

2022

## Optimize The NIU Split Hopkinson Pressure Bar Apparatus For Conducting High Strain Rate Tensile Test on S304 Foils

Jiale Ji  
jjaleji.98@gmail.com

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## ABSTRACT

### OPTIMIZE THE NIU SPLIT HOPKINSON PRESSURE BAR APPARATUS FOR CONDUCTING HIGH STRAIN RATE TENSILE TEST ON S304 FOILS

Jiale Ji, M.S.  
Department of Mechanical Engineering  
Northern Illinois University, 2022  
Jenn-Terng Gau, Director

Split Hopkinson Bar (SHB) methods are widely used to experimentally characterize the mechanical properties of materials such as metals, concrete, and ceramics undergoing rapid deformation. NIU had a split-Hopkinson tensile apparatus that can conduct coarse tensile test, but it can't output the useful signal. The aim of this thesis is to optimize the existing apparatus, so the high strain rate tensile tests for the stainless steel 304 can be conducted and useful signal can be got.

The alignment of the bars is critical for a good signal that has less noise. To eliminate the noise signal, some accurate fixture is designed. A new reflective tensile test apparatus is used to get better data. Based on the one-dimensional wave propagation theory, when compression wave arrives at the interface of the two identical bars, it will propagate almost totally. However, a tension wave will still pull the bars to be split. A new specimen is also designed corresponding. This new specimen not only has no influence to the propagation of the compression wave, but also can bear the tensile wave. By testing with some samples, the feasibility of this device is proved. An open-source MATLAB code is immigrated to analyze the data outputted by the oscilloscope. The procedures and tips about the using of the code is introduced in the thesis.

NORTHERN ILLINOIS UNIVERSITY  
DE KALB, ILLINOIS

DECEMBER 2022

OPTIMIZE THE NIU SPLIT HOPKINSON PRESSURE BAR APPARATUS  
FOR CONDUCTING HIGH STRAIN RATE TENSILE TEST ON S304 FOILS

BY

JIALE JI  
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A THESIS SUBMITTED TO THE GRADUATE SCHOOL  
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS  
FOR THE DEGREE  
MASTER OF SCIENCE

DEPARTMENT OF MECHANICAL ENGINEERING

Thesis Director:  
Jenn-Terng Gau

## ACKNOWLEDGMENTS

I would like to express my thanks to the NIU Mechanical Engineering Department and the Machine Shop for the support on the lab resources. I really appreciate the help from Michael Reynolds, Jeremy Peters, and Greg Kleinprinz. Three gentlemen are workers in the Machine shop who have lots of machine experience and good personalities.

I would also like to express my deep gratitude to Professor Jenn-Terng Gau, my thesis advisor.

I also appreciate my teammates, Jawad Najjar, levee Callahan, and Arjun Patel.

Finally, I wish to thank my family members and friends for their support and encouragement throughout my pursuing the Master degree.

## DEDICATION

To my family

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## **CHAPTER 1. INTRODUCTION**

### **1.1 History of Split Hopkinson Pressure Bar**

The Hopkinson pressure bar was first proposed by Bertram Hopkinson in 1914 (Lang, 2012). His design consisted of a long steel bar, a short steel billet (test specimen), and a ballistic pendulum. Hopkinson would impact one end of the steel bar using an explosive charge, generating a compressive wave that would travel through the bar and into the steel billet. The idea was to generate pressures in the bar that would resemble pressures seen in an impact. From these experiments, Hopkinson was able to generate pressure-time curves that would describe an impact event.

The process of the Split Hopkinson Pressure Bar (SHPB) data has been extensively studied for over 73 years, beginning with Kolsky's modifications to the Hopkinson Pressure Bar. In 1949 Kolsky added a second pressure bar to Hopkinson's original design. Instead of putting a billet at the far end of the bar, he sandwiched it in between the bars. This split bar system is how the Hopkinson split bar apparatus got its name. This design has become the most common and widely used technique to determine dynamic material properties (Lang, 2012).

Since Harding was the first to develop the tensile Hopkinson-bar technique in 1960, various modifications have been made to generate a tensile loading pulse in the Split Hopkinson Bar (SHB) system (Huang, et al., 2006). Nicholas places a shoulder between the two Hopkinson Pressure Bars, so the initial compression wave will bypass the specimen and then be reflected in tension to

load the specimen. Staab and Gilat (1991) introduced a direct-tension Split Hopkinson-bar apparatus. The specimen is loaded by a tensile wave generated by the release of a prestressed section of the incident bar. In recent decades, direct impact on an impact block connected to the input bar was employed to generate a tension pulse. A weight bar tube or a gas gun with a strike tube was generally used for generating the load.

## 1.2 Background of SHPB

Split Hopkinson Bar (SHB) methods are widely used to experimentally characterize the mechanical properties of materials such as metals, concrete, and ceramics undergoing rapid deformation (Gerlach, Kettenbeil, & Petrinic, 2012). Strain rates are between 400 and 5,000  $s^{-1}$  in the usual SHPB tests. Also, the rise time of the stress pulse in conventional steel split Hopkinson pressure bar is typically less than 10  $\mu s$ . These mechanical properties determined in rapid deformation or high strain rate tests are also called dynamic properties. Nowadays, the SHB are generally classified into compression (SHCB), torsion (SHTOB), and tension (SHTB) split Hopkinson bars.

A conventional SHCB consists of a striker bar, an incident bar, and a transmission bar as schematically illustrated in Fig. 1-1 (Chen, Zhang, & Forrestal, 1999). A specimen is placed between the incident bar and the transmission bar. When the striker bar impacts the incident bar, an elastic compressive stress pulse, referred to as the incident pulse  $\varepsilon_i$ , is generated and propagates along the incident bar toward the specimen. When the incident pulse reaches the specimen, part of the pulse  $\varepsilon_r$  is reflected backward into the incident bar due to the impedance mismatch at the bar-specimen interface. The remaining part of the pulse  $\varepsilon_t$  is transmitted into the specimen and,

eventually, into the transmission bar. Axial strain gages mounted on the surfaces of the incident and transmission bars provide time-resolved measures of the elastic strain pulses in the bars. The specimen (a short, small diameter cylinder) is placed between the incident and transmitter bars. Usually, a lubricant is applied at the contact surfaces, and it is assumed that the specimen is loaded only by an axial compression force, since the lubricant prevents any shear traction on the end surfaces as the specimen is deformed.

