ghost propagator terms since the Goldstone bosons are massless. The contribution is,

$$-\frac{3}{2}(-i2\lambda v)(-i2\lambda v)\int \frac{d^dp}{(2\pi)^d}\frac{-i}{p^2}\frac{-i}{(p-q)^2}$$
$$= \quad -6\lambda^2 v^2 B(0,0). \qquad (3.15)$$

3.4 Higgs One Loop Contribution from the W Boson

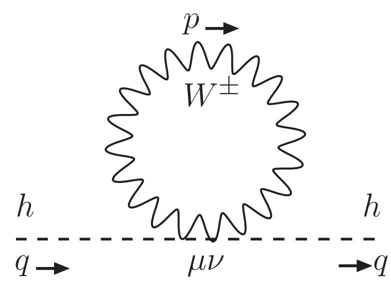


Figure 3.6: W boson loop with 1 vertex

First, we look at the W boson loop with one vertex. The vertex factor is $(-ig^2/2\eta_{\mu\nu})$ and we get the propagator term from the Feynman rules in the Appendix. Following the definition of the Passarino-Veltman integrals we get an $\boldsymbol{A}(x)$ function along with the $A(x)$ function, since one of the integrals is over an ϵ term. By using the relation, $\boldsymbol{A}(x) = A(x) - x/\epsilon + \mathcal{O}(\epsilon)$ and then taking $\epsilon \to 0$, one of the terms with ϵ remains finite and the other term goes to infinity. We keep the infinite term as it is for now but will set it equal to the counterterms in the end so that they are canceled out. The contribution is calculated as follows,

$$
\begin{aligned}
& -(-i)\frac{g^2}{2}\eta_{\mu\nu}\int\frac{d^dp}{(2\pi)^d}(-i)\left[\frac{\eta^{\mu\nu}}{p^2+m_W{}^2}+\frac{p^\mu p^\nu}{m_W{}^2}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right)\right] \\
= \; & \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{d}{p^2+m_W{}^2}+\frac{p^2}{m_W{}^2}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right)\right] \\
= \; & \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{d}{p^2+m_W{}^2}+\frac{p^2+m_W{}^2-m_W{}^2}{m_W{}^2(p^2+m_W{}^2)}-\frac{1}{m_W{}^2}\right] \\
= \; & \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{4-2\epsilon}{p^2+m_W{}^2}+\frac{1}{m_W{}^2}-\frac{1}{(p^2+m_W{}^2)}-\frac{1}{m_W{}^2}\right] \\
= \; & \frac{g^2}{2}(4-2\epsilon)\mathbf{A}(W)-\frac{g^2}{2}[A(W)] \\
= \; & \frac{g^2}{2}(4-2\epsilon)[A(W)-\frac{W}{\epsilon}]-\frac{g^2}{2}[A(W)]
\end{aligned}
$$

$$\begin{aligned} &= \frac{g^2}{2}[4A(W) - \frac{4W}{\epsilon} - 2\epsilon A(W) + 2W] - \frac{g^2}{2}[A(W)] \\ &= g^2\left[-\frac{2W}{\epsilon} + \frac{3}{2}A(W) + W - \epsilon A(W)\right] \\ &\approx g^2\left[\frac{3}{2}A(W) + W\right] + g^2\left[-\frac{2W}{\epsilon}\right]. \end{aligned} \tag{3.16}$$

Next, we look at the loop with two vertices. The vertex factor and propagator terms stay the same.

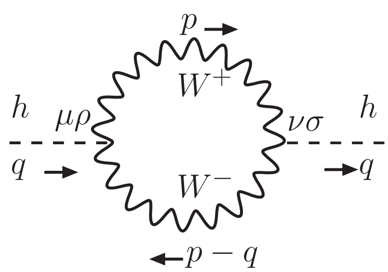


Figure 3.7: W-Boson loop with 2 vertices

We use B.3 in the second line of the calculation and set ${m_W}^2 = W$ towards the end. The calculation is pretty long and thus divided into different parts where individual terms are calculated separately. Throughout the calculations, we utilize some properties of the integral functions such as

$$\int \frac{d^dp}{(2\pi)^d}\frac{p\cdot q}{p^2 + {m_W}^2} = 0 \tag{3.17}$$

$$\int \frac{d^dp}{(2\pi)^d}\frac{1}{p^2} = A(0) = 0. \tag{3.18}$$

We start with the Feynman rules,

$$
\begin{aligned}
& -(-ig^2\frac{v}{2})(-ig^2\frac{v}{2})\eta_{\mu\rho}\eta_{\nu\sigma}\int\frac{d^dp}{(2\pi)^d}(-i)\left[\frac{\eta^{\mu\nu}}{p^2+m_W{}^2}+\frac{p^\mu p^\nu}{m_W{}^2}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right)\right] \\
& (-i)\left[\frac{\eta^{\rho\sigma}}{(p-q)^2+m_W{}^2}+\frac{(p-q)^\rho(p-q)^\sigma}{m_W{}^2}\left(\frac{1}{(p-q)^2+m_W{}^2}-\frac{1}{(p-q)^2}\right)\right] \\
= & -\left(g^2\frac{v^2}{4}\right)g^2\int\frac{d^dp}{(2\pi)^d}\left[\frac{\eta_{\mu\rho}\eta_{\nu\sigma}\eta^{\mu\nu}}{p^2+m_W{}^2}+\frac{\eta_{\mu\rho}\eta_{\nu\sigma}p^\mu p^\nu}{m_W{}^2}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right)\right] \\
& \left[\frac{\eta^{\rho\sigma}}{(p-q)^2+m_W{}^2}+\frac{(p-q)^\rho(p-q)^\sigma}{m_W{}^2}\left(\frac{1}{(p-q)^2+m_W{}^2}-\frac{1}{(p-q)^2}\right)\right] \\
= & -m_W{}^2g^2\int\frac{d^dp}{(2\pi)^d}\left[\frac{\eta_{\mu\rho}\eta_{\nu\sigma}\eta^{\mu\nu}\eta^{\rho\sigma}}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}+\frac{\eta_{\mu\rho}\eta_{\nu\sigma}\eta^{\mu\nu}(p-q)^\rho(p-q)^\sigma}{m_W{}^2(p^2+m_W{}^2)}\right. \\
& \left(\frac{1}{(p-q)^2+m_W{}^2}-\frac{1}{(p-q)^2}\right)+\frac{\eta_{\mu\rho}\eta_{\nu\sigma}p^\mu p^\nu\eta^{\rho\sigma}}{m_W{}^2[(p-q)^2+m_W{}^2]}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right) \\
& \left.+\frac{\eta_{\mu\rho}\eta_{\nu\sigma}p^\mu p^\nu(p-q)^\rho(p-q)^\sigma}{m_W{}^4}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right)\left(\frac{1}{(p-q)^2+m_W{}^2}-\frac{1}{(p-q)^2}\right)\right] \\
= & -g^2\int\frac{d^dp}{(2\pi)^d}\left[\frac{dm_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}+\frac{(p-q)^2}{(p^2+m_W{}^2)}\left(\frac{1}{(p-q)^2+m_W{}^2}\right.\right. \\
& \left.-\frac{1}{(p-q)^2}\right)+\frac{p^2}{[(p-q)^2+m_W{}^2]}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right) \\
& \left.+\frac{[p.(p-q)]^2}{m_W{}^2}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right)\left(\frac{1}{(p-q)^2+m_W{}^2}-\frac{1}{(p-q)^2}\right)\right]. \qquad (3.19)
\end{aligned}
$$

From the 1^{st} term in 3.19 we get ,

$$
\begin{aligned}
& -g^2\int\frac{d^dp}{(2\pi)^d}\frac{dm_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} \\
= & -g^2\int\frac{d^dp}{(2\pi)^d}\frac{(4-2\epsilon)m_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} \\
= & -g^2W(4-2\epsilon)\mathbf{B}(W,W) \\
= & -g^2W(4-2\epsilon)[\frac{1}{\epsilon}+B(W,W)] \\
= & -g^2[\frac{4}{\epsilon}W+4WB(W,W)-2W-2\epsilon WB(W,W)] \\
\approx & \; g^2[-\frac{4}{\epsilon}W-4WB(W,W)+2W]. \qquad (3.20)
\end{aligned}
$$

Here, we got a $\boldsymbol{B}(x)$ function along with the $B(x)$ function, since one of the integrals is over an ϵ term. By using the relation, $\boldsymbol{B}(x) = 1/\epsilon + B(x) + \mathcal{O}(\epsilon)$ and then taking $\epsilon \to 0$, one of the terms with ϵ is finite and the other term goes to infinity. We keep the infinite term as it is for now but will set it equal to the counterterms in the end so that they are canceled out. From the 2^{nd} term in 3.19 we get,

$$
\begin{aligned}
&-g^2 \int \frac{d^dp}{(2\pi)^d} \frac{(p-q)^2}{(p^2+m_W{}^2)} \left(\frac{1}{(p-q)^2+m_W{}^2} - \frac{1}{(p-q)^2} \right) \\
=&\ -g^2 \int \frac{d^dp}{(2\pi)^d} \left[\frac{(p-q)^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} - \frac{1}{(p^2+m_W{}^2)} \right] \\
=&\ -g^2 \int \frac{d^dp}{(2\pi)^d} \left[\frac{[(p-q)^2+m_W{}^2]-m_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} - \frac{1}{(p^2+m_W{}^2)} \right] \\
=&\ -g^2 \int \frac{d^dp}{(2\pi)^d} \left[\frac{1}{p^2+m_W{}^2} - \frac{m_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} - \frac{1}{(p^2+m_W{}^2)} \right] \\
=&\ -g^2 \int \frac{d^dp}{(2\pi)^d} \left[\frac{-m_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} \right] \\
=&\ g^2 \left[WB(W,W) \right].
\end{aligned} \tag{3.21}
$$

From the 3^{rd} term in 3.19 we get,

$$
\begin{aligned}
&-g^2 \int \frac{d^dp}{(2\pi)^d} \frac{p^2}{(p-q)^2+m_W{}^2} \left(\frac{1}{p^2+m_W{}^2} - \frac{1}{p^2} \right) \\
=&\ -g^2 \int \frac{d^dp}{(2\pi)^d} \left[\frac{p^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} - \frac{1}{(p-q)^2+m_W{}^2]} \right] \\
=&\ -g^2 \int \frac{d^dp}{(2\pi)^d} \left[\frac{(p^2+m_W{}^2)-m_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} - \frac{1}{[(p-q)^2+m_W{}^2]} \right] \\
=&\ -g^2 \int \frac{d^dp}{(2\pi)^d} \left[\frac{1}{[(p-q)^2+m_W{}^2]} - \frac{m_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} - \frac{1}{[(p-q)^2+m_W{}^2]} \right] \\
=&\ -g^2 \int \frac{d^dp}{(2\pi)^d} \left[-\frac{m_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]} \right] \\
=&\ g^2 \left[WB(W,W) \right].
\end{aligned} \tag{3.22}
$$

From the 4^{th} term in 3.19 we get,

$$
\begin{aligned}
& -g^2 \int \frac{d^d p}{(2\pi)^d} \frac{[p.(p-q)]^2}{m_W{}^2} \left(\frac{1}{p^2 + m_W{}^2} - \frac{1}{p^2} \right) \left(\frac{1}{(p-q)^2 + m_W{}^2} - \frac{1}{(p-q)^2} \right) \\
= \; & \frac{g^2}{m_W{}^2} \int \frac{d^d p}{(2\pi)^d} [p.(p-q)]^2 \left[- \frac{1}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \right. \\
& \left. + \frac{1}{(p^2 + m_W{}^2)(p-q)^2} + \frac{1}{p^2[(p-q)^2 + m_W{}^2]} - \frac{1}{p^2(p-q)^2} \right]. \qquad (3.23)
\end{aligned}
$$

Let's look at the 1^{st} term in 3.23,

$$
\begin{aligned}
& -\frac{g^2}{m_W{}^2} \int \frac{d^d p}{(2\pi)^d} \frac{[p.(p-q)]^2}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \\
= \; & -\frac{g^2}{m_W{}^2} \int \frac{d^d p}{(2\pi)^d} \frac{p^4 - 2p^2(p \cdot q) + (p \cdot q)^2}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \\
= \; & -\frac{g^2}{m_W{}^2} \int \frac{d^d p}{(2\pi)^d} \left[\frac{p^4 - 2p^2(p \cdot q)}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \right. \\
& \left. + \frac{(p \cdot q)^2}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \right]. \qquad (3.24)
\end{aligned}
$$

Here,

$$
\begin{aligned}
& -\frac{g^2}{m_W{}^2} \int \frac{d^d p}{(2\pi)^d} \left[\frac{p^4 - 2p^2(p \cdot q)}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \right] \\
= \; & -\frac{g^2}{m_W{}^2} \int \frac{d^d p}{(2\pi)^d} \left[\frac{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2] - q^2 p^2 - p^2 m_W{}^2 - m_W{}^2 (p-q)^2}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \right. \\
& \left. - \frac{m_W{}^4}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \right] \\
= \; & -\frac{g^2}{m_W{}^2} \int \frac{d^d p}{(2\pi)^d} \left[1 - \frac{q^2(p^2 + m_W{}^2)}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} + \frac{q^2 m_W{}^2}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \right. \\
& - \frac{m_W{}^2 (p^2 + m_W{}^2)}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} - \frac{m_W{}^2 [(p-q)^2 + m_W{}^2]}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \\
& \left. + \frac{m_W{}^4}{(p^2 + m_W{}^2)[(p-q)^2 + m_W{}^2]} \right]
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[1-\frac{q^2}{[(p-q)^2+m_W{}^2]}+\frac{q^2 m_W{}^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right.\\
&\quad\left.-\frac{m_W{}^2}{[(p-q)^2+m_W{}^2]}-\frac{m_W{}^2}{(p^2+m_W{}^2)}+\frac{m_W{}^4}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right]\\
&= -\frac{g^2}{W}\left[sA(W)-sWB(W,W)-WA(W)-WA(W)+W^2B(W,W)\right]\\
&= \frac{g^2}{W}\left[-W^2B(W,W)+sWB(W,W)+2WA(W)-sA(W)\right]\\
&= g^2\left[-WB(W,W)+sB(W,W)+2A(W)-\frac{s}{W}A(W)\right].
\end{aligned}
\tag{3.25}
$$

And,

$$
\begin{aligned}
&-\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{(p\cdot q)^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right]\\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right](p\cdot q)\\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{[(p-q)^2+m_W{}^2]}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right.\\
&\quad\left.+\frac{1}{2}\frac{(p^2+m_W{}^2)}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}+\frac{1}{2}\frac{q^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right](p\cdot q)\\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{p\cdot q}{p^2+m_W{}^2}+\frac{1}{2}\frac{p\cdot q}{[(p-q)^2+m_W{}^2]}\right.\\
&\quad\left.+\frac{1}{2}\frac{q^2(p\cdot q)}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right]\\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left(\frac{1}{2}\right)q^2\left[-\frac{1}{2}\frac{[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right]\\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left(\frac{1}{2}\right)q^2\left[-\frac{1}{2}\frac{[(p-q)^2+m_W{}^2]}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right.\\
&\quad\left.+\frac{1}{2}\frac{(p^2+m_W{}^2)}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}+\frac{1}{2}\frac{q^2}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right]\\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{4}\frac{q^2}{p^2+m_W{}^2}+\frac{1}{4}\frac{q^2}{[(p-q)^2+m_W{}^2]}\right.\\
&\quad\left.+\frac{1}{4}\frac{q^4}{(p^2+m_W{}^2)[(p-q)^2+m_W{}^2]}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{g^2}{W}\left[-\frac{1}{4}sA(W)+\frac{1}{4}sA(W)-\frac{1}{4}s^2B(W,W)\right] \\
&= g^2\left[-\frac{s^2}{4W}B(W,W)\right]. \qquad (3.26)
\end{aligned}
$$

Adding 3.20, 3.21, 3.22, 3.25 and 3.26, we get,

$$
\begin{aligned}
& g^2\left[-\frac{4}{\epsilon}W-4WB(W,W)+2W+WB(W,W)+WB(W,W)\right. \\
& \left.-WB(W,W)+sB(W,W)+2A(W)-\frac{s}{W}A(W)-\frac{s^2}{4W}B(W,W)\right] \\
= \; & g^2\left[\left(s-3W-\frac{s^2}{4W}B(W,W)\right)+2W+2A(W)-\frac{s}{W}A(W)\right. \\
& \left.-\frac{4}{\epsilon}W\right]. \qquad (3.27)
\end{aligned}
$$

Let's look at the 2^{nd} term in 3.23,

$$
\begin{aligned}
& \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{[p.(p-q)]^2}{(p^2+m_W{}^2)(p-q)^2} \\
= \; & \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{p^4-2p^2(p\cdot q)+(p\cdot q)^2}{(p^2+m_W{}^2)(p-q)^2} \\
= \; & \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{p^4-2p^2(p\cdot q)}{(p^2+m_W{}^2)(p-q)^2}+\frac{(p\cdot q)^2}{(p^2+m_W{}^2)(p-q)^2}\right]. \qquad (3.28)
\end{aligned}
$$