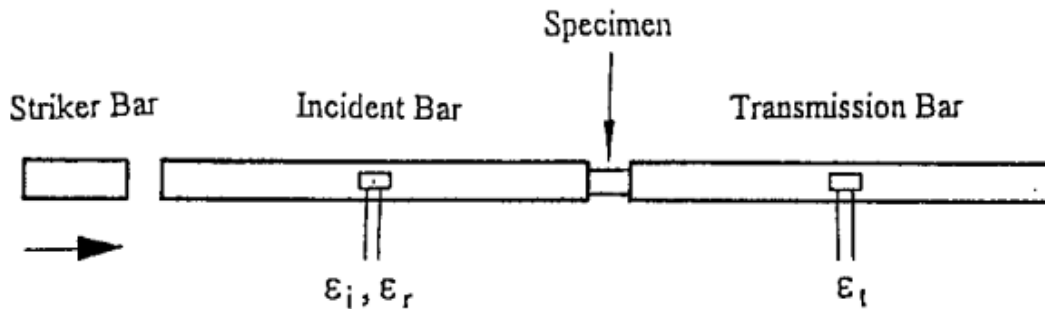


Fig. 1-1: Schematic of a conventional Split Hopkinson pressure bar

The most important feature of SHPB is that it avoids the difficulty of directly measuring the strain-stress of an object under a high strain rate loading. Using elastic wave theory, the force (and hence the stress), strain rate, and strain in the specimen can be determined from the stress waves in the bars. The technique was introduced with compression loading by Kolsky and has subsequently been modified for tension and torsion applications. It uses those three pulses  $\epsilon_i$ ,  $\epsilon_r$ , and  $\epsilon_t$  to calculate the stress, strain, and strain rate of the specimen. Because of using three pulses or waves, this method is named the three-wave theory. Below are the formulas based on three-wave theory (discussed in detail in chapter 2):

$$\varepsilon = \frac{c}{l_s} \int_0^t (\varepsilon_i - \varepsilon_r - \varepsilon_t) dt \quad (1.2.1)$$

$$\sigma = \frac{EA(\varepsilon_i + \varepsilon_r + \varepsilon_t)}{2A_s} \quad (1.2.2)$$

$$\dot{\varepsilon} = \frac{c}{l_s} (\varepsilon_i - \varepsilon_r - \varepsilon_t) \quad (1.2.3)$$

Where  $\varepsilon$  is strain,  $c$  is the speed of stress wave in the bars,  $l_s$  is the gauge length of the specimen,  $\sigma$  is the stress,  $E$  is the Young's Modulus of the bars,  $A$  is the area of the bars,  $A_s$  is the area of the specimen,  $\dot{\varepsilon}$  is the strain rate. The strain rate is the differential of strain respect to time.

The dimensions of the bars, the specimen, and the amplitude of the loading wave are designed such that, during the test, the specimen is loaded beyond the elastic limit. It is still in the plastic field while the bars remain elastic. When the amplitude of the stress pulse exceeds the dynamic yield strength of the soft specimen within this rise time, homogeneous deformation in the specimen cannot be reached before failure occurs due to the low elastic wave velocities in the low-impedance materials. Upon loading by the incident pulse, the specimen will deform plastically near the impact end and the deformation will remain small near the other end. This non-homogeneous deformation results in a non-equilibrium stress state. To reach equilibrium in the specimen, the loading pulse should travel back and forth inside the specimen more than three times.

After the stress wave passes through and back the bars three or more times, it is assumed that the deformation is uniform, and the stress and strain at both ends of the specimen are equal (showed by eq. 1.2.4). Therefore, the whole specimen is under the uniform state of uniaxial compression throughout the test. When the results are under a uniform state, the two-wave theory (discussed more in chapter 2) will be used to analyze the properties. In the two-wave theory, the

stress in the specimen is calculated by dividing the force in the specimen (determined from the transmitted wave) by the cross-sectional area. The strain rate is the difference between the velocities of the end surfaces of the specimen (can get only from the wave that is reflected from the specimen to the incident bar, because of the uniform state) divided by its length. The strain is determined by integrating the strain rate. The main equations for two-wave theory are shown below:

$$\varepsilon_i + \varepsilon_r = \varepsilon_t \quad (1.2.4)$$

$$\sigma = \frac{EA\varepsilon_t}{A_s} \quad (1.2.5)$$

$$\varepsilon = -\frac{2c}{l_s} \int_0^t \varepsilon_r dt \quad (1.2.6)$$

$$\dot{\varepsilon} = -\frac{2c}{l_s} \varepsilon_r \quad (1.2.7)$$

Where  $\varepsilon$  is strain,  $c$  is the speed of stress wave in the bars,  $l_s$  is the gauge length of the specimen,  $\sigma$  is the stress,  $E$  is the Young's Modulus of the bars,  $A$  is the area of the bars,  $A_s$  is the area of the specimen,  $\dot{\varepsilon}$  is the strain rate. Substitute Eq. (1.2.4) into eq. (1.2.1), (1.2.2) and (1.2.3); eq. (1.2.5), (1.2.6) and (1.2.7) can be derived.

The experiments show that up to about 30% axial strain, the specimen remains cylindrical in shape; however, at larger strains, the specimens start to barrel, which means that the specimen at that stage is no longer under a uniform state of uniaxial compression.



### 1.3 Split Hopkinson Tensile Bar (SHTB)

Tensile SHB tests are more difficult to conduct and analyze. A typical specimen for a tensile test has a dog bone geometry, with a middle section of a small cross-sectional area (gage section) and ends with a larger area (Gilat, Schmidt, & Walker, 2008). Rounded fillets comprise the transition from the gage section to the larger ends, which are attached to the bars of the SHTB apparatus. The strain distribution within the gage section and in the rounded fillets adjacent to the gage section depends on the exact geometry and properties (yield stress and strain hardening rate) of the specimen. Therefore, the specimen must be designed such that most of the gage section is under a state of uniaxial tension.

In the compression test, the stress is calculated by dividing the force in the specimen (determined from the transmitted wave) by the cross-sectional area. The approach to determine the strain rate in an SHTB test is more involved because the portion of the specimen coupon between the ends of the bars includes the central gage section and the rounded fillets. If the deformation is confined only to the gage section, then the strain rate can be accurately calculated by dividing the difference between the velocities of the end of the bars by the length of the gage section. However, at least some of the relative motion between the bars is due to deformation within the fillets.

The full-field strain measurement provides means for examining the validity and accuracy of the tests. In tests where the deforming section of the specimen is well defined and the deformation is uniform, the strains measured with the image correlation technique agree with the average strain that is determined from the split Hopkinson bar wave records. If significant deformation is taking place outside the gage section, and when necking develops, the strains

determined from the waves are not valid, but the image correlation method provides an accurate full-field strain history.

Take a usual SHTB test with the image correlation method as an example (Gilat, Schmidt, & Walker, 2008), the axial strain measured with the image correlation method along the specimen is shown in Fig. 1-2. The figure displays the strain at every fifth frame, or approximately every  $44.5 \mu\text{s}$ . This waterfall plot shows that in the early part of the test, the strain is nearly uniform along 70% of the gage length. As the specimen deforms, the length of the portion of the gage section that is under uniform deformation decreases gradually until the necking develops. The strain near the ends of the gage section is smaller. Figure. 1-2 shows also that some plastic deformation is taking place outside the gage section in the rounded fillets.

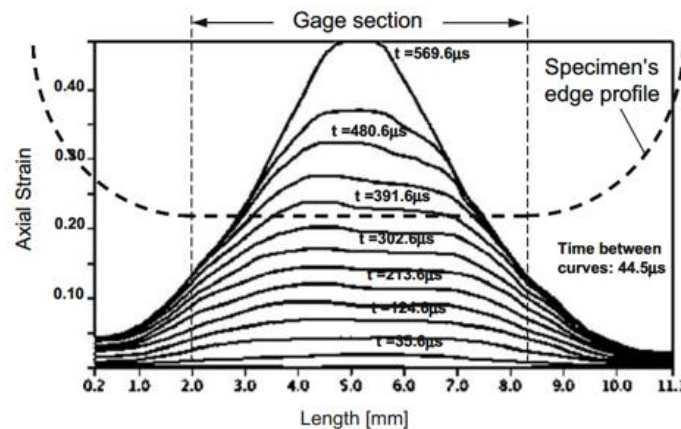


Fig. 1-2: Progression of strain distribution along with the specimen in the SHTB test

## 1.4 Signal Analysis

Transforms and filters are tools for processing and analyzing discrete data and are commonly used in signal-processing applications and computational mathematics. When data is

represented as a function of time or space, the Fourier transform decomposes the data into frequency components. The Fast Fourier Transformation (FFT) function uses a fast Fourier transform algorithm that reduces its computational cost compared to other direct implementations. The convolution and filter functions are also useful tools for modifying the amplitude or phase of input data using a transfer function.

The frequency of the stress wave pulse signal is lower than the noise signals, and thus the FFT and Wavelet Transform (WT) are commonly adopted to act as a filter. Nevertheless, since the stress wave pulse is a kind of transient and non-stationary signal with a noise ratio and fast mutation process, the filter methods based on FFT, and WT cannot remove the noisy interference while retaining the original signal. Compared with signal de-noising methods based on FFT and WT, the Hilbert–Huang Transform (HHT) method has a better performance in dealing with such short-time abrupt and high-noise signals as well as being easier to calculate (Ai, Zhao, Wang, & Li, 2019). The basic computational process of HHT can be summarized as decomposing the signal into Intrinsic Mode Functions (IMF) by means of Empirical Mode Decomposition (EMD) and then performing HHT on the IMF. Therefore, the instantaneous frequency of the non-stationary signal is obtained. After the high-frequency instantaneous noise is removed, the remaining IMF components are reconstructed to obtain the final filtered result. To ensure that the IMF decomposed by the EMD retains the actual physical meaning of the instantaneous frequency and amplitude, it is necessary to set a stop filtering condition to control the number of iteration selections in the decomposition calculation. The standard deviation (SD) commonly used in the EMD decomposition is as the following expression:

$$SD = \sum_{t=0}^T \frac{|h_{k-1}(t) - h_k(t)|^2}{h_{k-1}(t)^2} \quad (1.4.1)$$

Where  $h_{k-1}(t)$  and  $h_k(t)$  are IMF components of the original data. When the SD is smaller than a pre-given value, the filtering process will stop. To further verify and analyze the filtering results, FFT energy spectrum of the original and filtered signals were adopted to verify the filtering effect.

## **CHAPTER 2. FORMULA DERIVATION AND MATHEMATICS**

### **2.1 Introduction of Stress Wave**

Stress waves produced by an impact load are typically characterized by fast rise and fall times, high amplitude, and short duration (Iskander, Omidvar, & Bless, 2015). Stress waves transmitted through an elastic-plastic material can be separated into two distinct waves, an elastic wave with a magnitude in the Hugoniot elastic limit (HEL) and a plastic wave.

When a localized disturbance is applied suddenly into a medium, it will propagate to other parts of this medium (Haddad, 2000). The local excitation is not detected at the other positions of the medium instantaneously, as some time would be necessary for the disturbance to propagate from its source to other parts of the medium. This fact constitutes a general basis for the interesting subject of "wave propagation". For instance, the transmission of sound in air, the propagation of a seismic disturbance in the earth, and the transmission of radio waves, among others. In the case, when the suddenly applied disturbance is mechanical, e.g., an impact force, the resulting waves in the medium are due to mechanical stress effects and, thus, these waves are referred to as "mechanical stress waves", or simply "stress waves".

In rigid body dynamics, it is assumed that, when an external force is applied to any point of the body, the resulting effect sets every other point of the body instantaneously in motion, and the applied force can be considered as producing a linear acceleration of the whole body, together with an angular acceleration about its centroid (Kelly, 2.2 One-dimensional Elastodynamics, 2014).

In the theory of deformable media, the body is in equilibrium under the action of the externally applied forces, and the occurring deformations are assumed to have reached their equilibrium static values (Haddad, 2000). This assumption could be sufficiently accurate for problems in which the time between the application of the force and the setting up of effective equilibrium is short compared with the time in which the observation is made. However, if the external force is applied for only a short period of time, or it is changing rapidly, the resulting effect must be considered from the point of view of stress wave motion.

Mechanical stress waves originate due to a forced motion of a portion of a deformable medium. As the other parts of the medium are deformed, because of such motion, the disturbance is transmitted from one point of the medium to the next and the disturbance, or wave, progresses through the medium. In this process, the resistance offered to deformation by the consistency of the medium, as well as the resistance to motion due to the inertia, must be overcome. As the disturbance propagates through the medium, it carries along with various amounts of kinetic and potential energies. Energy can be transmitted over considerable distances by wave motion. The transmission of energy is affected because motion is passed on from one particle to the next and not by any sustained bulk motion of the entire medium. Mechanical waves are characterized by the transport of energy through motions of particles about an equilibrium position. Thus, bulk types of motion of a medium such as those that occur in the turbulence of fluid are not classified as wave motion (Haddad, 2000).

## 2.2 One-dimensional Elastostatics and Elastodynamics

In elastostatics problems, it is not necessary to know how the load was applied, or how the material particles moved to reach the stressed state; it is necessary only that the load was applied slowly enough so that the accelerations are zero, or that it was applied sufficiently long ago that any vibrations have died away and movements have ceased. There are three main equations in the elastostatics problems:

$$\frac{d\sigma}{dx} + b = 0 \quad \text{Equation of equilibrium (2.2.1)}$$

$$\varepsilon = \frac{du}{dx} \quad \text{Strain-Displacement Relationship (2.2.2)}$$

$$\sigma = E\varepsilon \quad \text{Constitutive Equation (2.2.3)}$$

Where E is Young's Modulus, b is a body force (per unit volume). The unknowns in these three equations are the stress  $\sigma$ , strain  $\varepsilon$ , and displacement u. These equations can be combined to give a second-order differential equation in u, called Navier's Equation:

$$\frac{d^2u}{dx^2} + \frac{b}{E} = 0 \quad \text{1-D Navier's Equation (2.2.4)}$$

This equation requires two boundary conditions to obtain a solution (Kelly, 2.1 One-dimensional Elastostatics, 2014).

In rigid body dynamics, it is assumed that when a force is applied to one point of an object, every other point in the object is set in motion simultaneously (Kelly, 2.2 One-dimensional Elastodynamics, 2014). In static elasticity, it is assumed that the object is at rest and is in

equilibrium under the action of the applied forces; the material may well have undergone considerable changes in deformation when first struck, but one is only concerned with the final static equilibrium state of the object. Elastostatics and rigid body dynamics are sufficiently accurate for many problems but when one is considering the effects of forces that are applied rapidly, or for very short periods of time, the effects must be considered in terms of the propagation of stress waves. The analysis presented below is for one-dimensional deformations. The assumptions are that (1) material properties are uniform over a plane perpendicular to the longitudinal direction, (2) plane sections remain plane and perpendicular to the longitudinal direction, and (3) there is no transverse displacement.

In the case of elastodynamics,  $u=u(x, t)$  and consider the governing equations:

$$\frac{\partial \sigma}{\partial x} + b = \rho a \quad \text{Equation of Motion (2.2.5)}$$

Where  $a$  is the acceleration. Expressing the acceleration in terms of the displacement, combining eq. (1.2.3), (1.2.4) and (1.2.5), the dynamic version of Navier's equation can be derived:

$$E \frac{\partial^2 u}{\partial x^2} + b = \rho \frac{\partial^2 u}{\partial t^2} \quad (2.2.6)$$

In most situations, the body forces will be negligible, and so consider the partial differential equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \quad \text{1-D Wave Equation (2.2.7)}$$

Where  $c = \sqrt{\frac{E}{\rho}}$ , is the speed of wave propagating in the materials. It will be proved in Appendix A.