Here,

$$
\begin{aligned}
& \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{p^4-2p^2(p\cdot q)}{(p^2+m_W{}^2)(p-q)^2} \\
= \; & \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{(p^2+m_W{}^2)(p-q)^2-q^2(p^2+m_W{}^2)-m_W{}^2(p-q)^2+q^2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2} \\
= \; & \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[1-\frac{q^2}{(p-q)^2}-\frac{m_W{}^2}{p^2+m_W{}^2}+\frac{q^2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}\right] \\
= \; & \frac{g^2}{W}[-WA(W)-sWB(W,0)]. \qquad (3.29)
\end{aligned}
$$

Now,

$$\begin{aligned}
&\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{(p\cdot q)^2}{(p^2+m_W{}^2)(p-q)^2}\\
=\;&\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]}{(p^2+m_W{}^2)(p-q)^2}\right](p\cdot q)\\
=\;&\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{[(p-q)^2+m_W{}^2]}{(p^2+m_W{}^2)(p-q)^2}+\frac{1}{2}\frac{(p^2+m_W{}^2)}{(p^2+m_W{}^2)(p-q)^2}\right.\\
&\left.+\frac{1}{2}\frac{q^2}{(p^2+m_W{}^2)(p-q)^2}\right](p\cdot q)\\
=\;&\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{1}{p^2+m_W{}^2}-\frac{1}{2}\frac{m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}+\frac{1}{2}\frac{1}{(p-q)^2}\right.\\
&\left.+\frac{1}{2}\frac{q^2}{(p^2+m_W{}^2)(p-q)^2}\right](p\cdot q)\\
=\;&\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{(p\cdot q)}{p^2+m_W{}^2}-\frac{1}{2}\frac{m_W{}^2(p\cdot q)}{(p^2+m_W{}^2)(p-q)^2}\right.\\
&\left.+\frac{1}{2}\frac{q^2(p\cdot q)}{(p^2+m_W{}^2)(p-q)^2}\right]\\
=\;&\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{m_W{}^2[-\frac{1}{2}[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]]}{(p^2+m_W{}^2)(p-q)^2}\right.\\
&\left.+\frac{1}{2}\frac{q^2[-\frac{1}{2}[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]]}{(p^2+m_W{}^2)(p-q)^2}\right]\\
=\;&\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{1}{4}m_W{}^2\left[\frac{1}{p^2+m_W{}^2}+\frac{m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{1}{(p-q)^2}\right.\right.\\
&\left.-\frac{q^2}{(p^2+m_W{}^2)(p-q)^2}\right]-\frac{1}{4}q^2\left[\frac{1}{p^2+m_W{}^2}+\frac{m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{1}{(p-q)^2}\right.\\
&\left.\left.-\frac{q^2}{(p^2+m_W{}^2)(p-q)^2}\right]\right]\\
=\;&\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{1}{4}\left[\frac{m_W{}^2}{p^2+m_W{}^2}+\frac{m_W{}^4}{(p^2+m_W{}^2)(p-q)^2}-\frac{m_W{}^2}{(p-q)^2}\right.\right.\\
&\left.-\frac{q^2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}\right]-\frac{1}{4}\left[\frac{q^2}{p^2+m_W{}^2}+\frac{m_W{}^2q^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{q^2}{(p-q)^2}\right.\\
&\left.\left.-\frac{q^4}{(p^2+m_W{}^2)(p-q)^2}\right]\right]
\end{aligned}$$

$$
\begin{aligned}
&= \frac{g^2}{W}\left[\frac{1}{4}WA(W) + \frac{1}{4}W^2B(W,0) + \frac{1}{4}sWB(W,0)\right. \\
&\quad \left. + \frac{1}{4}sA(W) + \frac{1}{4}sWB(W,0) + \frac{1}{4}s^2B(W,0)\right] \\
&= \frac{g^2}{W}\left[\frac{1}{4}WA(W) + \frac{1}{4}sA(W) + \frac{1}{4}(s+W)^2B(W,0)\right]. \qquad (3.30)
\end{aligned}
$$

Let's look at the 3^{rd} term in 3.23,

$$
\begin{aligned}
&\frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{[p.(p-q)]^2}{p^2[(p-q)^2+m_W{}^2]} \\
&= \frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{p^4-2p^2(p\cdot q)+(p\cdot q)^2}{p^2[(p-q)^2+m_W{}^2]} \\
&= \frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\left[\frac{p^4-2p^2(p\cdot q)}{p^2[(p-q)^2+m_W{}^2]} + \frac{(p\cdot q)^2}{p^2[(p-q)^2+m_W{}^2]}\right]. \qquad (3.31)
\end{aligned}
$$

Looking at the 1^{st} part of 3.31

$$
\begin{aligned}
&\frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{p^4-2p^2(p\cdot q)}{p^2[(p-q)^2+m_W{}^2]} \\
&= \frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{p^4-2p^2(p\cdot q)+p^2q^2+p^2m_W{}^2-p^2q^2-p^2m_W{}^2}{p^2[(p-q)^2+m_W{}^2]} \\
&= \frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{p^2[(p-q)^2+m_W{}^2]-p^2q^2-p^2m_W{}^2}{p^2[(p-q)^2+m_W{}^2]} \\
&= \frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\left[1-\frac{q^2}{[(p-q)^2+m_W{}^2]}-\frac{m_W{}^2}{[(p-q)^2+m_W{}^2]}\right] \\
&= \frac{g^2}{W}[sA(W)-WA(W)]. \qquad (3.32)
\end{aligned}
$$

Looking at the 2nd part of 3.31,

$$
\begin{aligned}
&\frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{(p\cdot q)^2}{p^2[(p-q)^2+m_W{}^2]} \\
&= \frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]}{p^2[(p-q)^2+m_W{}^2]}\right](p\cdot q)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{[(p-q)^2+m_W{}^2]}{p^2[(p-q)^2+m_W{}^2]}+\frac{1}{2}\frac{(p^2+m_W{}^2)}{p^2[(p-q)^2+m_W{}^2]}\right.\\
&\left.+\frac{1}{2}\frac{q^2}{p^2[(p-q)^2+m_W{}^2]}\right](p\cdot q)\\
&= \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{1}{p^2}+\frac{1}{2}\frac{1}{[(p-q)^2+m_W{}^2]}+\frac{1}{2}\frac{m_W{}^2}{p^2[(p-q)^2+m_W{}^2]}\right.\\
&\left.+\frac{1}{2}\frac{q^2}{p^2[(p-q)^2+m_W{}^2]}\right](p\cdot q)\\
&= \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{(p\cdot q)}{p^2}+\frac{1}{2}\frac{(p\cdot q)}{[(p-q)^2+m_W{}^2]}+\frac{1}{2}\frac{m_W{}^2(p\cdot q)}{p^2[(p-q)^2+m_W{}^2]}\right.\\
&\left.+\frac{1}{2}\frac{q^2(p\cdot q)}{p^2[(p-q)^2+m_W{}^2]}\right]\\
&= \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{1}{2}\frac{m_W{}^2[-\frac{1}{2}[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]}{p^2[(p-q)^2+m_W{}^2]}]\right.\\
&\left.+\frac{1}{2}\frac{q^2[-\frac{1}{2}[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]}{p^2[(p-q)^2+m_W{}^2]}\right]\\
&= \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{1}{4}m_W{}^2\left[-\frac{1}{p^2}+\frac{1}{[(p-q)^2+m_W{}^2]}+\frac{m_W{}^2}{p^2[(p-q)^2+m_W{}^2]}\right.\right.\\
&\left.+\frac{q^2}{p^2[(p-q)^2+m_W{}^2]}\right]+\frac{1}{4}q^2\left[-\frac{1}{p^2}+\frac{1}{[(p-q)^2+m_W{}^2]}+\frac{m_W{}^2}{p^2[(p-q)^2+m_W{}^2]}\right.\\
&\left.\left.+\frac{q^2}{p^2[(p-q)^2+m_W{}^2]}\right]\right]\\
&= \frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{1}{4}\left[-\frac{m_W{}^2}{p^2}+\frac{m_W{}^2}{[(p-q)^2+m_W{}^2]}+\frac{m_W{}^4}{p^2[(p-q)^2+m_W{}^2]}\right.\right.\\
&\left.+\frac{m_W{}^2q^2}{p^2[(p-q)^2+m_W{}^2]}\right]+\frac{1}{4}\left[-\frac{q^2}{p^2}+\frac{q^2}{[(p-q)^2+m_W{}^2]}+\frac{m_W{}^2q^2}{p^2[(p-q)^2+m_W{}^2]}\right.\\
&\left.\left.+\frac{q^4}{p^2[(p-q)^2+m_W{}^2]}\right]\right]\\
&= \frac{g^2}{W}\left[\frac{1}{4}WA(W)+\frac{1}{4}W^2B(W,0)-\frac{1}{4}sWB(W,0)\right.\\
&\left.-\frac{1}{4}sA(W)-\frac{1}{4}sWB(W,0)+\frac{1}{4}s^2B(W,0)\right]\\
&= \frac{g^2}{W}\left[\frac{1}{4}WA(W)-\frac{1}{4}sA(W)+\frac{1}{4}(s-W)^2B(W,0)\right]. \qquad (3.33)
\end{aligned}
$$

Let's look at the 4^{th} term in 3.23,

$$
\begin{aligned}
& -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{[p.(p-q)]^2}{p^2(p-q)^2} \\
= & -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{p^4-2p^2(p\cdot q)+(p\cdot q)^2}{p^2(p-q)^2} \\
= & -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{p^4-2p^2(p\cdot q)}{p^2(p-q)^2}+\frac{(p\cdot q)^2}{p^2(p-q)^2}\right].
\end{aligned}
\tag{3.34}
$$

Looking at the $1st$ part of 3.34,

$$
\begin{aligned}
& -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{p^4-2p^2(p\cdot q)}{p^2(p-q)^2}\right] \\
= & -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{p^2}{(p-q)^2}-\frac{2(p\cdot q)}{(p-q)^2}\right] \\
= & \ 0.
\end{aligned}
\tag{3.35}
$$

Looking at the $2nd$ part of 3.34,

$$
\begin{aligned}
= & -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{(p\cdot q)^2}{p^2(p-q)^2}\right] \\
= & -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]}{p^2(p-q)^2}\right](p\cdot q) \\
= & -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{[(p-q)^2+m_W{}^2]}{p^2(p-q)^2}+\frac{1}{2}\frac{(p^2+m_W{}^2)}{p^2(p-q)^2}\right. \\
& \left.+\frac{1}{2}\frac{q^2}{p^2(p-q)^2}\right](p\cdot q) \\
= & -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{(p-q)^2}{p^2(p-q)^2}-\frac{1}{2}\frac{m_W{}^2}{p^2(p-q)^2}+\frac{1}{2}\frac{p^2}{p^2(p-q)^2}+\frac{1}{2}\frac{m_W{}^2}{p^2(p-q)^2}\right. \\
& \left.+\frac{1}{2}\frac{q^2}{p^2(p-q)^2}\right](p\cdot q) \\
= & -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{1}{2}\frac{(p\cdot q)}{p^2}-\frac{1}{2}\frac{m_W{}^2(p\cdot q)}{p^2(p-q)^2}+\frac{1}{2}\frac{(p\cdot q)}{(p-q)^2}+\frac{1}{2}\frac{m_W{}^2(p\cdot q)}{p^2(p-q)^2}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\frac{q^2(p\cdot q)}{p^2(p-q)^2}\Bigg] \\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{1}{2}q^2\left[-\frac{1}{2}\frac{[[(p-q)^2+m_W{}^2]-(p^2+m_W{}^2)-q^2]}{p^2(p-q)^2}\right] \\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{1}{2}q^2\left[-\frac{1}{2}\frac{1}{p^2}-\frac{1}{2}\frac{m_W{}^2}{p^2(p-q)^2}+\frac{1}{2}\frac{1}{(p-q)^2}+\frac{1}{2}\frac{m_W{}^2}{p^2(p-q)^2}\right. \\
&\left.+\frac{1}{2}\frac{q^2}{p^2(p-q)^2}\right] \\
&= -\frac{g^2}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{1}{4}\frac{q^4}{p^2(p-q)^2} \\
&= \frac{g^2}{W}\left[-\frac{1}{4}s^2B(0,0)\right]. \qquad (3.36)
\end{aligned}
$$

Adding 3.29, 3.30, 3.32, 3.33, 3.35 and 3.36 we get,

$$
\begin{aligned}
&\frac{g^2}{W}\left[-WA(W)-sWB(W,0)+\frac{1}{4}WA(W)+\frac{1}{4}sA(W)\right. \\
&+\frac{1}{4}(s+W)^2B(W,0)+sA(W)-WA(W)+\frac{1}{4}WA(W)-\frac{1}{4}sA(W) \\
&\left.+\frac{1}{4}(s-W)^2B(W,0)+0+\frac{1}{4}s^2B(0,0)\right] \\
&= \frac{g^2}{W}\left[\frac{1}{2}(s-W)^2B(W,0)-\frac{3}{2}WA(W)+sA(W)-\frac{1}{4}s^2B(0,0)\right] \\
&= g^2\left[\frac{1}{2W}(s-W)^2B(W,0)-\frac{3}{2}A(W)+\frac{s}{W}A(W)-\frac{1}{4W}s^2B(0,0)\right]. \qquad (3.37)
\end{aligned}
$$

Adding 3.27 and 3.37, we get,

$$
\begin{aligned}
&g^2\left[\left(s-3W-\frac{s^2}{4W}\right)B(W,W)+2W+2A(W)-\frac{s}{W}A(W)\right. \\
&\left.-\frac{4}{\epsilon}W+\frac{1}{2W}(s-W)^2B(W,0)-\frac{3}{2}A(W)+\frac{s}{W}A(W)-\frac{1}{4W}s^2B(0,0)\right] \\
&= g^2\left[\left(s-3W-\frac{s^2}{4W}\right)B(W,W)+\frac{1}{2W}(s-W)^2B(W,0)+2W+\frac{1}{2}A(W)\right.
\end{aligned}
$$

$$-\frac{1}{4}s^2B(0,0) - \frac{4}{\epsilon}W\bigg]. \tag{3.38}$$

3.5 Higgs One Loop Contribution from the W and Goldstone Boson

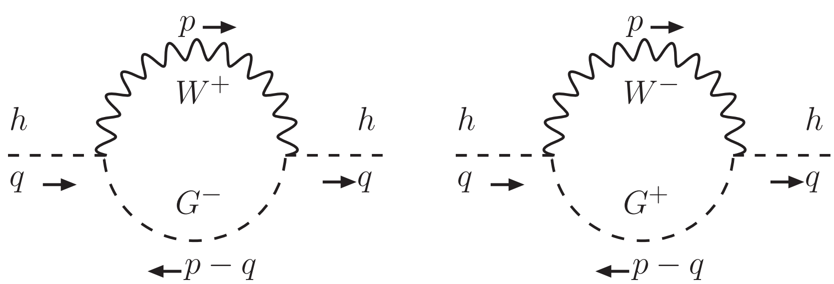


Figure 3.8: W Boson and Goldstone loop with 2 vertices

We get two diagrams for the one loop contribution from the W and Goldstone boson mixed loops, one with a W^+ and G^- and the other with a W^- and G^+. We put a factor of 2 in our calculations since the two diagrams have separate and equal contributions. We again start from the Feynman rules and do the calculation in steps by considering individual terms one by one,

$$\begin{aligned} & -2(-i\frac{g}{2})(i\frac{g}{2})\int \frac{d^dp}{(2\pi)^d}[q+(p-q)]_\mu[-q-(p-q)]_\nu(-i)\left[\frac{\eta^{\mu\nu}}{p^2+m_W{}^2}\right. \\ & \left. +\frac{p^\mu p^\nu}{m_W{}^2}\left(\frac{1}{p^2+m_W{}^2}-\frac{1}{p^2}\right)\right]\frac{-i}{(p-q)^2} \\ = & \ \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[(2q-p)_\mu(p-2q)_\nu\left[\frac{\eta^{\mu\nu}}{(p^2+m_W{}^2)(p-q)^2}\right.\right. \\ & \left.\left. +\frac{p^\mu p^\nu}{m_W{}^2}\left(\frac{1}{(p^2+m_W{}^2)(p-q)^2}-\frac{1}{p^2(p-q)^2}\right)\right]\right] \end{aligned}$$

$$
\begin{aligned}
&= \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{(2q-p)(p-2q)}{(p^2+m_W{}^2)(p-q)^2}\right. \\
&\quad \left.+\frac{[p(2q-p)][p(p-2q)]}{m_W{}^2}\left(\frac{1}{(p^2+m_W{}^2)(p-q)^2}-\frac{1}{p^2(p-q)^2}\right)\right] \\
&= \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{4(p\cdot q)-p^2-4q^2}{(p^2+m_W{}^2)(p-q)^2}\right. \\
&\quad \left.-\frac{p^4-4p^2(p\cdot q)+4(p\cdot q)^2}{m_W{}^2}\left(\frac{1}{(p^2+m_W{}^2)(p-q)^2}-\frac{1}{p^2(p-q)^2}\right)\right] \\
&= \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\frac{4(p\cdot q)-p^2-4q^2}{(p^2+m_W{}^2)(p-q)^2} \\
&\quad -\frac{g^2}{2}\frac{1}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{p^4-4p^2(p\cdot q)+4(p\cdot q)^2}{(p^2+m_W{}^2)(p-q)^2} \\
&\quad +\frac{g^2}{2}\frac{1}{m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{p^4-4p^2(p\cdot q)+4(p\cdot q)^2}{p^2(p-q)^2}. \qquad (3.39)
\end{aligned}
$$

Looking at the 1^{st} term in equation 3.39,

$$
\begin{aligned}
&\frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\frac{4(p\cdot q)-p^2-4q^2}{(p^2+m_W{}^2)(p-q)^2} \\
&= \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{4(p\cdot q)}{(p^2+m_W{}^2)(p-q)^2}-\frac{p^2+m_W{}^2-m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{4q^2}{(p^2+m_W{}^2)(p-q)^2}\right] \\
&= \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{2[(p-q)^2+m_W{}^2]}{(p^2+m_W{}^2)(p-q)^2}+\frac{2(p^2+m_W{}^2)}{(p^2+m_W{}^2)(p-q)^2}+\frac{2q^2}{(p^2+m_W{}^2)(p-q)^2}\right. \\
&\quad \left.-\frac{1}{(p-q)^2}+\frac{m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{4q^2}{(p^2+m_W{}^2)(p-q)^2}\right] \\
&= \frac{g^2}{2}\int\frac{d^dp}{(2\pi)^d}\left[-\frac{2}{p^2+m_W{}^2}-\frac{2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}+\frac{2}{(p-q)^2}+\frac{2q^2}{(p^2+m_W{}^2)(p-q)^2}\right. \\
&\quad \left.-\frac{1}{(p-q)^2}+\frac{m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{4q^2}{(p^2+m_W{}^2)(p-q)^2}\right] \\
&= \frac{g^2}{2}[-2A(W)-2WB(W,0)-2sB(W,0)+WB(W,0)+4sB(W,0)] \\
&= \frac{g^2}{2}[-2A(W)-WB(W,0)+2sB(W,0)] \\
&= \frac{g^2}{W}[-WA(W)-\frac{1}{2}W^2B(W,0)+sWB(W,0)]. \qquad (3.40)
\end{aligned}
$$

Looking at the 2^{nd} term in equation 3.39,

$$-\frac{g^2}{2m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{p^4-4p^2(p\cdot q)+4(p\cdot q)^2}{(p^2+m_W{}^2)(p-q)^2}. \tag{3.41}$$

Here,

$$\begin{aligned}
&-\frac{g^2}{2m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\frac{p^4-4p^2(p\cdot q)}{(p^2+m_W{}^2)(p-q)^2}\\
=\;&-\frac{g^2}{2m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{p^4-2p^2(p\cdot q)}{(p^2+m_W{}^2)(p-q)^2}-\frac{2p^2(p\cdot q)}{(p^2+m_W{}^2)(p-q)^2}\right]\\
=\;&-\frac{g^2}{2m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[\frac{(p^2+m_W{}^2)(p-q)^2-p^2q^2-q^2m_W{}^2+q^2m_W{}^2-m_W{}^2(p-q)^2}{(p^2+m_W{}^2)(p-q)^2}\right.\\
&\left.+p^2\left[\frac{(p-q)^2+m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{p^2+m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{q^2}{(p^2+m_W{}^2)(p-q)^2}\right]\right]\\
=\;&-\frac{g^2}{2m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[1-\frac{q^2(p^2+m_W{}^2)}{(p^2+m_W{}^2)(p-q)^2}+\frac{q^2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}\right.\\
&-\frac{m_W{}^2(p-q)^2}{(p^2+m_W{}^2)(p-q)^2}+p^2\left[\frac{(p-q)^2}{(p^2+m_W{}^2)(p-q)^2}+\frac{m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{1}{(p-q)^2}\right.\\
&\left.\left.-\frac{q^2}{(p^2+m_W{}^2)(p-q)^2}\right]\right]\\
=\;&-\frac{g^2}{2m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[1-\frac{q^2}{(p-q)^2}+\frac{q^2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{m_W{}^2}{(p^2+m_W{}^2)}\right.\\
&\left.+\left[\frac{p^2}{(p^2+m_W{}^2)}+\frac{p^2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{p^2}{(p-q)^2}-\frac{p^2q^2}{(p^2+m_W{}^2)(p-q)^2}\right]\right]\\
=\;&-\frac{g^2}{2m_W{}^2}\int\frac{d^dp}{(2\pi)^d}\left[1-\frac{q^2}{(p-q)^2}+\frac{q^2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}-\frac{m_W{}^2}{(p^2+m_W{}^2)}\right.\\
&+\left[\frac{p^2+m_W{}^2}{(p^2+m_W{}^2)}-\frac{m_W{}^2}{(p^2+m_W{}^2)}+\frac{m_W{}^2(p^2+m_W{}^2)}{(p^2+m_W{}^2)(p-q)^2}-\frac{m_W{}^4}{(p^2+m_W{}^2)(p-q)^2}\right.\\
&\left.\left.-\frac{p^2}{(p-q)^2}-\frac{q^2(p^2+m_W{}^2)}{(p^2+m_W{}^2)(p-q)^2}+\frac{q^2m_W{}^2}{(p^2+m_W{}^2)(p-q)^2}\right]\right]\\
=\;&-\frac{g^2}{2W}[-sWB(W,0)-WA(W)-WA(W)-W^2B(W,0)-sWB(W,0)]
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{g^2}{2W}[-2sWB(W,0) - W^2B(W,0) - 2WA(W)] \\
&= \frac{g^2}{W}\left[\frac{1}{2}W^2B(W,0) + sWB(W,0) + WA(W)\right]. \qquad (3.42)
\end{aligned}
$$

And,

$$
\begin{aligned}
&-\frac{g^2}{2m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{4(p\cdot q)^2}{(p^2+m_W{}^2)(p-q)^2} \\
&= -\frac{2g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{(p\cdot q)^2}{(p^2+m_W{}^2)(p-q)^2} \\
&= -\frac{2g^2}{W}\left[\frac{1}{4}WA(W) + \frac{1}{4}sA(W) + \frac{1}{4}(s+W)^2B(W,0)\right] \\
&= \frac{g^2}{W}\left[-\frac{1}{2}WA(W) - \frac{1}{2}sA(W) - \frac{1}{2}(s+W)^2B(W,0)\right]. \qquad (3.43)
\end{aligned}
$$

Looking at the 3^{rd} term in equation 3.39,

$$
\frac{g^2}{2}\frac{1}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{p^4 - 4p^2(p\cdot q) + 4(p\cdot q)^2}{p^2(p-q)^2}. \qquad (3.44)
$$

Here, the 1^{st} part is similar to 3.35,

$$
\begin{aligned}
&\frac{g^2}{2}\frac{1}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{p^4 - 4p^2(p\cdot q)}{p^2(p-q)^2} \\
&= 0. \qquad (3.45)
\end{aligned}
$$

And, for the 2^{nd} part which is similar to 3.36,

$$
\begin{aligned}
&\frac{g^2}{2}\frac{1}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{4(p\cdot q)^2}{p^2(p-q)^2} \\
&= 2\frac{g^2}{m_W{}^2}\int \frac{d^dp}{(2\pi)^d}\frac{(p\cdot q)^2}{p^2(p-q)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\frac{g^2}{W}\left[\frac{1}{4}s^2 B(0,0)\right] \\
&= \frac{g^2}{W}\left[\frac{1}{2}s^2 B(0,0)\right]. \qquad (3.46)
\end{aligned}
$$

Adding 3.40, 3.42, 3.43, 3.45 and 3.46 we get,

$$
\begin{aligned}
&\frac{g^2}{W}\left[-\frac{1}{2}(s-W)^2 B(W,0) - \frac{1}{2}WA(W) - \frac{1}{2}sA(W) + \frac{1}{2}s^2 B(0,0)\right] \\
= \; & g^2\left[-\frac{1}{2W}(s-W)^2 B(W,0) - \frac{1}{2}A(W) - \frac{s}{2W}A(W) + \frac{1}{2}s^2 B(0,0)\right]. \qquad (3.47)
\end{aligned}
$$

3.6 Sum of all W and Goldstone Boson Contributions

We take the sum of all the terms we have obtained until now for the W and Goldstone boson loops. Adding 3.16, 3.38 and 3.47, we get,

$$
\begin{aligned}
&g^2\left[\frac{3}{2}A(W) + W\right] + g^2\left[-\frac{2W}{\epsilon}\right] + g^2\left[\left(s - 3W - \frac{s^2}{4W}\right)B(W,W)\right. \\
&+\frac{1}{2W}(s-W)^2 B(W,0) + 2W + \frac{1}{2}A(W) - \frac{1}{4W}s^2 B(0,0) - \frac{4}{\epsilon}W \\
&\left.-\frac{1}{2W}(s-W)^2 B(W,0) - \frac{1}{2}A(W) - \frac{s}{2W}A(W) + \frac{1}{2W}s^2 B(0,0)\right] \\
= \; & g^2\left[\frac{3}{2}A(W) + W\right] + g^2\left[-\frac{2W}{\epsilon}\right] + g^2\left[\left(s - 3W - \frac{s^2}{4W}\right)B(W,W)\right. \\
&\left.-\frac{s}{2W}A(W) + 2W + \frac{1}{4W}s^2 B(0,0) - \frac{4}{\epsilon}W\right]. \qquad (3.48)
\end{aligned}
$$

3.7 Higgs One-loop Contributions from the Z and Goldstone boson

The vertex term for calculations involving the Z boson is $(g^2+g'^2)v/2$. We get Feynman diagrams exactly similar to those for the W boson but with the W replaced by Z.

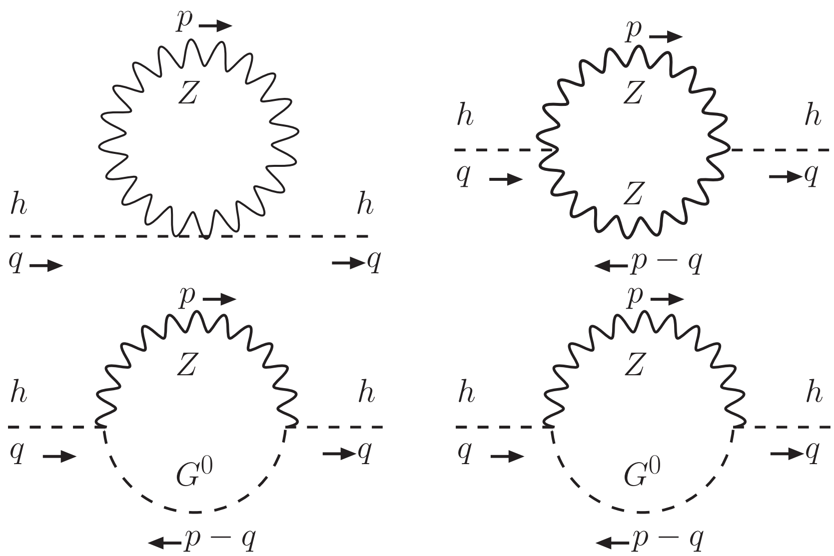


Figure 3.9: One loop contributions from the Z anf Goldstone boson

The contributions are found to be exactly similar to the terms for the W Boson in 3.48, but with an extra symmetry factor of 1/2 since the Z is its own antiparticle and with the appropriate coupling term. For the Z boson, we get,

$$\frac{(g^2+g'^2)}{2}\left[\frac{3}{2}A(Z)+Z\right]+\frac{(g^2+g'^2)}{2}\left[-\frac{2Z}{\epsilon}\right]+\frac{(g^2+g'^2)}{2}\left[\left(s-3Z-\frac{s^2}{4Z}\right)B(Z,Z)\right.$$
$$\left.-\frac{s}{2Z}A(Z)+2Z+\frac{1}{4Z}s^2B(0,0)-\frac{4}{\epsilon}Z\right]. \tag{3.49}$$

3.8 Sum of all the Contributions

We take the sum of all the terms we have calculated till now, including contributions from the Higgs self-interaction, the top quark, Goldstone self-interations, W, Z and Goldstone bosons. Adding 3.8, 3.10, 3.14, 3.15, 3.16, 3.38, 3.48 and 3.49, we get,

$$
\begin{aligned}
&-18\lambda^2 v^2 B(h,h) + 3\lambda A(h) + 3{y_t}^2[(4t-s)B(t,t)] - 6{y_t}^2 A(t) \\
&-6\lambda^2 v^2 B(0,0) + g^2\left[\frac{3}{2}A(W) + W\right] + g^2\left[-\frac{2W}{\epsilon}\right] \\
&+g^2\left[\left(s - 3W - \frac{s^2}{4W}\right)B(W,W) - \frac{s}{2W}A(W) + 2W + \frac{1}{4W}s^2 B(0,0)\right. \\
&\left.-\frac{4}{\epsilon}W\right] + \frac{(g^2+g'^2)}{2}\left[\frac{3}{2}A(Z) + Z\right] + \frac{(g^2+g'^2)}{2}\left[-\frac{2Z}{\epsilon}\right] \\
&+\frac{(g^2+g'^2)}{2}\left[\left(s - 3Z - \frac{s^2}{4Z}\right)B(Z,Z) - \frac{s}{2Z}A(Z) + 2Z + \frac{1}{4Z}s^2 B(0,0)\right] \\
&\left.-\frac{4}{\epsilon}Z\right]. \qquad (3.50)
\end{aligned}
$$

We see that amazingly 3.50 has all the terms in 3.5 which is exactly what we need for the minimization condition of the Effective Potential. So we exclude those terms and we set the terms with $1/\epsilon$ to be equal to the negative of the counterterms and then they cancel out. We also have terms with $B(0,0)$ and by utilizing the relation, $h = 2\lambda v^2$ and equations, B.3 and B.4 they can be expressed as,

$$
\begin{aligned}
&-6\lambda^2 v^2 B(0,0) + g^2\left[\frac{1}{4W}s^2 B(0,0)\right] + (g^2+g'^2)\frac{1}{2}\left[\frac{1}{4Z}s^2 B(0,0)\right] \\
=\ &\left[-6\lambda^2\frac{h}{2\lambda}\lambda^2 + \frac{4g^2}{4g^2v^2}s^2 + \frac{1}{2}\frac{4(g^2+g'^2)}{4(g^2+g'^2)v^2}s^2\right]B(0,0) \\
=\ &\left[-3\lambda h + \frac{1}{v^2}s^2 + \frac{1}{2v^2}s^2\right]B(0,0)
\end{aligned}
$$

$$
\begin{aligned}
&= \left[-3\lambda h + 2\frac{\lambda}{h}s^2 + \frac{\lambda}{h}s^2\right] B(0,0) \\
&= 3\frac{\lambda}{h}(s^2 - h^2)B(0,0). \qquad (3.51)
\end{aligned}
$$

Referring again to [35], it has been shown that the term $(3\lambda/h)(s^2 - h^2)B(0,0)$ can be considered alongside terms coming from 2-loop calculations and it cancels out against other terms. Thus in the end, we are left with the final result of 3.7 and we can obtain the Higgs squared pole mass M_h at 1-loop order from 3.6.

CHAPTER 4

DERIVATION OF THE INTERPOLATION FORMULAS

4.1 Preamble

As mentioned before, the interpolation formulas relate the $\overline{\text{MS}}$ inputs to on-shell observables. The Standard Model $\overline{\text{MS}}$ Lagrangian parameters to be evaluated include:

$$\begin{aligned}
\text{Higgs sector:} \quad & \lambda, m^2, \\
\text{gauge couplings:} \quad & g_3,\ g,\ g', \\
\text{quark Yukawa couplings:} \quad & y_t,\ y_b,\ y_c,\ y_s,\ y_u,\ y_d, \\
\text{lepton Yukawa couplings:} \quad & y_\tau,\ y_\mu,\ y_e,
\end{aligned} \tag{4.1}$$

where we have neglected the neutrino sector. These will be our outputs for the interpolation formulas we will derive. Also omitted here are the four physical parameters (three flavor-mixing angles and one CP-violating phase angles) associated with the Cabibbo-Kobayashi-Maskawa (CKM) matrix for quarks, which can be considered separately and decouple from the discussion below to a high degree of accuracy due to unitarity of the CKM matrix.

Each of the 14 quantities in eq. (4.1) is a running parameter, dependent on the choice of $\overline{\text{MS}}$ renormalization scale Q, governed by renormalization group equations that are now known [36]-[62] with some effects through 5-loop order. The normalization convention of the Higgs sector parameters is the now-standard one such that the tree-level potential for the canonically normalized real neutral component of the Higgs field is $V = \frac{1}{2}m^2H^2 + \frac{\lambda}{4}H^4$.