### 2.3 Particle velocities and Stress wave speed

A stress wave travels at speed  $c$  through the material from the point of disturbance, e.g., applied load (Kelly, 2.2 One-dimensional Elastodynamics, 2014). When the stress wave reaches a given material particle, the particle vibrates about an equilibrium position. When the stress wave passes away from the particle, the particle goes back to its original position. As for the parts that have not been disturbed yet, they just keep in equilibrium. Fig. 2-1 shows the situation of the whole bar that is disturbed. Since the material is elastic, no energy is lost, and the solution predicts that the particle will vibrate indefinitely, without damping or decay, unless that energy is transferred to a neighboring particle. This type of wave, where the disturbance (particle vibration) is in the same direction as the direction of wave propagation, is called a longitudinal wave. The wave equation is solved subject to the initial conditions and boundary conditions. The initial conditions are that the displacement  $u$  and the particle velocity  $\frac{\partial u}{\partial t}$  are specified at  $t = 0$  (for all  $x$ ). The boundary conditions are that the displacement  $u$  and the first derivative  $\frac{\partial u}{\partial x}$  are specified (for all  $t$ ). This latter derivative is the strain, which is proportional to the stress. In problems where there is no boundary (an infinite medium), no boundary conditions are explicitly applied. A semi-infinite medium will have one boundary. For a rod of finite length, there will be two boundaries and a boundary condition will be applied to each boundary.

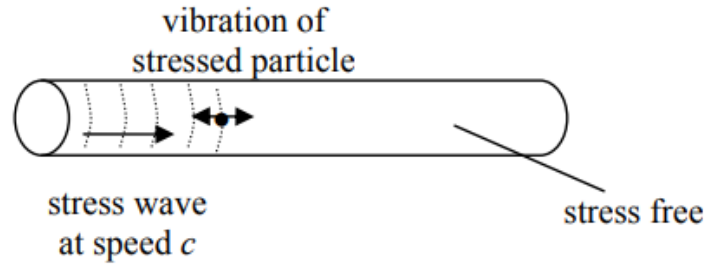


Figure 2-1: Stress wave travelling at speed  $c$  through an elastic rod

Rewrite the 1-D Wave Equation (2.2.7) as:

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t} \quad (2.3.1)$$

Where  $v$  is the particle velocity. Consider an element of material which has just been reached by the stress wave, seen in Fig. 2-2. The length of material passed by the stress wave in a time interval  $\Delta t$  is  $c\Delta t$ . During this time interval, the stressed material at the left-hand side of the element moves at (average) velocity  $v$  and so moves an amount  $v\Delta t$ . The strain of the element is then the change in length divided by the original length:

$$\varepsilon = -\frac{v}{c} \quad (2.3.2)$$

Under the small strain assumption, this implies that  $v \ll c$ , also that the density of the element will change as it is compressed, but again this change in density is small and can be neglected in the linear elastic theory. This formula will be introduced in detail in the next section.

The stress on the free side of the element is zero. Then eq. (2.3.1) leads to:

$$\frac{\Delta\sigma}{c\Delta t} = \rho \frac{v}{\Delta t} \quad (2.3.3)$$

And so:

$$\Delta\sigma = \rho cv \quad (2.3.4)$$

where  $\Delta\sigma$  is the discontinuity in the stress across the wave front.

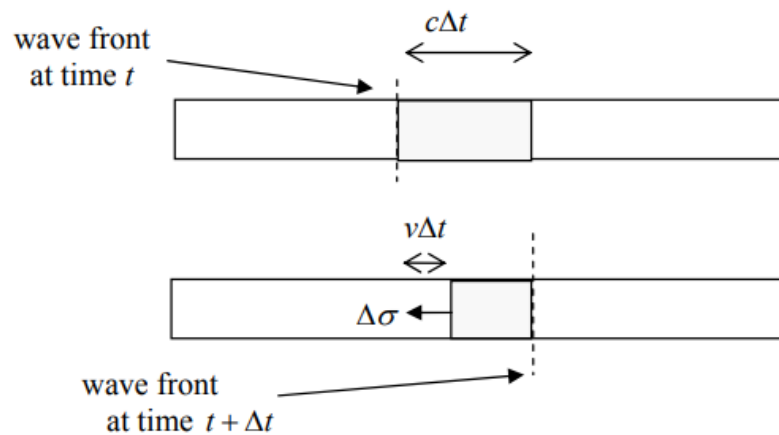


Figure 2-2: Stress wave passing through a material element

Since  $\Delta\sigma = E\varepsilon$ , one has  $c = \sqrt{\frac{E}{\rho}}$ . The wave speeds for some materials are given in Table 2-1. The

wave speeds for typical engineering materials are in the order of km/s. Therefore, when load is in the elastic field, the particle velocities will be in the range 0 – 50 m/s.

Table 2-1: Elastic Wave Speeds for Several Materials

Material	$\rho \left( \frac{kg}{m^3} \right)$	E (GPa)	C(m/s)
Aluminum Alloy	2700	70	5092
Brass	8300	95	3383
Copper	8500	114	3662
Lead	11300	17.5	1244
Steel	7800	210	5189
Glass	1870	55	5300
Granite	2700	26	3120
Limestone	2600	63	4920

Consider steel: the velocity at which the material ceases to behave linearly elastic (taking the yield stress to be 400MPa) is  $v = Y / c_p \approx 10\text{m/s}$ .

Fig. 2-3 shows a schematic diagram of SHPB; specifically, a split Hopkinson pressure compression bar. The working principle is that the striker impacts the incident bar first, then elastic stress wave will be produced in the incident bar. Then, the wave will propagate along with the incident bar, specimen, and transmitted bar. Since the material of the specimen is different from those two bars', there will be a reflected wave that passes back through the incident bar.

After the impact, a compressive stress wave with a magnitude of  $\sigma = \rho cv/2$  develops in the input bar. In eq. (2.3.4),  $\Delta\sigma = \rho cv$ . Thus, here is the derivation of the magnitude of  $\sigma = \rho cv/2$ :



Fig. 2-3: The moment before impact

In this figure, it shows the moment before the impact. The left bar is the striker, and the right bar is the incident bar. Their Young's modulus, density, cross-sectional area, and wave speed are  $E_{st}, \rho_{st}, A_{st}, C_{st}$  and  $E_i, \rho_i, A_i, C_i$ , respectively.

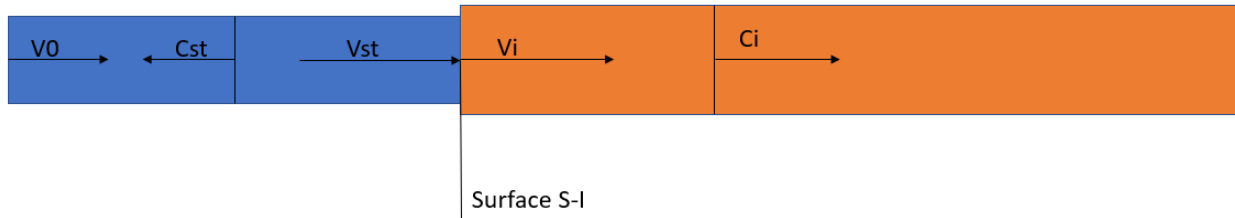


Fig.2-4: The moment at impact

Figure 2-4 shows the moment at impact. The  $v_i$  and  $v_{st}$  are the velocities of the particles at the incident bar and striker, respectively. The  $c_i$  and  $c_{st}$  are the wave speed in the incident bar and striker, respectively. At surface S-I, from continuity condition, there will be  $v_{st}$  equals to  $v_i$ .

According to Newton's third law:

$$-\Delta\sigma_{st}A_{st} = \Delta\sigma_iA_i \text{ (the direction of stress needs to be inspected)} \quad (2.3.5)$$

It means the action force equals the reaction force. If taking the right direction as positive, in other word, the increasing stress equals to that of the decreasing.

From equation (2.3.4), the  $\Delta\sigma_{st}$  is from the change of velocity of the particle. So,  $\Delta\sigma_{st}$  should be:

$$\Delta\sigma_{st} = \rho_{st} \times c_{st} \times v_{st} - \rho_{st} \times c_{st} \times v_0 \quad (2.3.6)$$

Similarly, the  $\sigma_i$  equals to:

$$\Delta\sigma_i = \rho_i \times c_i \times v_i - \rho_i \times c_i \times 0 \quad (2.3.7)$$

Then it becomes:

$$-A_{st} \times (\rho_{st} \times c_{st} \times v_{st} - \rho_{st} \times c_{st} \times v_0) = (\rho_i \times c_i \times v_i - \rho_i \times c_i \times 0) \times A_i \quad (2.3.8)$$

Since  $v_{st}$  equals to  $v_i$ :

$$v_i = v_{st} = \frac{\rho_{st} \times c_{st} \times v_0 \times A_{st}}{\rho_{st} \times c_{st} \times A_{st} + \rho_i \times c_i \times A_i} \quad (2.3.9)$$

Substitute eq. (2.3.9) into the eq. (2.3.6) and eq. (2.3.7):

$$\Delta\sigma_{st} = \rho_{st} \times c_{st} \left( \frac{\rho_{st} \times c_{st} \times v_0 \times A_{st}}{\rho_{st} \times c_{st} \times A_{st} + \rho_i \times c_i \times A_i} - v_0 \right) = -\rho_{st} c_{st} \frac{\rho_i \times c_i \times v_0 \times A_i}{\rho_{st} \times c_{st} \times A_{st} + \rho_i \times c_i \times A_i} \quad (2.3.10)$$

$$\Delta\sigma_i = \rho_i \times c_i \times \frac{\rho_{st} \times c_{st} \times v_0 \times A_{st}}{\rho_{st} \times c_{st} \times A_{st} + \rho_i \times c_i \times A_i} \quad (2.3.11)$$

If the striker and incident bar are identical materials and have the same cross-sectional area, that means:  $\rho_{st} = \rho_i = \rho$ ,  $c_i = c_{st} = c$ ,  $A_i = A_{st} = A$ , therefore:

$$\Delta\sigma_{st} = -\frac{\rho cv_0}{2} \quad (2.3.12)$$

$$\Delta\sigma_i = \frac{\rho cv_0}{2} \quad (2.3.13)$$

So, when the striker and incident bar are identical material and have the same cross-sectional area, the magnitude of stress wave in the incident bar is  $\frac{\rho cv_0}{2}$ . The magnitude has a linear relationship with respect to the velocity of the striker. In other words, if one wants to get a higher stress, he can increase the velocity of the striker linearly.

#### 2.4 The way to calculate strain of strain gauge from the Wheatstone bridge circuit

For this experiment, strain gauges are used to measure the strain that occur in the bars. However, strain gauges can't output any signals actively; they just change their resistances when they have deformation. Therefore, some tools are needed to record the resistances change history. A normal ohmmeter can't work well in this situation because the resistances vary very quickly, people even can't see the change from the ohmmeter. An oscilloscope with trigger function and a high sampling rate can record the fast variations. On the other hand, if oscilloscope is used in this system, one must convert the variations of resistances into electric signals like voltage or current. Thus, a Wheatstone Bridge Circuit is used because it can convert the resistance change into voltage change and enlarge it so that it can be tracked by high-speed oscilloscopes (Zhu, 2019). A typical Wheatstone quarter bridge setup is shown in Figure 2-5. R1, R2, and R3 have the same resistance as the strain gauge at zero loading conditions. An external voltage would be applied between points A and B. When the bridge is balanced, there would be no output between points C and D. Once

the strain gauge is loaded, the change of resistance would break the balance, and a potential difference would be generated between points C and D, which would be recorded by the oscilloscope.

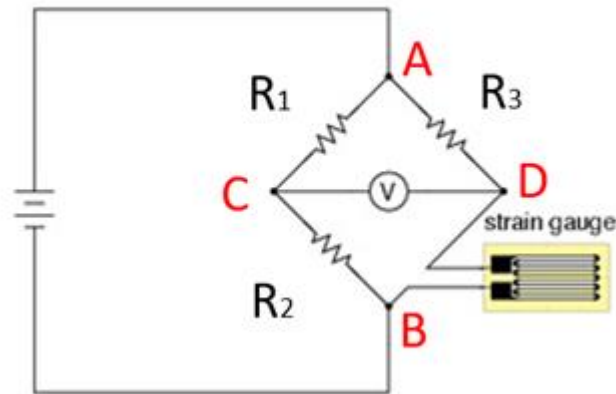


Fig. 2-5: Schematic of Wheatstone Bridge Circuit (quarter bridge) (Zhu, 2019)

However, in the real test, lead wires are used to connect the strain gauges with the Wheatstone Bridge Circuit. If using a two-wire strain gauge, it cannot build up a balanced bridge circuit because the wires have their own resistances, shown in the Figure 2-6.

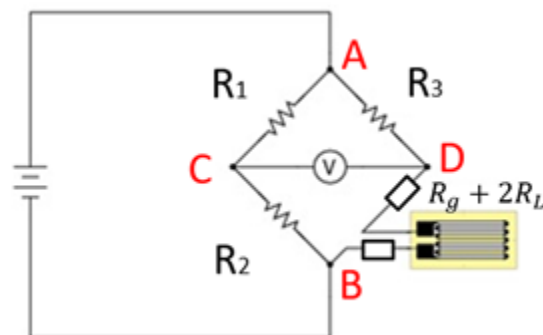


Fig. 2-6: Two-wire Strain gauge circuit



The resistance of the unstrained strain gauge is identical to  $R_1$ ,  $R_2$  and  $R_3$ . So, it has:

$$\frac{R_1}{R_2} = \frac{R_3}{R_g} \quad (2.4.1)$$

But it must take the resistances of lead wires into the calculation, so it gets:

$$\frac{R_1}{R_2} \neq \frac{R_3}{R_g + 2R_L} \quad (2.4.2)$$

It breaks the balance of equation (2.4.1). In other words, this circuit cannot balance either. Thus, a three-wire strain gauge must be used. It is shown in Figure 2-7. It can be found that the balance remains when wires used have the same initial resistance, as shown in equation (2.4.3).