In particular, the Higgs squared mass parameter m^2 is negative, as required by electroweak symmetry breaking, and can be traded for the vacuum expectation value (VEV) v for H, defined as the minimum of the all-orders effective potential in Landau gauge, so that the sum of all tadpole diagrams (including the tree-level tadpole) simply vanishes. In practice, the effective potential is known fully at 2-loop [63, 64] and 3-loop [65, 66, 67] orders, supplemented by the QCD 4-loop contributions [60].

For the purposes of matching to ultraviolet new physics proposals, one should work in a non-decoupling scheme, in which all of the Standard Model particles including the top quark are propagating degrees of freedom. For this reason, we choose as a benchmark $\overline{\rm MS}$ renormalization scale the value $Q = 200$ GeV, which is somewhat arbitrary but has the advantages of being a round number, safely above the top-quark mass, and probably well below the scale of new physics. Furthermore, taking a fixed scale (rather than, say, the experimental top-quark mass, which is subject to uncertainty and change) provides for better numerical stability. The results given here can then be evolved to any desired matching scale by the Standard Model renormalization group equations.

The standard reference for important experimental results in high energy physics, the RPP [12] published by the PDG, instead (so far, at least) summarizes our knowledge of the Standard Model in terms of what we will refer to as the on-shell quantities. The RPP quantities that are in the most direct correspondence to the Lagrangian parameters in eq. (4.1) are:

fine-structure constant:	$\alpha = 1/137.035999084\ldots$ and $\Delta\alpha^{(5)}_{\rm had}(M_Z)$,
Fermi decay constant:	G_F,
5-quark QCD coupling:	$\alpha_S^{(5)}(M_Z)$,
heavy particle physical masses:	M_t, M_h, M_Z, M_W,

$$\text{running light quark masses:} \quad m_b(m_b),\, m_c(m_c),\, m_s(2\text{ GeV}), \qquad (4.2)$$

$$m_u(2\text{ GeV}),\, m_d(2\text{ GeV}),$$

$$\text{lepton pole masses:} \quad M_\tau,\, M_\mu,\, M_e. \qquad (4.3)$$

These will be our inputs for the interpolation formulas. It should be noted that in this work M_Z and M_W are the on-shell masses in the PDG parameterization; these are related to the gauge-invariant complex pole squared masses $s_p = (M_p - i\Gamma_p/2)^2$ by $M = M_p(1+\delta)/\sqrt{1-\delta}$, where $\delta = \Gamma_p^2/4M_p^2$ in each case. (For recent discussions, see refs. [68] and [137].) In principle, the hadronic contribution to the fine-structure constant, $\Delta\alpha^{(5)}_{\text{had}}(M_Z)$ is not independent, and could be determined in terms of the other quantities, but in practice, it cannot be perturbatively evaluated and therefore is taken as an independent experimental input. The W boson mass, M_W, can also be determined in terms of the others. The other 14 on-shell quantities in eq. (4.3) are dual to those of the 14 independent $\overline{\text{MS}}$ parameters in eq. (4.1). This means that one can take the $\overline{\text{MS}}$ parameters as theoretical inputs with the on-shell quantities as outputs, or one can take the on-shell quantities as experimental inputs and view the $\overline{\text{MS}}$ parameters as the outputs.

A great deal of effort has gone into relating the two sets of parameters, and to evaluating M_W in terms of the others. For a necessarily incomplete set of earlier references, see [69]-[132] and papers discussed therein. The present work is based on the computer code `SMDR` [8, 33], which implements calculations in refs. [35, 133, 134, 135, 136, 137] in the tadpole-free pure $\overline{\text{MS}}$ scheme. This code contains command-line utilities and library programs for fitting the on-shell parameters in terms of the $\overline{\text{MS}}$ parameters, and vice versa, and for performing the renormalization group running in the Standard Model, implementing the state-of-the-art calculations.

The purpose of the present work is to provide simple and convenient interpolation formulas that accurately provide the $\overline{\text{MS}}$ parameters in eq. (4.1) and M_W in terms of the on-shell parameters in eq. (4.3), obtained by doing a fit to the results of `SMDR`. The simplicity and accuracy of the interpolation formulas is aided by the fact that the allowed parameters in the Standard Model are now all experimentally restricted to rather narrow ranges.

We now discuss the organization and notation of the interpolation formulas below. First, define a set of benchmark (denoted by subscript 0) on-shell inputs, using the values from the most recent 2022 PDG data:

$$\begin{aligned}
&\alpha_0 = 1/137.035999084, \qquad \Delta\alpha^{(5)}_{\text{had}}(M_Z)_0 = 0.027660,\\
&G_{F0} = 1.1663787 \times 10^{-5}, \qquad \alpha^{(5)}_{S0}(M_Z) = 0.1179,\\
&M_{t0} = 172.5 \text{ GeV}, \qquad M_{h0} = 125.25 \text{ GeV}, \qquad M_{Z0} = 91.1876 \text{ GeV},\\
&m_b(m_b)_0 = 4.18 \text{ GeV}, \qquad m_c(m_c)_0 = 1.27 \text{ GeV}, \qquad m_s(2\text{ GeV})_0 = 93 \text{ MeV},\\
&m_u(2\text{ GeV})_0 = 2.16 \text{ MeV}, \qquad m_d(2\text{ GeV})_0 = 4.67 \text{ MeV},\\
&M_{\tau 0} = 1.77686 \text{ GeV}, \qquad M_{\mu 0} = 0.1056583745 \text{ GeV},\\
&M_{e0} = 0.5109989461 \text{ MeV}.
\end{aligned} \tag{4.4}$$

The Sommerfeld fine structure constant α is very accurately known compared to the others, with a fractional uncertainty of 1.5×10^{-10}, so no variation in it will be considered.

For the other on-shell parameters, we next define the following dimensionless quantities:

$$\delta_Z = (M_Z - M_{Z0})/(0.001 \text{ GeV}), \tag{4.5}$$

$$\delta_t = (M_t - M_{t0})/(1 \text{ GeV}), \tag{4.6}$$

$$\delta_h = (M_h - M_{h0})/(0.1 \text{ GeV}), \tag{4.7}$$

$$\delta_S = 1000 \left[\alpha^{(5)}_S(M_Z) - \alpha^{(5)}_{S0}(M_Z)\right], \tag{4.8}$$

$$\delta_a = 10^4 \left[\Delta\alpha^{(5)}_{\text{had}}(M_Z) - \Delta\alpha^{(5)}_{\text{had},0}(M_Z) \right], \tag{4.9}$$

as measures of the deviation from the benchmark model. The normalizations of these five quantities are chosen so that a change in the on-shell input by an amount of order the present experimental uncertainty will correspond very roughly to an order 1 change in the corresponding δ. In the interpolation formulas found below, we will often give at least the contributions linear in the five δ's above, even when they are numerically too small to be practically significant, in order to quantitatively illustrate their contributions to the parametric errors.

In the interpolation formulas for the Yukawa couplings at $Q = 200$ GeV, we will also make use of the variations in the fermion masses as needed, parameterized by

$$\Delta_f = \frac{m_f}{m_{f0}} - 1, \tag{4.10}$$

for $f = b, c, s, u, d, \tau, \mu, e$, where the m_f are running $\overline{\text{MS}}$ masses $m_b(m_b)$, $m_c(m_c)$, $m_s(2 \text{ GeV})$, $m_u(2 \text{ GeV})$, and $m_d(2 \text{ GeV})$ for the light quarks, and pole (on-shell) masses for the leptons $m_f = M_\tau$, M_μ, and M_e. Also, in a few of the interpolation formulas, we will include the small effect due to a possible deviation in the Fermi decay constant from its benchmark central value, parameterized by

$$\Delta_{G_F} = \frac{G_F}{G_{F0}} - 1. \tag{4.11}$$

The effects of this are typically expected to be very small since the fractional uncertainty in G_F given in the RPP is about 5×10^{-7}.

4.2 Procedure

The interpolation formulas presented below were obtained by running v1.2 of `SMDR` with its default choices repeatedly for points in parameter space on grids that cover the plausible allowed ranges of inputs, and then performing a fit to obtain the coefficients, which were then validated on more parameter space grids.

For instance, we could consider n input quantities x_i labeled by an index $i = 1, \ldots, n$. For some output quantity Y (the quantities in eq. 4.1, as well as M_W), then we would have a candidate interpolating formula,

$$Y = \sum_{i=1}^{n} c_i x_i, \tag{4.12}$$

and our goal was to fit the numerical coefficients c_i. The x_i are the δs from eqs. (4.5 - 4.9) and in some cases, will also include Δ_f and Δ_{G_F} from eqs. 4.10 and 4.11 respectively. The x_i include quadratic and cubic monomials on the deltas, not just the deltas. To accomplish our goal, we could obtain N data points labeled by an index $k = 1, \ldots, N$. For each of these data points, the input quantities $x_i^{(k)}$ and the output $Y^{(k)}$ could therefore be obtained, by running `SMDR`. Then we could define the best-fit coefficients c_i as the ones that minimized the quantity

$$\chi^2 = \sum_{k=1}^{N} \left(Y^{(k)} - \sum_{i=1}^{n} c_i x_i^{(k)} \right)^2 . \tag{4.13}$$

The minimum would occur when the following n quantities vanish:

$$\frac{d\chi^2}{dc_i} = 0, \tag{4.14}$$

which would imply

$$\sum_{k=1}^{N} x_i^{(k)} \left(Y^{(k)} - \sum_{j=1}^{n} c_j x_j^{(k)} \right) = 0. \tag{4.15}$$

To put this into a nicer form, we could define

$$v_i = \sum_{k=1}^{N} x_i^{(k)} Y^{(k)} \tag{4.16}$$

$$A_{ij} = \sum_{k=1}^{N} x_i^{(k)} x_j^{(k)}. \tag{4.17}$$

In terms of the vector v_i and the matrix A_{ij}, eq. (4.15) can be written (following the Einstein summation convention) as,

$$A_{ij} c_j = v_i, \tag{4.18}$$

which has the solution for the best-fit coefficients

$$c_j = \left(A^{-1}\right)_{ji} v_i. \tag{4.19}$$

It is to be noted that x_i does not only include linear terms, it may also include quadratic and cubic terms. The x_i will include linear inputs like δ_Z, δ_t, δ_h, δ_S, δ_a, and in some cases, quadratic inputs like δ_Z^2, δ_t^2, δ_h^2 and so on. In some cases, for instance, for λ and the light quarks, we even had cubic inputs like δ_t^3 and δ_S^3. And, the value of n in eq. 4.12 could vary depending on the number of inputs we want to keep.

For our data set, we formed a grid of $N = 5^5 = 3125$ inputs by taking all possible combinations of 5 different values of the 5 on-shell experimental quantities, M_Z, M_t, M_h, $\alpha_S^{(5)}(M_Z)$ and $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ when it came to finding interpolation formulas for M_W, λ, m^2,

g_3, g, g' and y_t. The data set was created by running 5 different iterations of `SMDR` in parallel on the NICADD compute cluster [138] and each iteration would take approximately 36 hours. So, a single run of SMDR to produce our dataset of 3125 values would have taken approximately 180 hours. The 5 different values were chosen by varying the values in 4.4 by increments of 2.5 times the experimental uncertainty σ both in the positive and negative directions. For example, for M_t, the value of the experimental uncertainty σ (obtained from 2022 PDG data) is 0.7 GeV. So, the 5 values chosen were,

$$\begin{aligned}
M_{t0} - 5\sigma &= 169 \text{ GeV}, \\
M_{t0} - 2.5\sigma &= 170.75 \text{ GeV}, \\
M_{t0} &= 172.5 \text{ GeV}, \\
M_{t0} + 2.5\sigma &= 174.25 \text{ GeV}, \\
M_{t0} + 5\sigma &= 176 \text{ GeV}.
\end{aligned} \tag{4.20}$$

When it came to the creating a data set for the other parameters, namely the Yukawa couplings of lights quarks and leptons, we formed a grid $3^5 \times 5 = 1215$ inputs. The approach was slightly different this time since we were also using the mass of the respective quark or lepton as an input. We took all possible combinations of 3 different values of the 5 on-shell experimental quantities and 5 different values of the respective masses. The 3 different values were chosen by varying the central values by increments of 5 times the experimental uncertainty σ this time and for the 5 different values of masses, we followed the exact approach of M_t.

We aim to provide results accurate to well under the experimental and theoretical uncertainties, for deviations of the on-shell inputs by up to 5 times their RPP quoted experimental uncertainties. Contributions quadratic in the deviations therefore will also be included when necessary to achieve relative precision goals for each quantity as stated below. For each output parameter, we will quote a conservative fractional precision, which in this work refers to the fractional difference between the interpolation formula result and the output of `SMDR` with default scale-setting choices, obtained as all on-shell inputs are varied over ranges such that the total deviation from the experimental central values, added in quadrature, is $\leq 5\sigma$. Here we interpret the uncertainties quoted in the RPP as 1σ, even though a Gaussian distribution of errors may not be the appropriate description. It should be recognized that the actual theoretical uncertainty and the parametric uncertainty are both always much larger than this fractional precision. We have attempted to err on the side of including coefficients even when they are only significant for rather large deviations from the experimental central values.

We now provide the benchmark output results obtained using v1.2 of `SMDR` with default choices. We give many more significant digits than justified by the theoretical and parametric uncertainties, merely for the sake of reproducibility. The benchmark running $\overline{\text{MS}}$ parameters evaluated at the scale $Q = 200$ GeV are:

$$g_{30} = 1.1525136966, \tag{4.21}$$

$$g_0 = 0.64683244428, \tag{4.22}$$

$$g'_0 = 0.35885152738, \tag{4.23}$$

$$\lambda_0 = 0.12353343830, \tag{4.24}$$

$$m_0^2 = -(93.126827678 \text{ GeV})^2, \tag{4.25}$$

$$y_{t0} = 0.92377763013, \tag{4.26}$$

$$y_{b0} = 0.0153349059085, \tag{4.27}$$

$$y_{c0} = 0.00336181598480, \tag{4.28}$$

$$y_{s0} = 2.8885955612 \times 10^{-4}, \tag{4.29}$$

$$y_{d0} = 1.4505079604 \times 10^{-5}, \tag{4.30}$$

$$y_{u0} = 6.6738103560 \times 10^{-6}, \tag{4.31}$$

$$y_{\tau 0} = 0.0100065524355, \tag{4.32}$$

$$y_{\mu 0} = 5.8908805223 \times 10^{-4}, \tag{4.33}$$

$$y_{e0} = 2.7963423115 \times 10^{-6}. \tag{4.34}$$

Also, the physical W boson mass in the PDG parameterization is found to be, for this benchmark set of parameters,

$$M_{W0} = 80.352476 \text{ GeV}, \tag{4.35}$$

where we have used the `SMDR` default by computing the W boson pole mass in terms of the running parameters at $Q = 160$ GeV. The values in eqs. (4.21)-(4.35) will be used in the interpolation formulas below, as they give the results when all of the δ's vanish, by definition.

4.3 Interpolation formula for the W-boson mass

The result for M_W has recently become of heightened interest because of a report [139] from the Fermilab Tevatron's CDF collaboration which is incompatible with the Standard Model prediction, and in strong tension with other experimental results [12]. For the W-boson physical mass in the PDG convention, we find,

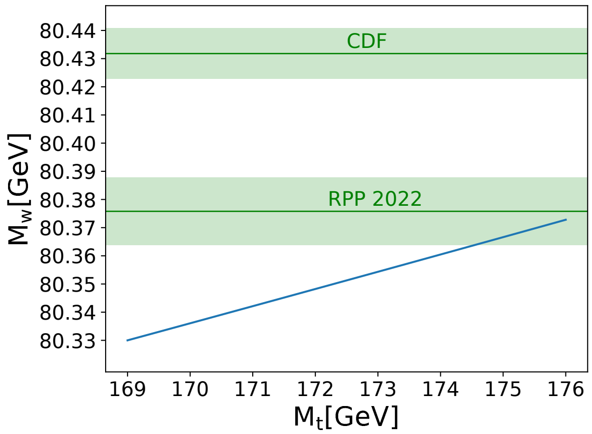
$$M_W = M_{W0} \left(1 + c^{t}_{M_W} \delta_t + c^{Z}_{M_W} \delta_Z + c^{a}_{M_W} \delta_a + c^{S}_{M_W} \delta_S + c^{h}_{M_W} \delta_h + c^{tt}_{M_W} \delta_t^2\right), \tag{4.36}$$

where M_{W0} was given in eq. (4.35), and the other potentially significant coefficients are

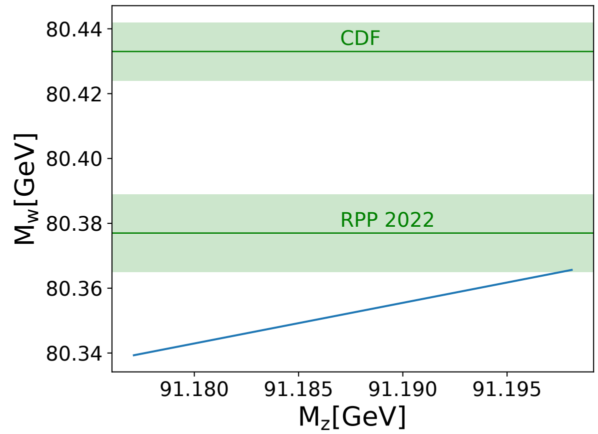
$$c^t_{M_W} = 7.61 \times 10^{-5}, \qquad c^Z_{M_W} = 1.56 \times 10^{-5}, \qquad c^a_{M_W} = -2.29 \times 10^{-5},$$
$$c^S_{M_W} = -8.8 \times 10^{-6}, \qquad c^h_{M_W} = -5.9 \times 10^{-7}, \qquad c^{tt}_{M_W} = 1.3 \times 10^{-7}. \tag{4.37}$$

This interpolation formula reproduces the results of `SMDR` (with its default scale-setting choices) to better than 0.1 MeV, which is much smaller than the current theoretical and experimental uncertainties when the input on-shell parameters are varied such that the total deviation from the central values, added in quadrature, is $\leq 5\sigma$. We include some plots obtained by using the interpolation formulas to show the change of M_W with respect to small changes in M_t, M_h and $\Delta\alpha^{(5)}_{had}$ respectively within the $\pm 5\sigma$ uncertainty. For comparison, we also produced 100 values of M_W (which would be required to make such plots) by running `SMDR` and it took approximately 6 hours. In our plots, we are also including the current experimental values of M_W as suggested by CDF and RPP.