$$\frac{R_1}{R_2} = \frac{R_3 + R_L}{R_g + R_L} \quad (2.4.3)$$

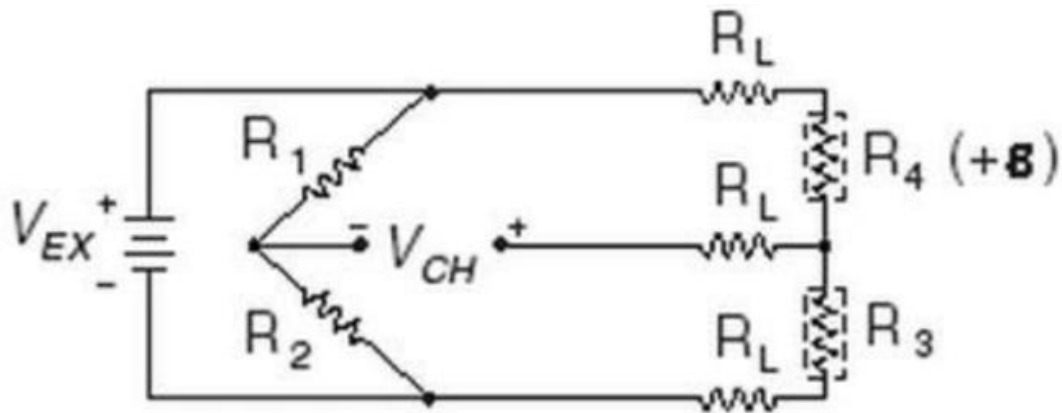


Fig.2-7: Three-wire Wheatstone Bridge Circuit

The formula to get strain is (Lang, 2012):

$$\text{strain}(\varepsilon) = \frac{-4V_r}{GF(1+2V_r)} \left( 1 + \frac{R_L}{R_g} \right) \quad (2.4.4)$$

This formula will be explained in Appendix B. In the eq. 2.4.4,  $R_L$  is the resistance of lead wire and  $R_g$  is the nominal resistance of the gauge. GF is the gauge factor and it is from the strain gauge's manual. As for the physical meaning, it is:

$$GF = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\varepsilon} \quad (2.4.5)$$

$V_r$  is the voltage ratio and is defined as:

$$V_r = \frac{V_{ch}(\text{strained}) - V_{ch}(\text{unstrained})}{V_{ex}} \quad (2.4.6)$$

where  $V_{ex}$  is the external voltage and  $V_{ch}$  is the output voltage. Resistors R1 and R2 are half bridge completion resistors, R3 is the dummy gauge, and R4 is the active strain sensing gauge. So, through the three wave signals: incident wave, reflected wave and the transmitted wave, have strains denoted as  $\varepsilon_i$ ,  $\varepsilon_r$  and  $\varepsilon_t$ , respectively. Since the voltage signal is a function of time, the strain from the strain gauge should also be a function of time. At a particular time  $t_a$ , the incident strain should be  $\varepsilon_i(t_a)$ . Similarly, the reflected strain and transmitted strain are  $\varepsilon_r(t_a)$  and  $\varepsilon_t(t_a)$ , respectively.

## 2.5 Calculate the stress and strain rate with the strain gotten

From the signals in the oscilloscope, the direction of oscillation (compression or tension) of the reflected wave is always opposite to the original wave. For instance, when the incident wave

is a compression wave, the reflected wave back to the incident bar must be a tensile wave. Meanwhile, these two directions of wave propagation are opposite too. Some examples are shown in the following figures:

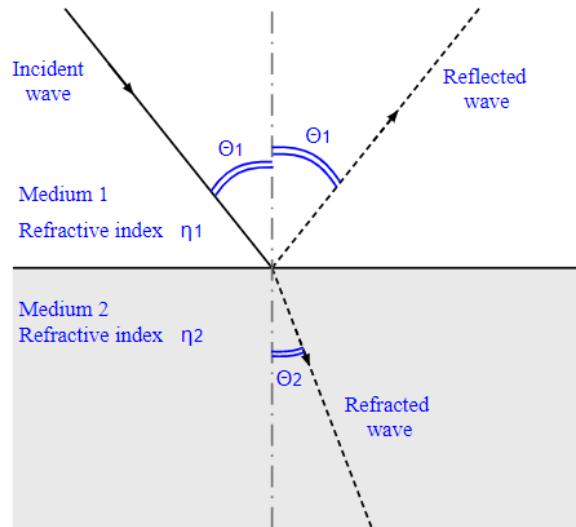


Fig. 2-8: Propagation of the reflected & refracted waves

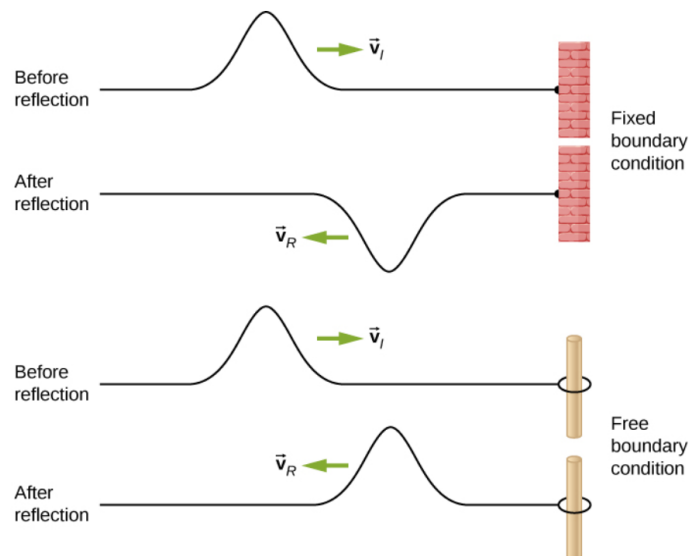


Fig. 2-9: Phenomenon shows the difference between the reflected and original waves

From eq. 2.3.2,  $\vec{v} = -\varepsilon \times \vec{c}$ ; where  $\vec{v}$  is the velocity of the particles in the bar,  $\varepsilon$  is the strain of bars, and  $\vec{c}$  is the velocity of the wave. One can get  $\varepsilon$  from the signals recorded by the oscilloscope. The magnitude of  $\vec{c}$  is known, so one can figure out the magnitude of  $\vec{v}$  by this formula. It is important to get the magnitude of  $\vec{v}$ , because by integrating  $\vec{v}$  respect to time, one will get the displacement  $\vec{u}$  of bars. And regarding to conditions talked about previously, the particles in the bar only oscillate when there is a disturbance and will go back equilibrium position afterwards. At the beginning, the disturbance will propagate in the bar without energy loss, so the strain at the position of the strain gauges will repeat at the end of the bar. Therefore, the strain from the strain gauges are used to describe the strain at the end of bar. In other words, the  $\vec{v}$  and  $\vec{u}$  is from  $\varepsilon$ , therefore, the  $\vec{v}$  and  $\vec{u}$  at the end of bar are known as well. By subtracting the displacements of both ends of the bars, the deformation of the specimen can be determined. In other words, the strain of the specimen, because one knows the original length of strain gauge. So, vector  $\vec{v}$  is derived below.