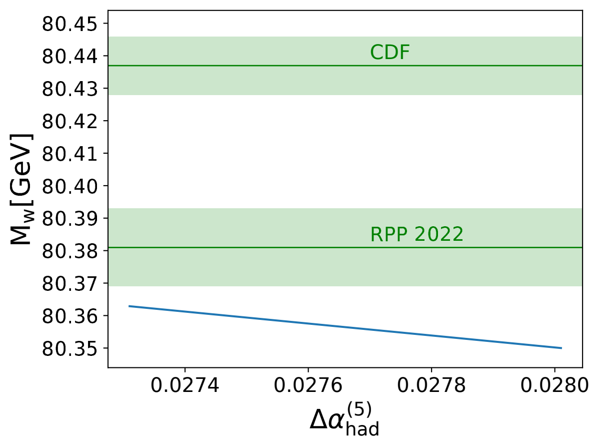
The results are based on the pure $\overline{\text{MS}}$ scheme used by `SMDR`, and can be compared with similar interpolation formula results based on on-shell [106] and hybrid [124] scheme calculations, which both used fits to a much wider range for the Higgs mass. A numerical comparison between the results from these three different approaches was made in ref. [137] (see in particular Figures 4.1 and 4.2), showing that they agree well within the theoretical uncertainty due to renormalization scale dependence, and supporting a theoretical error estimate of perhaps ± 4 MeV. This is less than the parametric error, coming principally from the top-quark mass, of about $6.11\delta_t + 1.25\delta_Z - 1.84\delta_a - 0.71\delta_S - 0.047\delta_h + 0.010\delta_t^2$, in MeV, which can be read off from eq. (4.37). The relatively large uncertainty associated with the top-quark mass is difficult to reduce since it is due in large part to the problems in connecting hadron collider measurements and simulations to a well-defined short-distance top-quark mass or Yukawa coupling.



(a) M_W vs M_t



(b) M_W vs M_Z



(c) M_W vs $\Delta\alpha^{(5)}_{had}$

Figure 4.1: Plots for the W-boson mass M_W at $Q = 200$ GeV obtained from eq. 4.36. The CDF and RPP 2022 bands are the current experimental measurements of M_W, as published by CDF and PDG respectively, including the experimental uncertainty of 1σ.

4.4 Interpolation formulas for the $\overline{\text{MS}}$ parameters

4.4.1 Higgs sector

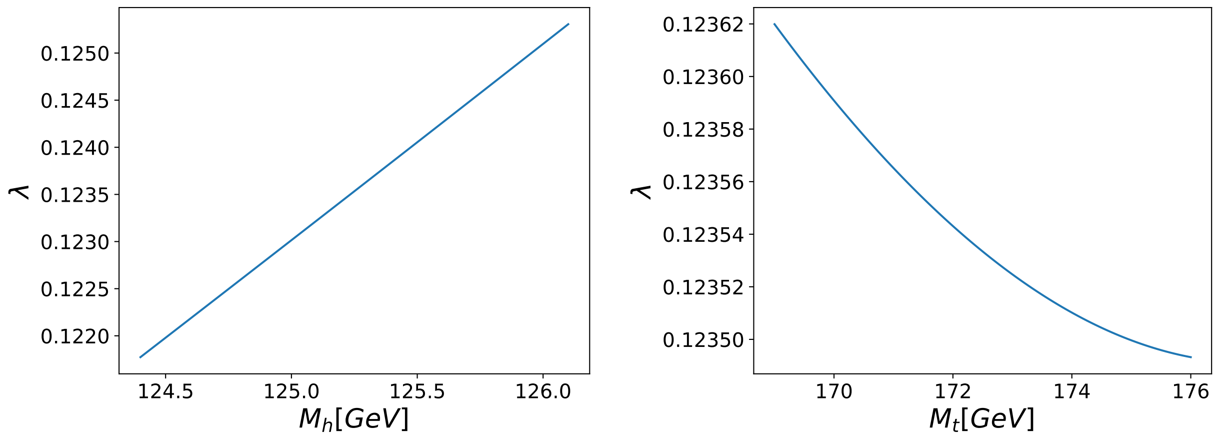
For the Higgs self-coupling λ at $Q = 200$ GeV, we find

$$\begin{aligned}\lambda &= \lambda_0 \big[1 + c_\lambda^h \delta_h + c_\lambda^t \delta_t + c_\lambda^Z \delta_Z + c_\lambda^S \delta_S + c_\lambda^a \delta_a + c_\lambda^{tt} \delta_t^2 + c_\lambda^{tS} \delta_t \delta_S + c_\lambda^{hh} \delta_h^2 \\ &\quad + c_\lambda^{ht} \delta_h \delta_t + c_\lambda^{SS} \delta_S^2 + c_\lambda^{hS} \delta_h \delta_S + c_\lambda^{ttt}\, \delta_t^3 + c_\lambda^{ttS}\, \delta_t^2 \delta_S + c_\lambda^b\, \Delta_b + c_\lambda^{G_F}\, \Delta_{G_F}\big], \qquad (4.38)\end{aligned}$$

where λ_0 was given in eq. (4.24), and the other coefficients are

$$\begin{aligned}&c_\lambda^h = 1.6823 \times 10^{-3}, \quad c_\lambda^t = -1.488 \times 10^{-4}, \quad c_\lambda^Z = -3.5 \times 10^{-7}, \\ &c_\lambda^S = -2.2 \times 10^{-7}, \quad c_\lambda^a = 3.4 \times 10^{-7}, \quad c_\lambda^{tt} = 1.528 \times 10^{-5}, \\ &c_\lambda^{tS} = -4.02 \times 10^{-6}, \quad c_\lambda^{hh} = 7.0 \times 10^{-7}, \quad c_\lambda^{ht} = -6.1 \times 10^{-7}, \\ &c_\lambda^{SS} = 3.0 \times 10^{-7}, \quad c_\lambda^{hS} = 6.4 \times 10^{-8}, \quad c_\lambda^{ttt} = 1.9 \times 10^{-7}, \\ &c_\lambda^{ttS} = -7.6 \times 10^{-8}, \quad c_\lambda^b = 4.5 \times 10^{-5}, \quad c_\lambda^{G_F} = 0.95. \qquad (4.39)\end{aligned}$$

This formula, based on a fit to the best available calculation of the physical Higgs boson mass [35, 137], agrees with the results of `SMDR` to better than 10^{-6} fractional precision in λ as the input parameters are varied over ranges with a total deviation, added in quadrature, of 5σ from their central values. Again, the theoretical and parametric errors are much larger than this fractional precision, with the top-quark mass giving the largest contribution to the error budget other than the Higgs boson mass itself. We include some plots to show the change in λ with respect to the change in M_h and M_t respectively. For comparison, we again produced 100 values of λ (which would be required to make such plots) by running `SMDR` and it took approximately 6 hours.



(a) λ vs M_h (b) λ vs M_t

Figure 4.2: Plots for the Higgs self-coupling λ at $Q = 200$ GeV obtained from eq. 4.38.

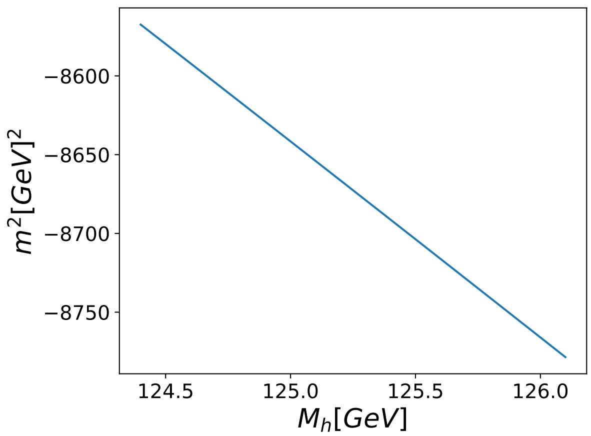
For the running Higgs squared mass parameter m^2 at $Q = 200$ GeV, we find

$$
\begin{aligned}
m^2 &= m_0^2 \big(1 + c_{m^2}^h \delta_h + c_{m^2}^t \delta_t + c_{m^2}^S \delta_S + c_{m^2}^Z \delta_Z + c_{m^2}^a \delta_a + c_{m^2}^{tt} \delta_t^2 + c_{m^2}^{tS} \delta_t \delta_S \\
&\quad + c_{m^2}^{hh} \delta_h^2 + c_{m^2}^{ht} \delta_h \delta_t\big), \qquad (4.40)
\end{aligned}
$$

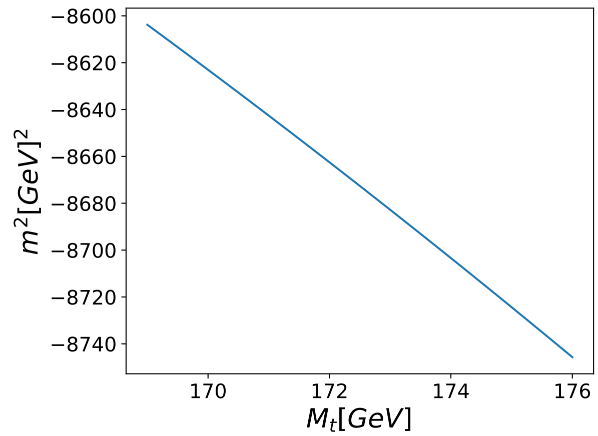
where m_0^2 was given in eq. (4.25), and the other significant coefficients are

$$
\begin{aligned}
&c_{m^2}^h = 1.4319 \times 10^{-3}, \quad && c_{m^2}^t = 2.337 \times 10^{-3}, \quad && c_{m^2}^S = -1.052 \times 10^{-4}, \\
&c_{m^2}^Z = -5.7 \times 10^{-7}, && c_{m^2}^a = 5.4 \times 10^{-7}, && c_{m^2}^{tt} = 2.02 \times 10^{-5}, \\
&c_{m^2}^{tS} = -2.45 \times 10^{-6}, && c_{m^2}^{hh} = 5.8 \times 10^{-7}, && c_{m^2}^{ht} = -4.3 \times 10^{-7}. \qquad (4.41)
\end{aligned}
$$

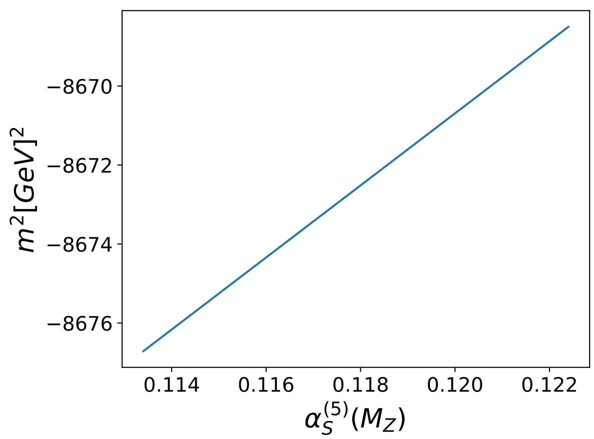
This formula provides agreement with the output of `SMDR` to a fractional precision of better than 10^{-5}. We include some plots to show the change in m^2 with respect to M_h, M_t and $\alpha_S^{(5)}(M_Z)$ respectively.



(a) m^2 vs M_h



(b) m^2 vs M_t



(c) m^2 vs $\alpha_S^{(5)}(M_Z)$

Figure 4.3: Plots for the Higgs squared mass parameter m^2 at $Q = 200$ GeV obtained from eq. 4.40.

4.4.2 Gauge couplings

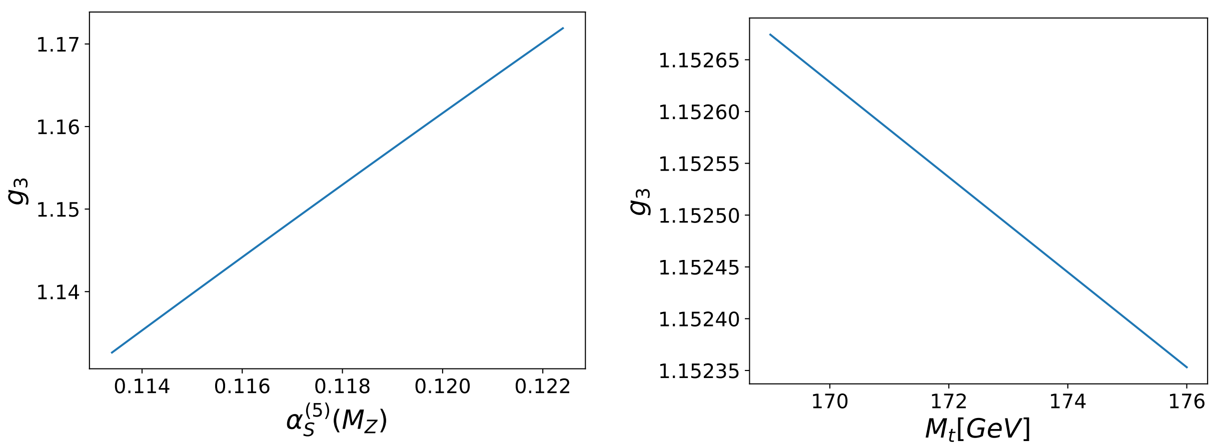
For the $SU(3)_c$ $\overline{\text{MS}}$ gauge coupling g_3 evaluated at $Q = 200$ GeV, we obtained the following interpolation formula:

$$g_3 \;=\; g_{30}\left(1 + c^S_{g_3}\delta_S + c^t_{g_3}\delta_t + c^{SS}_{g_3}\delta_S^2 + c^h_{g_3}\delta_h + c^Z_{g_3}\delta_Z + c^a_{g_3}\delta_a\right), \tag{4.42}$$

where g_{30} was given in eq. (4.21), and the coefficients are

$$c_{g_3}^S = 3.7875 \times 10^{-3}, \qquad c_{g_3}^t = -3.98 \times 10^{-5}, \qquad c_{g_3}^{SS} = -1.07 \times 10^{-5},$$
$$c_{g_3}^h = 2.5 \times 10^{-8}, \qquad c_{g_3}^Z = 2.7 \times 10^{-9}, \qquad c_{g_3}^a = -2.0 \times 10^{-9}. \qquad (4.43)$$

Note that the top-quark mass is significant here because we are relating the 5-quark QCD coupling $\alpha_S^{(5)}(M_Z)$ to the Standard Model QCD coupling g_3 with the top quark not decoupled. The three terms proportional to δ_S, δ_t, and δ_S^2 are sufficient to obtain a fractional precision compared to `SMDR` of better than 10^{-5}, but the linear deviation coefficients $c_{g_3}^h$, $c_{g_3}^Z$, and $c_{g_3}^a$ are also listed in order to illustrate the small size of the parametric errors. We include plots for g_3.



(a) g_3 vs $\alpha_S^{(5)}(M_Z)$ (b) g_3 vs M_t

Figure 4.4: Plots for the Standard Model QCD coupling g_3 at $Q = 200$ GeV obtained from eq. 4.42.