There are four kinds of wave propagation, the first one is a compression wave propagating along with the positive coordinate direction, as shown below:

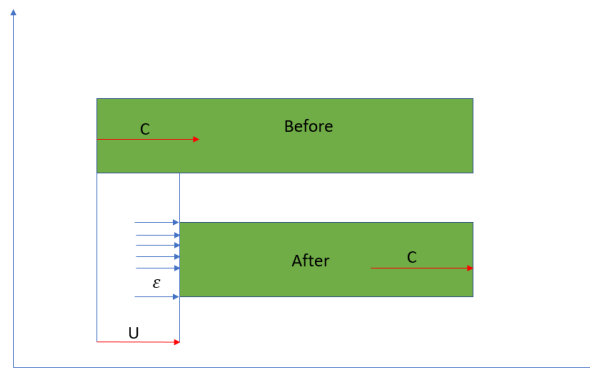


Fig. 2-10: Compression wave propagates along with positive X axis

An assumption here is shown in a micro-element in the bar. From Figure 2-9, after the time  $t$ , the wave propagates from the bar's left end to the right. So, the length of this element is  $\vec{L} = \vec{c} \times t$ , and the displacement of left end is  $\vec{u}$  whereas the deformation of the bar is  $\delta$ . Since this is a small element, the strains are uniform. That means  $\vec{u} = -\int_0^L \varepsilon dx$  can be transferred into  $\vec{u} = -\varepsilon \times \vec{L} = -\varepsilon \times \vec{c} \times t$ . The sign is inspected through Figure 2-9. The velocity of the particle is  $\vec{v} = \frac{d\vec{u}}{dt} = -\varepsilon \times \vec{c}$ . The three other situations are a tensile wave propagating along the positive X axis, a compression wave propagating along the negative X axis, and a tensile wave propagating along the negative X axis. They are shown in Figures 2-10, 2-11, and 2-12, respectively. By checking these three figures, it shows that  $\vec{v} = -\varepsilon \times \vec{c}$  is satisfied in every propagation situation.