For the $SU(2)_L$ gauge coupling g, we find:

$$g = g_0 \left(1 + c_g^t \delta_t + c_g^a \delta_a + c_g^Z \delta_Z + c_g^S \delta_S + c_g^h \delta_h + c_g^{tt} \delta_t^2 + c_g^{tS} \delta_t \delta_S + c_g^{G_F} \Delta_{G_F}\right), \quad (4.44)$$

where g_0 was given in eq. (4.22), and the other coefficients are

$$c_g^t = 5.735 \times 10^{-5}, \qquad c_g^a = -2.295 \times 10^{-5}, \qquad c_g^Z = 1.558 \times 10^{-5},$$
$$c_g^S = -5.97 \times 10^{-6}, \qquad c_g^h = -8.5 \times 10^{-7}, \qquad c_g^{tt} = 1.9 \times 10^{-7},$$
$$c_g^{tS} = -7.8 \times 10^{-8}, \qquad c_g^{G_F} = 0.71. \tag{4.45}$$

This formula provides a fractional precision of better than 10^{-6} as the input on-shell parameters are varied with $\leq 5\sigma$ total deviation from their central values, added in quadrature. We include some plots.

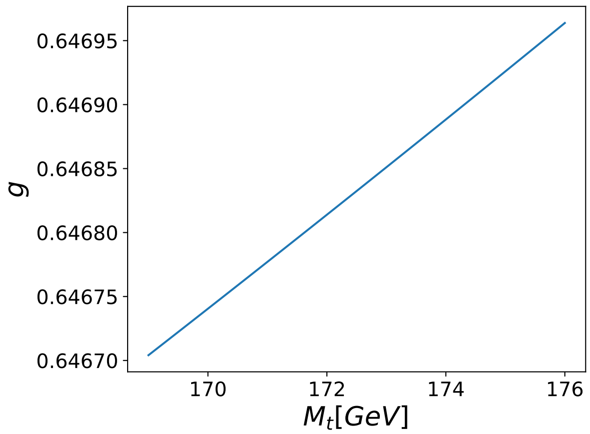
For the $U(1)_Y$ gauge coupling g', we find

$$g' = g'_0 \left(1 + c_{g'}^t \delta_t + c_{g'}^a \delta_a + c_{g'}^Z \delta_Z + c_{g'}^S \delta_S + c_{g'}^h \delta_h\right), \tag{4.46}$$

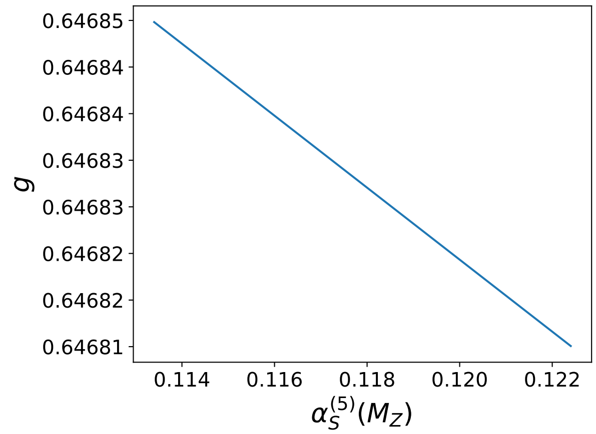
where g'_0 was given in eq. (4.23) and the other coefficients are

$$c_{g'}^t = -2.609 \times 10^{-5}, \qquad c_{g'}^a = 7.714 \times 10^{-5}, \qquad c_{g'}^Z = -4.70 \times 10^{-6},$$
$$c_{g'}^S = 3.29 \times 10^{-6}, \qquad c_{g'}^h = 2.6 \times 10^{-7}. \tag{4.47}$$

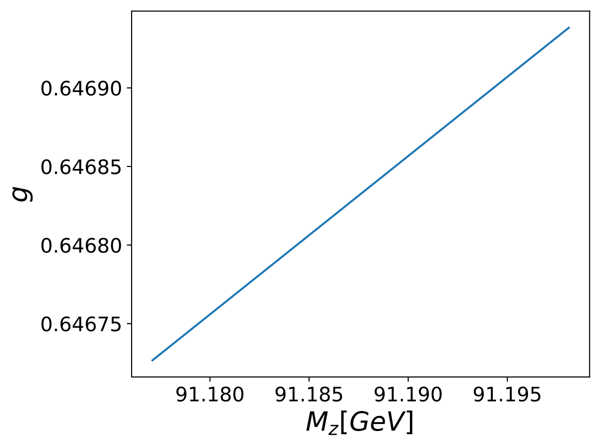
This formula again provides a fractional precision of better than 10^{-6} compared to `SMDR`. We include some plots.



(a) g vs M_t

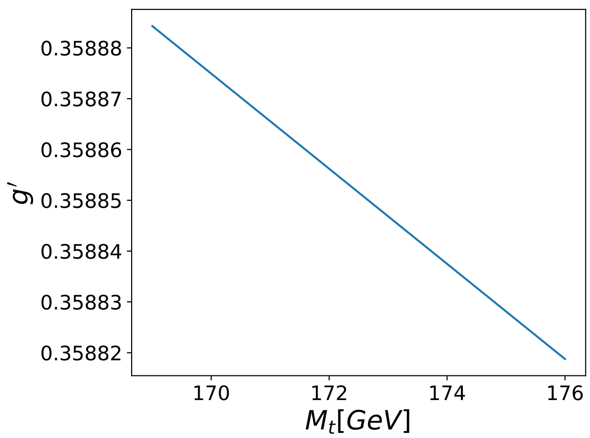


(b) g vs $\alpha_S^{(5)}(M_Z)$

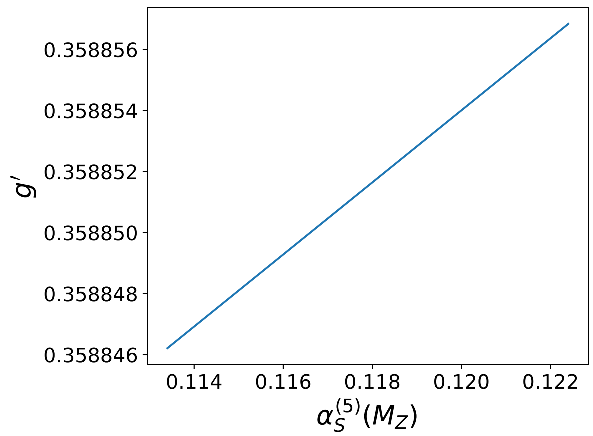


(c) g vs M_Z

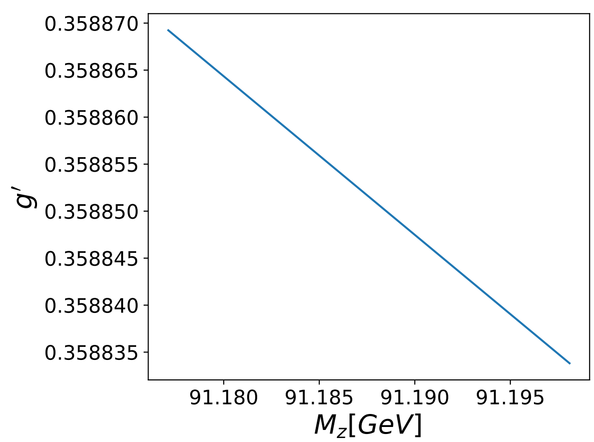
Figure 4.5: Plots for the $SU(2)_L$ gauge coupling g at $Q = 200$ GeV obtained from eq. 4.44.



(a) g' vs M_t



(b) g' vs $\alpha_S^{(5)}(M_Z)$



(c) g' vs M_Z

Figure 4.6: Plots for the $U(1)_Y$ gauge coupling g' at $Q = 200$ GeV obtained from eq. 4.46.

4.4.3 Top-quark Yukawa coupling

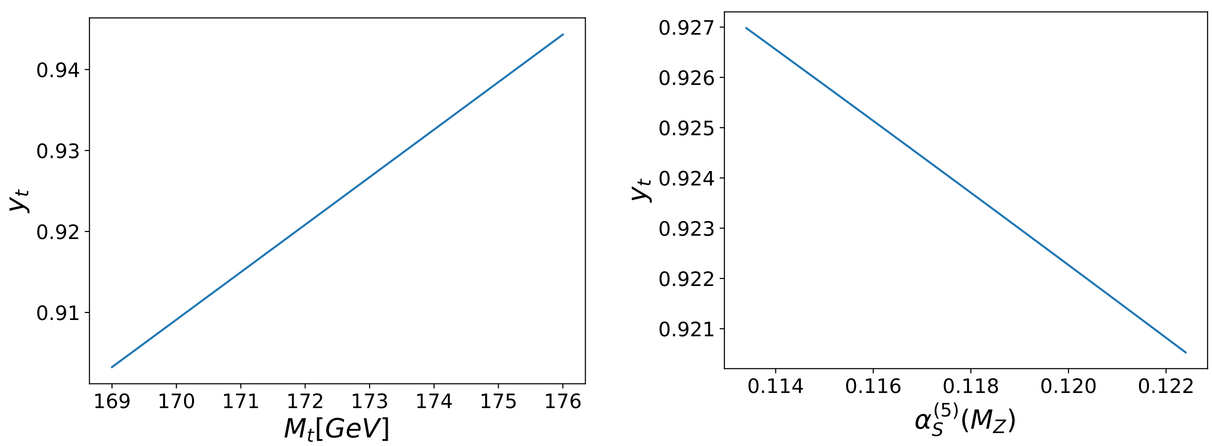
For the top-quark Yukawa coupling y_t at $Q = 200$ GeV, we find

$$y_t = y_{t0}\left(1 + c^t_{y_t}\delta_t + c^S_{y_t}\delta_S + c^h_{y_t}\delta_h + c^{tt}_{y_t}\delta_t^2 + c^{SS}_{y_t}\delta_S^2 + c^Z_{y_t}\delta_Z + c^a_{y_t}\delta_a\right), \qquad (4.48)$$

where y_{t0} was given in eq. (4.26), and the other coefficients are

$$c_{y_t}^{t} = 6.352 \times 10^{-3}, \qquad c_{y_t}^{S} = -7.76 \times 10^{-4}, \qquad c_{y_t}^{h} = -2.36 \times 10^{-6},$$
$$c_{y_t}^{tt} = 8.9 \times 10^{-7}, \qquad c_{y_t}^{SS} = -1.23 \times 10^{-6}, \qquad c_{y_t}^{Z} = -1.6 \times 10^{-7},$$
$$c_{y_t}^{a} = 2.2 \times 10^{-8}. \tag{4.49}$$

The five terms proportional to δ_t, δ_S, δ_h, δ_t^2, and δ_S^2 are sufficient to obtain a fractional precision better than 10^{-5}, and the linear deviation coefficients $c_{y_t}^{Z}$ and $c_{y_t}^{a}$ are also included in order to show their small contribution to the parametric error budget. We include some plots.



(a) y_t vs M_t

(b) y_t vs $\alpha_S^{(5)}(M_Z)$

Figure 4.7: Plots for the top-quark Yukawa coupling y_t at $Q = 200$ GeV obtained from eq. 4.48.

4.4.4 Yukawa couplings of light quarks

In the interpolation formulas for light-quark Yukawa couplings in the present subsection, the quantities δ_a, δ_h, and δ_Z make a relatively insignificant difference and are therefore omitted.

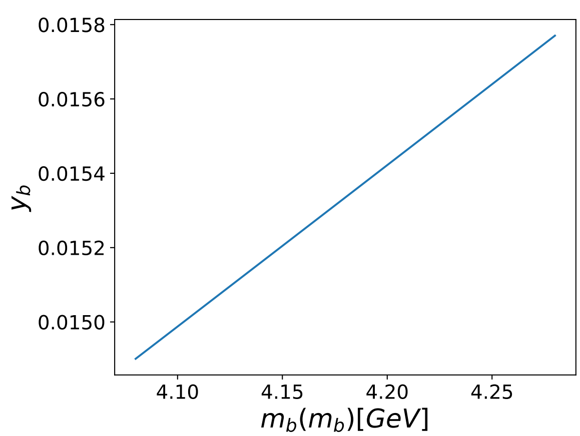
For the bottom-quark Yukawa coupling y_b at $Q = 200$ GeV, we find

$$y_b \;=\; y_{b0}\left(1 + c^{b}_{y_b}\Delta_b + c^{bb}_{y_b}\Delta_b^2 + c^{bS}_{y_b}\Delta_b\delta_S + c^{S}_{y_b}\delta_S + c^{t}_{y_b}\delta_t + c^{SS}_{y_b}\delta_S^2 + c^{SSS}_{y_b}\delta_S^3\right), \quad (4.50)$$

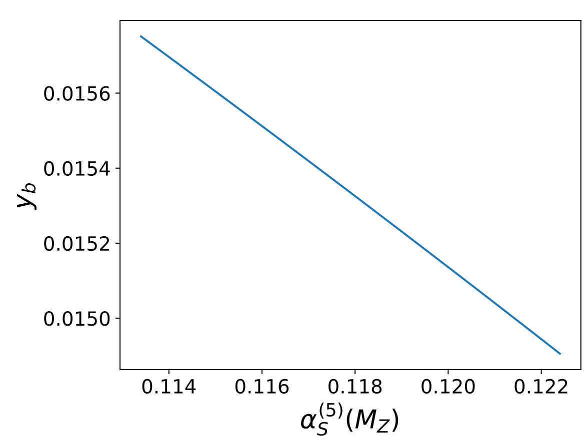
where y_{b0} was given in eq. (4.27), and the other coefficients are

$$c^{b}_{y_b} = 1.185, \qquad c^{bb}_{y_b} = 0.075, \qquad c^{bS}_{y_b} = -3.3\times 10^{-3}, \qquad c^{S}_{y_b} = -6.125\times 10^{-3},$$
$$c^{t}_{y_b} = -2.4\times 10^{-5}, \qquad c^{SS}_{y_b} = -2.1\times 10^{-5}, \qquad c^{SSS}_{y_b} = -1.5\times 10^{-7}. \quad (4.51)$$

This agrees with the results of `SMDR` to a fractional precision of better than 10^{-4}. We include some plots.



(a) y_b vs $m_b(m_b)$



(b) y_b vs $\alpha_S^{(5)}(M_Z)$

Figure 4.8: Plots for the bottom-quark Yukawa coupling y_b at $Q = 200$ GeV obtained from eq. 4.50.

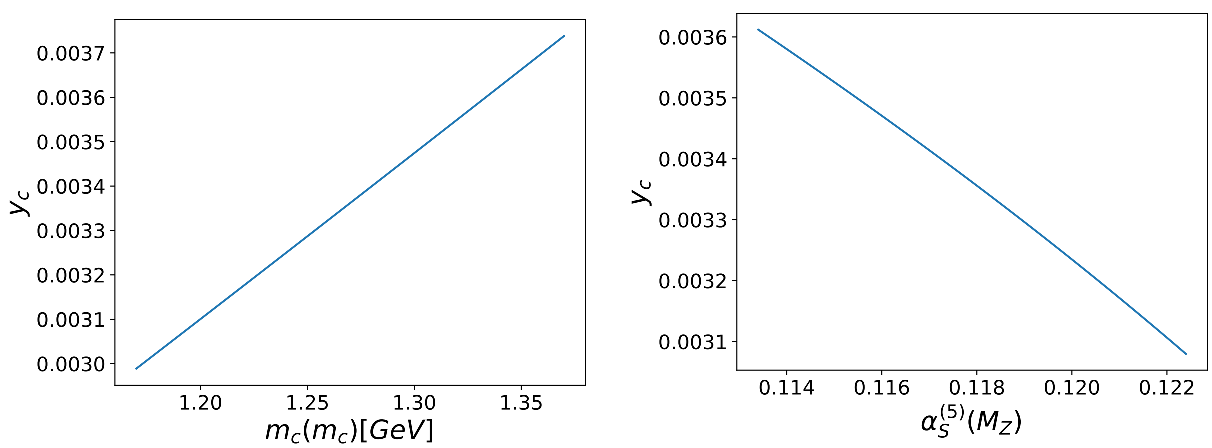
For the charm-quark Yukawa coupling y_c at $Q = 200$ GeV, we obtain

$$\begin{aligned} y_c &= y_{c0}\big(1 + c^c_{y_c}\Delta_c + c^{cc}_{y_c}\Delta_c^2 + c^{cS}_{y_c}\Delta_c\delta_S + c^S_{y_c}\delta_S + c^{SS}_{y_c}\delta_S^2 + c^{SSS}_{y_c}\delta_S^3 \\ &\quad + c^b_{y_c}\Delta_b + c^{bS}_{y_c}\Delta_b\delta_S + c^t_{y_c}\delta_t\big), \end{aligned} \tag{4.52}$$

where y_{c0} was given in eq. (4.28), and the other coefficients are

$$\begin{aligned} &c^c_{y_c} = 1.415, \qquad c^{cc}_{y_c} = 0.078, \qquad c^{cS}_{y_c} = -3.0\times 10^{-3}, \\ &c^S_{y_c} = -0.01746, \qquad c^{SS}_{y_c} = -2.34\times 10^{-4}, \qquad c^{SSS}_{y_c} = -6.5\times 10^{-6}, \\ &c^b_{y_c} = -0.027, \qquad c^{bS}_{y_c} = -1.6\times 10^{-3}, \qquad c^t_{y_c} = -1.5\times 10^{-5}. \end{aligned} \tag{4.53}$$

This result for the charm-quark Yukawa coupling agrees with `SMDR` to a fractional precision of better than 10^{-4}. We include plots.



(a) y_c vs $m_c(m_c)$

(b) y_c vs $\alpha_S^{(5)}(M_Z)$

Figure 4.9: Plots for the charm-quark Yukawa coupling y_c at $Q = 200$ GeV obtained from eq. 4.52.

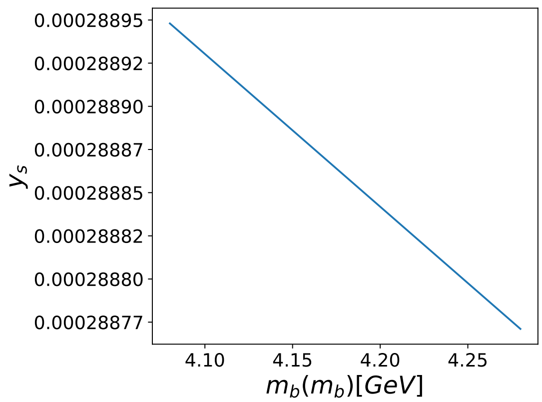
For the strange, down, and up Yukawa couplings, the interpolation formulas have a simpler, universal form, due to the fact that the "on-shell" input parameters from the RPP are actually running $\overline{\text{MS}}$ parameters determined at a common scale of $Q = 2$ GeV, so that the same QCD corrections apply to all three in the same way. For the Yukawa couplings at $Q = 200$ GeV, we find:

$$y_q = y_{q0}\left(1 + \Delta_q\right)\left(1 + c^S_{y_q}\delta_S + c^{SS}_{y_q}\delta_S^2 + c^{SSS}_{y_q}\delta_S^3 + c^b_{y_q}\Delta_b + c^t_{y_q}\delta_t\right), \tag{4.54}$$

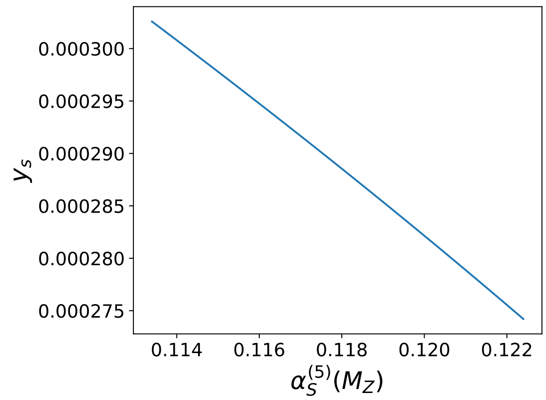
where the coefficients in all three cases ($q = s, d, u$) are approximated well by

$$c^S_{y_q} = -0.01089, \qquad c^{SS}_{y_q} = -7.93 \times 10^{-5}, \qquad c^{SSS}_{y_q} = -1.2 \times 10^{-6},$$
$$c^b_{y_q} = -0.0128, \qquad c^t_{y_q} = -1.5 \times 10^{-5}, \tag{4.55}$$

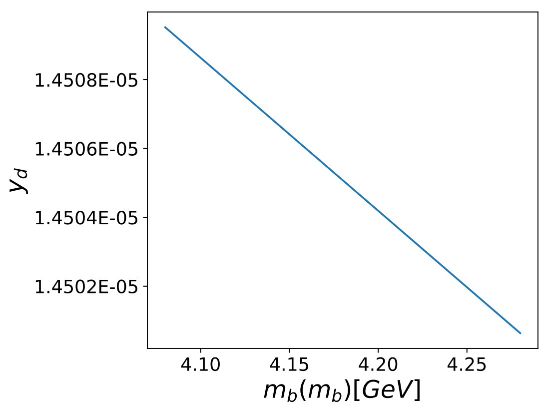
and y_{s0}, y_{d0}, and y_{u0} were given respectively in eqs. (4.29), (4.30), and (4.31). These formulas agree with those obtained by `SMDR` to a fractional precision of better than 10^{-4}. We include plots with respect to change in $m_b(m_b)$ and $\alpha_S^{(5)}(M_Z)$.



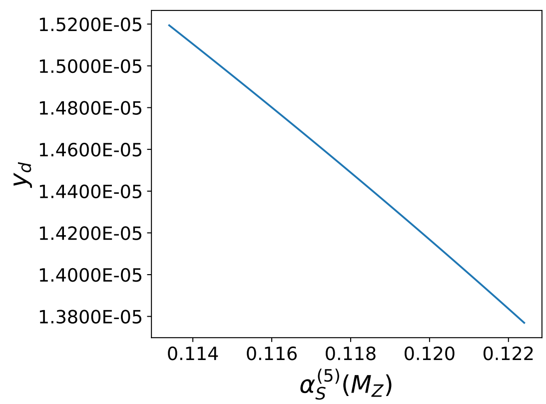
(a) y_s vs $m_b(m_b)$



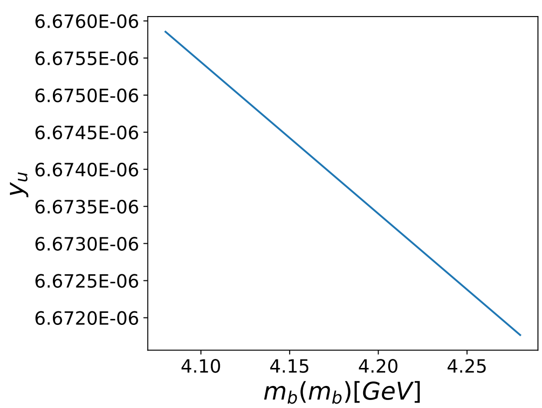
(b) y_s vs $\alpha_S^{(5)}(M_Z)$



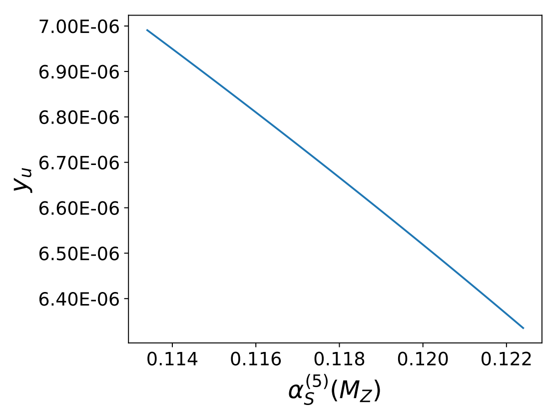
(c) y_d vs $m_b(m_b)$



(d) y_d vs $\alpha_S^{(5)}(M_Z)$



(e) y_u vs $m_b(m_b)$



(f) y_u vs $\alpha_S^{(5)}(M_Z)$

Figure 4.10: Plots for the strange, down and up Yukawa couplings at $Q = 200$ GeV obtained from eq. 4.54.

4.4.5 Yukawa couplings of leptons

For the tau-lepton Yukawa coupling at $Q = 200$ GeV, we obtain

$$y_\tau = y_{\tau 0}\big(1 + \Delta_\tau + 0.5\Delta_{G_F} + c^t_{y_\tau}\delta_t + c^S_{y_\tau}\delta_S + c^a_{y_\tau}\delta_a + c^h_{y_\tau}\delta_h + c^Z_{y_\tau}\delta_Z + c^{tt}_{y_\tau}\delta_t^2 + c^{tS}_{y_\tau}\delta_t\delta_S\big), \tag{4.56}$$

where $y_{\tau 0}$ was given in eq. (4.32), and the coefficients of Δ_τ and Δ_{G_F} are very close to 1 and 0.5 as indicated, and the other coefficients are

$$c^t_{y_\tau} = -1.252 \times 10^{-5}, \quad c^S_{y_\tau} = 2.63 \times 10^{-6}, \quad c^a_{y_\tau} = -1.83 \times 10^{-6}, \quad c^h_{y_\tau} = 1.74 \times 10^{-6}, \quad c^Z_{y_\tau} = -1.8 \times 10^{-7}, \quad c^{tt}_{y_\tau} = -6.9 \times 10^{-7}, \quad c^{tS}_{y_\tau} = 1.3 \times 10^{-7}. \tag{4.57}$$

This interpolation formula gives agreement with `SMDR` to a fractional precision of better than 10^{-7}.

The Yukawa couplings for $\ell = \mu, e$ at $Q = 200$ GeV are written in the common form:

$$y_\ell = y_{\ell 0}\big(1 + \Delta_\ell + 0.5\Delta_{G_F} + c^t_{y_\ell}\delta_t + c^S_{y_\ell}\delta_S + c^a_{y_\ell}\delta_a + c^h_{y_\ell}\delta_h + c^Z_{y_\ell}\delta_Z + c^{tt}_{y_\ell}\delta_t^2 + c^{tS}_{y_\ell}\delta_t\delta_S + c^c_{y_\ell}\Delta_c + c^b_{y_\ell}\Delta_b\big), \tag{4.58}$$

where $y_{\mu 0}$ and y_{e0} were given in eqs. (4.33) and (4.34), respectively. For the muon, the other coefficients are:

$$c^t_{y_\mu} = -1.3105 \times 10^{-5}, \quad c^S_{y_\mu} = 2.17 \times 10^{-6}, \quad c^a_{y_\mu} = -2.84 \times 10^{-6},$$

$$c^{h}_{y_\mu} = 1.73 \times 10^{-6}, \qquad c^{Z}_{y_\mu} = -1.78 \times 10^{-7}, \qquad c^{tt}_{y_\mu} = -6.93 \times 10^{-7},$$
$$c^{tS}_{y_\mu} = 1.26 \times 10^{-7}, \qquad c^{c}_{y_\mu} = -3.3 \times 10^{-5}, \qquad c^{b}_{y_\mu} = -4.1 \times 10^{-6}. \tag{4.59}$$

For the electron, the coefficients are

$$c^{t}_{y_e} = -1.312 \times 10^{-5}, \qquad c^{S}_{y_e} = 2.87 \times 10^{-6}, \qquad c^{a}_{y_e} = -4.72 \times 10^{-6},$$
$$c^{h}_{y_e} = 1.73 \times 10^{-6}, \qquad c^{Z}_{y_e} = -1.78 \times 10^{-7}, \qquad c^{tt}_{y_e} = -6.93 \times 10^{-7},$$
$$c^{tS}_{y_e} = 1.26 \times 10^{-7}, \qquad c^{c}_{y_e} = -8.1 \times 10^{-5}, \qquad c^{b}_{y_e} = -1.4 \times 10^{-5}. \tag{4.60}$$

The fractional precisions, compared to the results from `SMDR`, are less than 10^{-9}. Since the present fractional uncertainties in M_μ and M_e are about 2×10^{-8} and 6×10^{-9} respectively, we see that for each lepton, the bottleneck for obtaining the most accurate possible Yukawa coupling in the ultraviolet is not the uncertainty in the corresponding lepton mass, but rather the uncertainty associated with the top-quark mass, which is difficult to reduce as we have already mentioned.

CHAPTER 5

CONCLUSION

In this work, we have completed a thorough derivation of the mass of the Higgs boson M_h at one-loop order. But more importantly, we have presented simple interpolation formulas that provide the fundamental Lagrangian parameters for the Standard Model, given the corresponding on-shell experimental values as inputs. These results are an alternative to a more time-consuming and complicated evaluation using e.g. the computer code `SMDR`, on which our results are based. The structure of the interpolation formulas has been designed so as to avoid any numerically significant loss of precision, and are made to provide results at the $\overline{\text{MS}}$ renormalization scale $Q = 200$ GeV as a reference. For convenience, we have included a simple interactive command-line Python code `sm200.py` implementing the interpolation formulas above. The code listing is given in Appendix A and an electronic version is permanently available at https://arxiv.org/src/2211.08576v1/anc.

Besides satisfying basic curiosity about the fundamental parameters of the Standard Model, the results given here will have applications in matching to various candidate ultraviolet completions of the Standard Model, provided that the mass scales associated with new physics are sufficiently high that non-renormalizable terms in the effective theory can be neglected or corrected for. The results also can be viewed as providing the parametric error budget for the defining couplings of the Standard Model Lagrangian.

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APPENDIX A

PYTHON CODE

A basic Python code **sm200.py** was written which takes as inputs the on-shell quantities and gives the $\overline{\text{MS}}$ parameters as outputs by using the interpolation formulas presented in this work. We are including the code here.

```
'''
Run as,

    python sm200.py

Or,

    python3 sm200.py
'''

import numpy as np

#The Interpolation Constants for Mw are:
Mw_0 = 80.352476
ct_Mw = 7.61E-5
cZ_Mw = 1.56E-5
ch_Mw = -5.9E-7
cS_Mw = -8.8E-6
ca_Mw = -2.29E-5
ctt_Mw = 1.3E-7

#The Interpolation Constants for lambda are:
lambda_0 = 0.12353343830
ch_lam  =  1.6823E-3
ct_lam  =  -1.488E-4
cZ_lam  =  -3.5E-7
cS_lam  =  -2.2E-7
ca_lam  =  3.4E-7
```

```
ctt_lam  =  1.528E-5

ctS_lam  =  -4.02E-6

chh_lam  =  7.0E-7

cht_lam = -6.1E-7

cSS_lam =  3.0E-7

chS_lam  = 6.4E-8

cttt_lam  =  1.9E-7

cttS_lam  =  -7.6E-8

cb_lam = 4.5E-5

cG_F_lam = 0.95


#The Interpolation Constants for m2 are:

m2_0 = -93.126827678*93.126827678

ch_m2 =  1.4319E-3

ct_m2 =  2.337E-3

cS_m2 =  -1.052E-4

cZ_m2 =  -5.7E-7

ca_m2 = 5.4E-7

ctt_m2 = 2.02E-5

ctS_m2 = -2.45E-6

chh_m2 = 5.8E-7

cht_m2 = -4.3E-7


#The Interpolation Constants for g3 are:

g3_0 = 1.1525136966

cS_g3 = 3.7875E-03

ct_g3 = -3.98E-05

cSS_g3 = -1.07E-05

ch_g3 = 2.5E-8

cZ_g3 = 2.7E-9

ca_g3 = -2E-9
```

```
#The Interpolation Constants for g are:
g_0 = 0.64683244428
ct_g = 5.735E-05
cZ_g = 1.558E-05
ch_g = -8.5E-07
cS_g = -5.97E-06
ca_g = -2.295E-05
ctt_g = 1.9E-07
ctS_g = -7.8E-08
cGF_g = 0.71

#The Interpolation Constants for gp are:
gp_0 = 0.35885152738
ct_gp = -2.609E-05
cZ_gp = -4.70E-06
ch_gp = 2.6E-07
cS_gp = 3.29E-06
ca_gp = 7.714E-05

#The Interpolation Constants for yt are:
yt_0 = 0.92377763013
ct_yt = 6.352E-03
cZ_yt = -1.6E-07
ch_yt = -2.36E-06
cS_yt = -7.76E-04
ca_yt = 2.2E-08
ctt_yt = 8.9E-07
cSS_yt = -1.23E-06

#The Interpolation Constants for yb are:
```

```
yb_0 = 0.0153349059085

cS_yb = -6.125E-03

cmb_yb = 1.185

cSS_yb = -2.1E-05

cSmb_yb = -3.3E-03

cmbmb_yb = 0.075

ct_yb = -2.4E-5

cSSS_yb = -1.5E-7


#The Interpolation Constants for yc are:

yc_0 = 0.00336181598480

cmc_yc = 1.415

cmcmc_yc = 0.078

cSmc_yc = -3E-3

cS_yc = -0.01746

cSS_yc = -2.34E-04

cSSS_yc = -6.5E-6

cb_yc = -0.027

cbS_yc = -1.6E-3

ct_yc = -1.5E-5


#The Interpolation Constants for ys,yd and yu are:

ys_0 = 2.8885955612E-04

yd_0 = 1.4505079604E-05

yu_0 = 6.6738103560E-06

cS_q = -0.01089

cSS_q = -7.93E-05

cSSS_q = -1.2E-6

cb_q = -0.0128

ct_q = -1.5E-5
```

```
#The Interpolation Constants for ytau are:
ytau_0 = 0.0100065524355
ct_ytau = -1.252E-5
cS_ytau = 2.63E-6
ca_ytau = -1.83E-6
ch_ytau = 1.74E-6
cZ_ytau = -1.8E-7
ctt_ytau = -6.9E-7
ctS_ytau = 1.3E-7

#The Interpolation Constants for ymuon are:
ymuon_0 = 5.8908805223E-4
ct_ymuon = -1.3105E-5
cS_ymuon = 2.17E-6
ca_ymuon = -2.84E-6
ch_ymuon = 1.73E-6
cZ_ymuon = -1.78E-7
ctt_ymuon = -6.93E-7
ctS_ymuon = 1.26E-7
cmc_ymuon = -3.3E-5
cmb_ymuon = -4.1E-6

#The Interpolation Constants for ye are:
ye_0 = 2.7963423115E-06
ct_ye = -1.312E-5
cS_ye = 2.87E-6
ca_ye = -4.72E-6
ch_ye = 1.73E-6
cZ_ye = -1.78E-7
ctt_ye = -6.93E-7
ctS_ye = 1.26E-7
```

```
cmc_ye = -8.1E-5

cmb_ye = -1.4E-5


#Central Values

Mt0 = 172.5

Mz0 = 91.1876

Mh0 = 125.25

AS0 = 0.1179

DA0 = 0.027660

mb0 = 4.18

mc0 = 1.27

ms0 = 0.093

md0 = 0.00467

mu0 = 0.00216

Mtau0 = 1.77686

Mu0 = 0.1056583745

Me0 = 0.0005109989461

GF0 = 0.000011663787


#Experimental Uncertainties (from Particle Data Group 2021)

Mt_EXPT_UNC = 0.7

Mh_EXPT_UNC = 0.17

MZ_EXPT_UNC = 0.0021

alphaS_MZ_EXPT = 0.0009

Delta_alpha_had_5_MZ_EXPT_UNC = 0.00007

GFermi_EXPT_UNC = 0.000000000006

mbmb_EXPT_UNC_hi = 0.03

mbmb_EXPT_UNC_lo = 0.02

mcmc_EXPT_UNC = 0.02

ms_2GeV_EXPT_UNC_hi = 0.011

ms_2GeV_EXPT_UNC_lo = 0.005
```

```
mu_2GeV_EXPT_UNC_hi = 0.00049

mu_2GeV_EXPT_UNC_lo = 0.00026

md_2GeV_EXPT_UNC_hi = 0.00048

md_2GeV_EXPT_UNC_lo = 0.00017

Mtau_EXPT_UNC = 0.00012

Mmuon_EXPT_UNC = 0.0000000024

Melectron_EXPT_UNC = 0.0000000000031



print("This is sm200.py  (Standard Model at 200 GeV, version November 15, 2022).")

print("Interpolation formulas based on:")

print("SMDR (Standard Model in Dimensional Regularization) v1.2, arXiv:1907.02500")

print("")

print("At each prompt, enter a new value or [Return] to accept.")

if hasattr(__builtins__, 'raw_input'):

    input = raw_input

#input values

Mt = float(input("SMDR_Mt_pole = ? [" + str(Mt0) + "] ") or Mt0)

if (Mt < (Mt0 - 5*Mt_EXPT_UNC) or Mt > (Mt0 + 5*Mt_EXPT_UNC)):

    print("Warning! Value is not within 5 sigma of the central value!")

Mh = float(input("SMDR_Mh_pole = ? [" + str(Mh0) + "] ") or Mh0)

if (Mh < (Mh0 - 5*Mh_EXPT_UNC) or Mh > (Mh0 + 5*Mh_EXPT_UNC)):

    print("Warning! Value is not within 5 sigma of the central value!")

Mz = float(input("SMDR_MZ_PDG = ? [" + str(Mz0) + "] ") or Mz0)

if (Mz < (Mz0 - 5*MZ_EXPT_UNC) or Mz > (Mz0 + 5*MZ_EXPT_UNC)):

    print("Warning! Value is not within 5 sigma of the central value!")
```

```
AS = float(input("SMDR_alphaS_5_MZ = ? [" + str(AS0) + "] ") or AS0)
if (AS < (AS0 - 5*alphaS_MZ_EXPT) or AS > (AS0 + 5*alphaS_MZ_EXPT)):
    print("Warning! Value is not within 5 sigma of the central value!")

DA = float(input("SMDR_Delta_alpha_had_5_MZ_in = ? [" + str(DA0) + "] ") or DA0)
if(DA<(DA0-5*Delta_alpha_had_5_MZ_EXPT_UNC) or DA>(DA0+5*Delta_alpha_had_5_MZ_EXPT_UNC)):
    print("Warning! Value is not within 5 sigma of the central value!")

GF = float(input("SMDR_GFermi = ? [ %.12Lf ] " %GF0) or GF0)
if (GF < (GF0 - 5*GFermi_EXPT_UNC) or GF > (GF0 + 5*GFermi_EXPT_UNC)):
    print("Warning! Value is not within 5 sigma of the central value!")

mb = float(input("SMDR_mbmb = ? [" + str(mb0) + "] ") or mb0)
if (mb < (mb0 - 5*mbmb_EXPT_UNC_lo) or mb > (mb0 + 5*mbmb_EXPT_UNC_hi)):
    print("Warning! Value is not within 5 sigma of the central value!")

mc = float(input("SMDR_mcmc = ? [" + str(mc0) + "] ") or mc0)
if (mc < (mc0 - 5*mcmc_EXPT_UNC) or mc > (mc0 + 5*mcmc_EXPT_UNC)):
    print("Warning! Value is not within 5 sigma of the central value!")

ms = float(input("SMDR_ms_2GeV = ? [" + str(ms0) + "] ") or ms0)
if (ms < (ms0 - 5*ms_2GeV_EXPT_UNC_lo) or ms > (ms0 + 5*ms_2GeV_EXPT_UNC_hi)):
    print("Warning! Value is not within 5 sigma of the central value!")

md = float(input("SMDR_md_2GeV = ? [" + str(md0) + "] ") or md0)
if (md < (md0 - 5*md_2GeV_EXPT_UNC_lo) or md > (md0 + 5*md_2GeV_EXPT_UNC_hi)):
    print("Warning! Value is not within 5 sigma of the central value!")

mu = float(input("SMDR_mu_2GeV = ? [" + str(mu0) + "] ") or mu0)
if (mu < (mu0 - 5*mu_2GeV_EXPT_UNC_lo) or mu > (mu0 + 5*mu_2GeV_EXPT_UNC_hi)):
```

```
    print("Warning! Value is not within 5 sigma of the central value!")

Mtau = float(input("SMDR_Mtau_pole = ? [" + str(Mtau0) + "] ") or Mtau0)
if (Mtau < (Mtau0- 5*Mtau_EXPT_UNC) or Mtau > (Mtau0 + 5*Mtau_EXPT_UNC)):
    print("Warning! Value is not within 5 sigma of the central value!")

Mu = float(input("SMDR_Mmuon_pole = ? [" + str(Mu0) + "] ") or Mu0)
if (Mu < (Mu0 - 5*Mmuon_EXPT_UNC) or Mu > (Mu0 + 5*Mmuon_EXPT_UNC)):
    print("Warning! Value is not within 5 sigma of the central value!")

Me = float(input("SMDR_Melectron_pole = ? [" + str(Me0) + "] ") or Me0)
if (Me < (Me0 - 5*Melectron_EXPT_UNC ) or Me > (Me0 + 5*Melectron_EXPT_UNC)):
    print("Warning! Value is not within 5 sigma of the central value!")

print("\nOn-shell quantities read:\n")

print("Mt = %Lf;" % Mt)
print("Mh = %Lf;" %Mh)
print("MZ = %Lf;" %Mz)
print("alpha_S_5_MZ = %Lf;" %AS)
print("GFermi = %.8LE;" %GF)
print("alpha = 1/137.03599908;")
print("Delta_hadronic^(5) alpha(MZ) = %Lf;" %DA)
print("mb(mb) = %Lf;      mc(mc) = %Lf;" %(mb,mc))
print("ms(2 GeV) = %Lf;   mu(2 GeV) = %Lf;  md(2 GeV) = %Lf;" %(ms, mu, md))
print("Mtau = %Lf;        Mmuon = %.10Lf;  Melectron = %.13Lf;\n" %(Mtau, Mu, Me))

dt = (Mt - Mt0)
dz = (Mz - Mz0)*1000
dh = (Mh - Mh0)*10
dS = (AS - AS0)*1000
```

```
da = (DA - DA0)*10000

dmb = (mb/mb0 - 1)

dmc = (mc/mc0 - 1)

dms_2GeV = (ms/ms0 - 1)

dmd_2GeV = (md/md0 - 1)

dmu_2GeV = (mu/mu0 - 1)

dMtau = (Mtau/Mtau0 - 1)

dMu = (Mu/Mu0 - 1)

dMe = (Me/Me0 - 1)

dGF = (GF/GF0 - 1)


#interpolation formula

Mw = Mw_0 * (1 + ct_Mw *dt + cZ_Mw *dz + ch_Mw *dh + cS_Mw *dS + ca_Mw *da
     + ctt_Mw *dt*dt)


lambdaz = lambda_0 * (1 + ch_lam *dh + ct_lam *dt + cZ_lam *dz + cS_lam *dS
          + ca_lam *da + ctt_lam *dt*dt + ctS_lam *dt*dS + chh_lam *dh*dh
          + cht_lam *dh*dt + cSS_lam *dS*dS + chS_lam *dh*dS + cttt_lam *dt*dt*dt
          + cttS_lam *dt*dt*dS + cb_lam *dmb + cG_F_lam *dGF)


m2 = m2_0 * (1 + ch_m2 *dh + ct_m2 *dt + cS_m2 *dS + cZ_m2 *dz + ca_m2 *da
     + ctt_m2 *dt*dt + ctS_m2 *dt*dS + chh_m2 *dh*dh + cht_m2 *dh*dt)


g3 = g3_0 * (1 + ct_g3 *dt + cS_g3 *dS + cSS_g3 *dS*dS + ch_g3 *dh
     + cZ_g3 *dz + ca_g3 *da)


g = g_0 * (1 + ct_g *dt + cZ_g *dz + ch_g *dh + cS_g *dS + ca_g *da
    + ctt_g *dt*dt + ctS_g *dt*dS + cGF_g *dGF)


gp = gp_0 * (1 + ct_gp *dt + cZ_gp *dz + ch_gp *dh + cS_gp *dS + ca_gp *da)
```

```
yt = yt_0 * (1 + ct_yt *dt + cZ_yt *dz + ch_yt *dh + cS_yt *dS + ca_yt *da
     + ctt_yt *dt*dt + cSS_yt *dS*dS)

yb = yb_0 * (1 + cS_yb*dS + cmb_yb*dmb + cSS_yb *dS*dS + cSmb_yb *dmb*dS
     + cmbmb_yb *dmb*dmb + ct_yb * dt + cSSS_yb * dS * dS * dS)

yc = yc_0 * (1 + cmc_yc*dmc + cmcmc_yc*dmc*dmc +cSmc_yc *dmc*dS + cS_yc*dS
     + cSS_yc *dS*dS +cSSS_yc *dS*dS*dS +cb_yc*dmb +cbS_yc *dmb*dS + ct_yc * dt)

ys = ys_0 * (1 + dms_2GeV) * (1 + cS_q*dS  + cSS_q *dS*dS + cSSS_q*dS*dS*dS
     +cb_q*dmb + ct_q * dt)

yd = yd_0 * (1 + dmd_2GeV) * (1 + cS_q*dS  + cSS_q *dS*dS + cSSS_q*dS*dS*dS
     + cb_q*dmb + ct_q * dt)

yu = yu_0 * (1 + dmu_2GeV) * (1 + cS_q*dS  + cSS_q *dS*dS + cSSS_q*dS*dS*dS
     + cb_q*dmb + ct_q * dt)

ytau = ytau_0 * (1 + dMtau + 0.5*dGF + ct_ytau *dt + cZ_ytau *dz + ch_ytau *dh
       + cS_ytau *dS + ca_ytau *da + ctt_ytau *dt*dt + ctS_ytau *dt*dS)

ymuon = ymuon_0 * (1 + dMu + 0.5*dGF + ct_ymuon *dt + cZ_ymuon *dz + ch_ymuon *dh
        + cS_ymuon *dS + ca_ymuon *da + ctt_ymuon *dt*dt + ctS_ymuon *dt*dS
        + cmc_ymuon*dmc + cmb_ymuon*dmb)

ye = ye_0 * (1 + dMe + 0.5*dGF + ct_ye *dt + cZ_ye *dz + ch_ye *dh + cS_ye *dS
     + ca_ye *da + ctt_ye *dt*dt + ctS_ye *dt*dS + cmc_ye*dmc + cmb_ye*dmb)

print("Using interpolation formulas:\n")

print("MW = %Lf  (* PDG convention *)\n" %Mw)
```

```
print("MSbar parameters:")
print("Q = 200.000000;")
print("gauge  couplings: g3 = %Lf;      g = %Lf;      gp = %Lf;" %(g3, g, gp))
print("Yukawa couplings: yt = %Lf;     yb = %Lf;    ytau = %.8Lf;" %(yt, yb, ytau))
print("                  yc = %.7Lf;    ys = %.8Lf;   ymu = %.11Lf;" %(yc, ys, ymuon))
print("                  yu = %.10Lf; yd = %.9Lf;   ye = %.13Lf;" %(yu, yd, ye))
print("Higgs self-coupling lambda = %Lf;" %lambdaz)
print("Higgs mass^2 parameter m2 = %.6Lf = -(%f)^2" %(m2, np.sqrt(-m2)))
print("Delta_hadronic^(5) alpha(MZ) = %Lf\n" %DA)
```

APPENDIX B

FEYNMAN RULES FOR THE STANDARD MODEL HIGGS BOSON IN LANDAU GAUGE

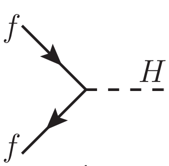
Tree-level masses

$$\begin{aligned} m_H^2 &= m^2 + 3\lambda v^2, && \text{(B.1)} \\ m_G^2 &= m^2 + \lambda v^2, && \text{(B.2)} \\ m_W^2 &= g^2 v^2/4, && \text{(B.3)} \\ m_Z^2 &= (g^2 + g'^2) v^2/4, && \text{(B.4)} \\ m_f^2 &= y_f^2 v^2/2, && \text{(B.5)} \\ v &\approx 246 \text{ GeV} && \text{(B.6)} \end{aligned}$$

Propagators

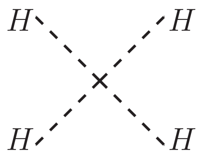
scalar:		$\frac{-i}{p^2+m^2}$
fermion:		$\frac{i(\not{p}-m)}{p^2+m^2}$
W, Z bosons:		$-i\left[\frac{\eta^{\mu\nu}}{p^2+m^2} + \frac{p^\mu p^\nu}{m^2}\left(\frac{1}{p^2+m^2} - \frac{1}{p^2}\right)\right]$
ghost:		$\frac{i}{p^2}$

Rules for scalar-fermion-fermion Yukawa couplings

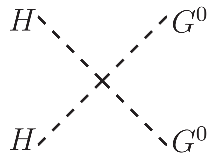


$$-\frac{i}{\sqrt{2}} y_f$$

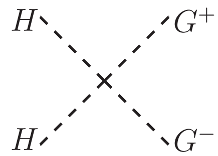
Rules for (scalar)^4 and (scalar)^3 couplings



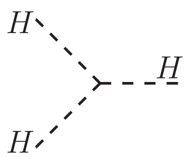
$-i6\lambda$



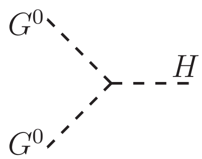
$-i2\lambda$



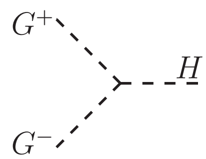
$-i2\lambda$



$-i6\lambda v$

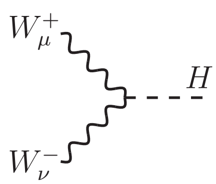


$-i2\lambda v$

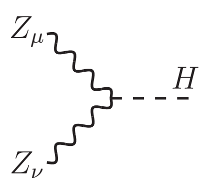


$-i2\lambda v$

Rules for vector-vector-scalar couplings



$-ig_{WWH}\eta_{\mu\nu}$



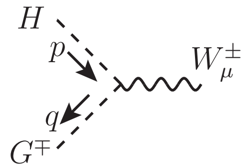
$-ig_{ZZH}\eta_{\mu\nu}$

(No G^0 coupling to W^+W^- or to ZZ.)

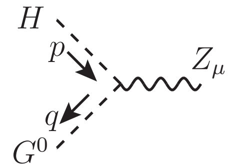
$$g_{WWH} = g^2 v/2, \tag{B.7}$$

$$g_{ZZH} = (g^2 + g'^2)v/2, \tag{B.8}$$

Rules for vector-scalar-scalar couplings



$\mp i g_{HGW}(p+q)_\mu$

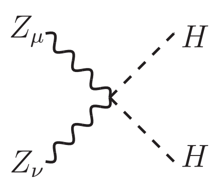


$g_{HGZ}(p+q)_\mu$

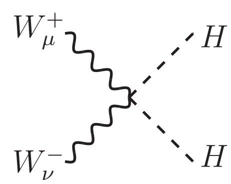
$$g_{HGW} = g/2 \quad \text{(B.9)}$$

$$g_{HGZ} = \sqrt{g^2 + g'^2}/2 \quad \text{(B.10)}$$

Rules for vector-vector-scalar-scalar couplings



$-i g_{ZZHH}\eta_{\mu\nu}$



$-i g_{WWHH}\eta_{\mu\nu}$

$$g_{ZZHH} = (g^2 + g'^2)/2, \quad \text{(B.11)}$$

$$g_{WWHH} = g^2/2 \quad \text{(B.12)}$$