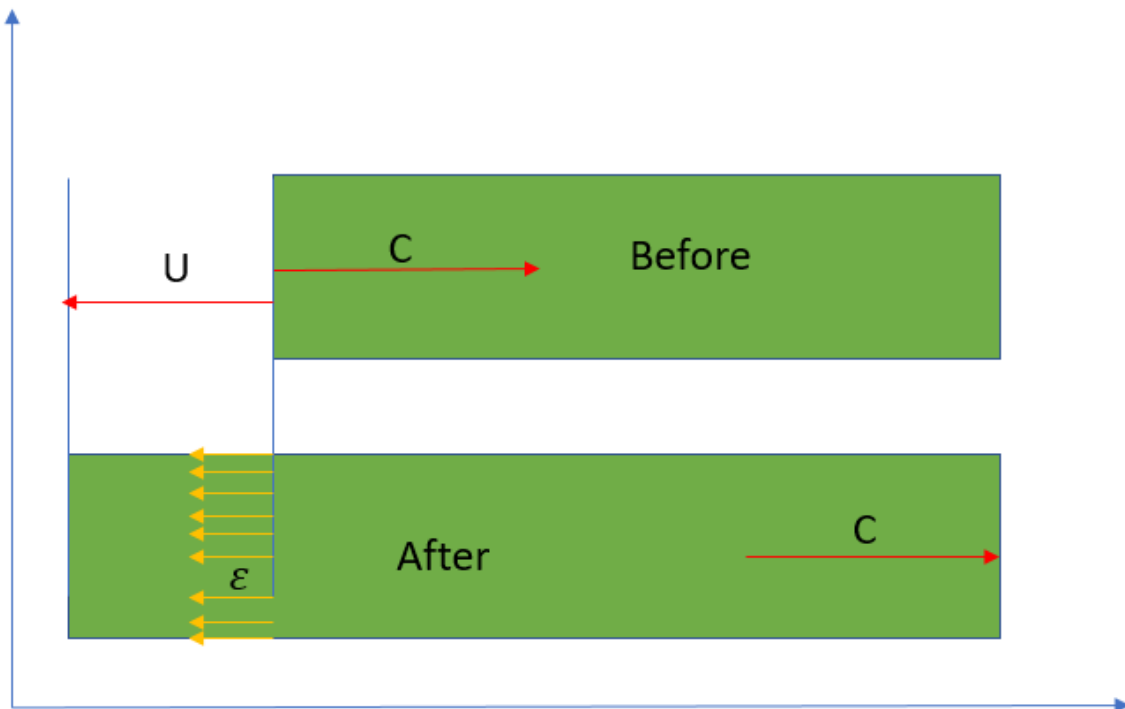


Fig. 2-11: A tensile wave propagating along the positive X axis

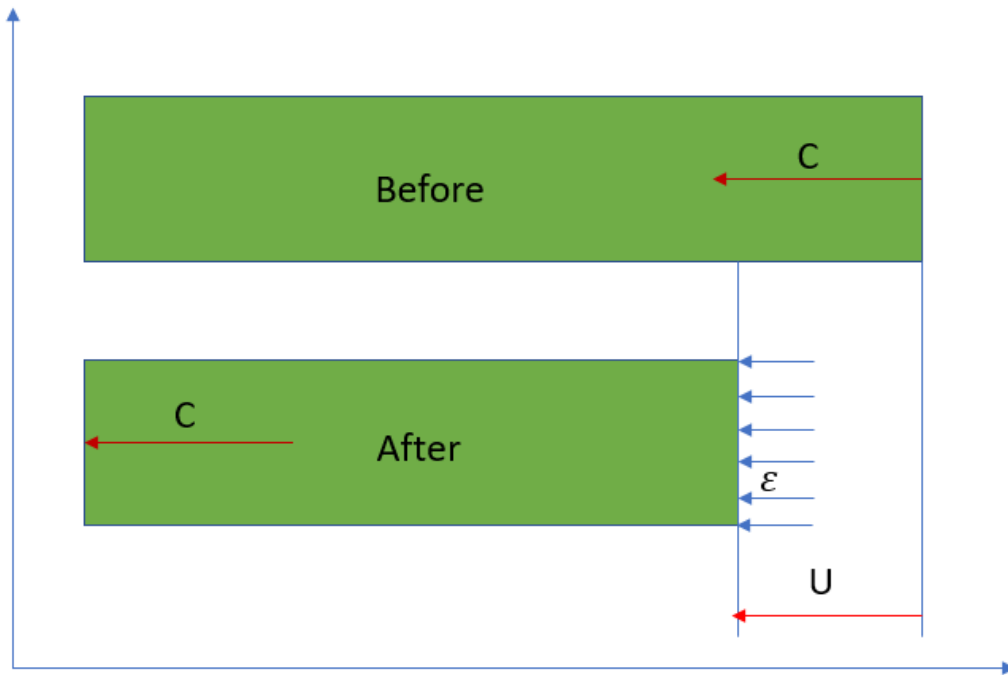


Fig.2-12: A compression wave propagating along the negative X axis

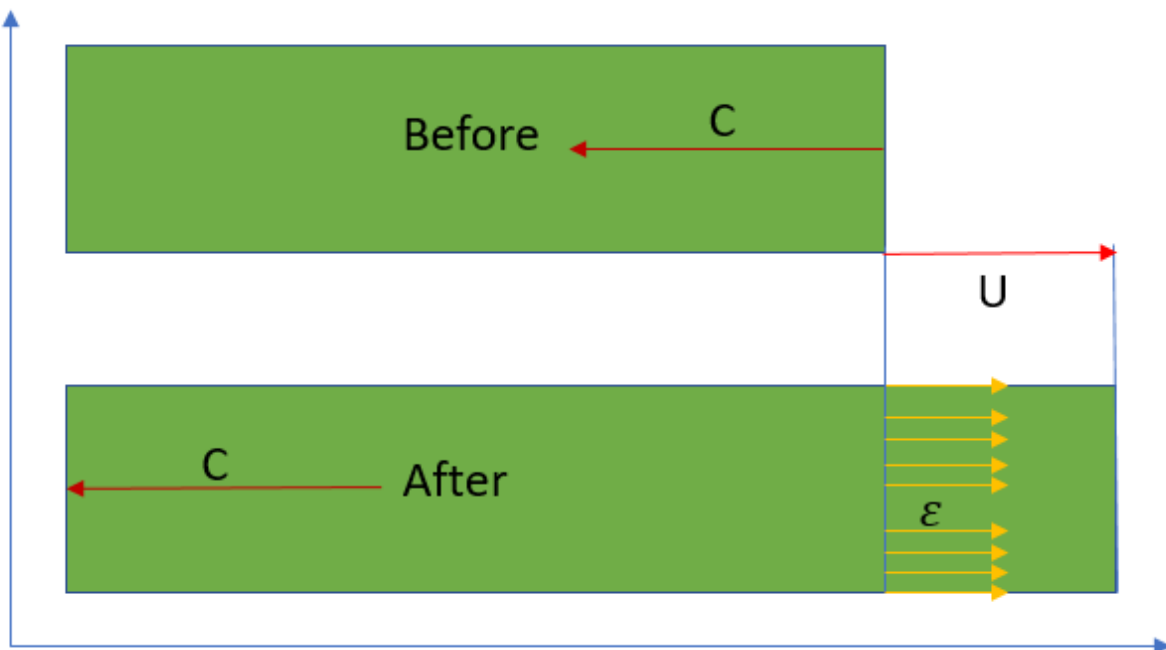


Fig.2-13: A tensile wave propagating along the negative X axis

Figure 2-13 shows the impact moment of the compression test. At the right end of the left bar, it includes two types of strain: incident strain ( $\epsilon_i$ ) and reflected strain ( $\epsilon_r$ ) at the same moment. However, they have opposite directions of propagation. At the left end of the right bar, there is only one transmitted strain which is  $\epsilon_t$ .  $C_i$ ,  $C_r$ , and  $C_t$  are the propagation directions of incident wave, reflected wave and transmitted wave, respectively. This figure has a detailed view of the right end of the left bar about the strain.  $\vec{u}_1$  is the displacement of the right end of the incident bar and  $\vec{u}_2$  is the displacement of the left end of the transmitted bar. Thus, the deformation of the specimen is  $\vec{u}_2 - \vec{u}_1$ .

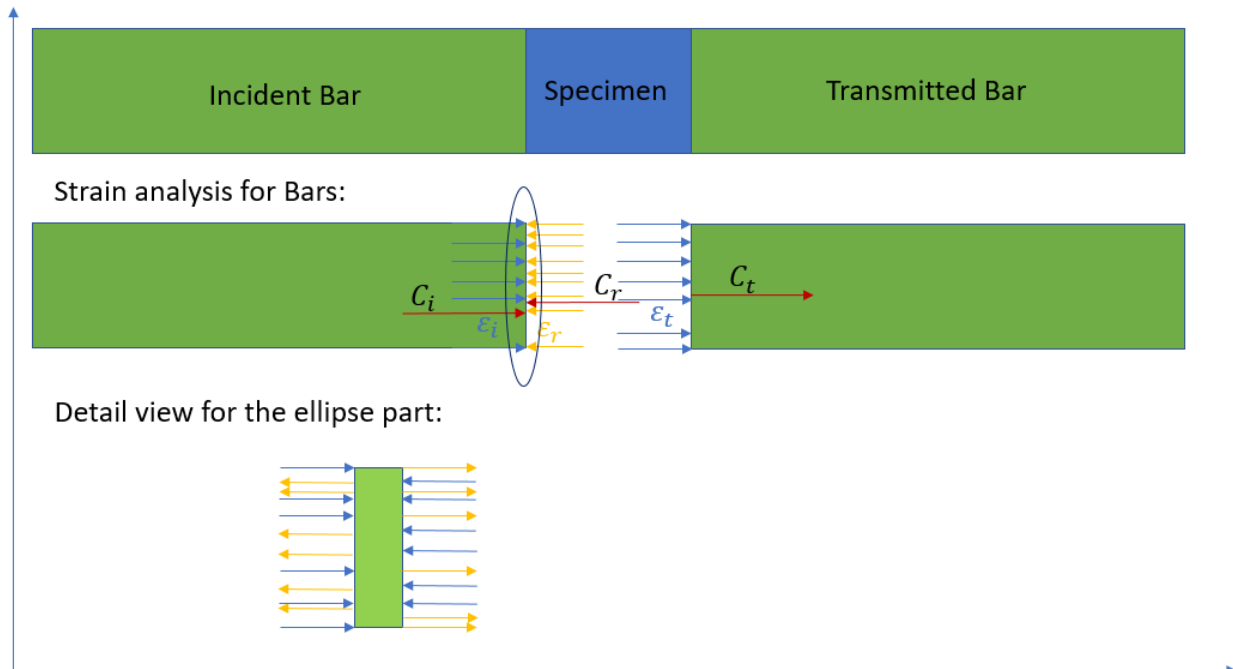


Fig. 2-14: The impact moment of compression test

$$\vec{u}_1 = \int_0^t \vec{v}_1 dt = \int_0^t (\vec{v}_i + \vec{v}_r) dt = \int_0^t (-\vec{c}_i \epsilon_i - \vec{c}_r \epsilon_r) dt \quad (2.5.1)$$

$\vec{v}_1$  is the velocity of particle at the end and it includes two parts: one is the velocity from the incident wave  $\vec{v}_i$  and the other one is the velocity from reflected wave  $\vec{v}_r$ . Through the formula  $\vec{v} = -\varepsilon \times \vec{c}$ , displacement can be derived by integrating the strain. The wave speed is stable as the material is the same, and one can find the direction of incident wave is opposite to the direction of reflected wave, so it is:

$$\vec{c} = \vec{c}_i = -\vec{c}_r \quad (2.5.2)$$

As talked in section 2.4, the strain of the particles is a function about time. Then, plug eq. 2.5.2 into eq. 2.5.1, it becomes:

$$\vec{u}_1 = \vec{c} \int_0^t (-\varepsilon_i + \varepsilon_r) dt \quad (2.5.3)$$

For the displacement of the left end of the transmitted bar, it is:

$$\vec{u}_2 = \int_0^t \vec{v}_t dt = \int_0^t -\vec{c}_t \varepsilon_t dt \quad (2.5.4)$$

Similarly,  $\vec{c}_t = \vec{c}$ , then:

$$\vec{u}_2 = \vec{c} \int_0^t -\varepsilon_t dt \quad (2.5.5)$$

The deformation of the specimen is:

$$\vec{u}_2 - \vec{u}_1 = \vec{c} \int_0^t (\varepsilon_i - \varepsilon_r - \varepsilon_t) dt \quad (2.5.6)$$

The strain of specimen is:



$$\varepsilon_s = \frac{\vec{u}_2 - \vec{u}_1}{L_s} = \frac{c}{L_s} \int_0^t (\varepsilon_i - \varepsilon_r - \varepsilon_t) dt \quad (2.5.7)$$

Here,  $\vec{c}$  is replaced by  $c$ .  $c$  is the scalar velocity of the wave, because all of the velocity waves have been transferred to the positive direction, which means  $c$  can represent  $\vec{c}$  right now.  $L_s$  is the original length of specimen.

The stress at the rightest surface of the incident bar is:

$$\sigma = E\vec{\varepsilon} = E(\varepsilon_i + \varepsilon_r) \quad (2.5.8)$$

Thus, the force of the right end of the incident bar is:

$$\vec{F}_{rl} = \sigma \times A = E(\varepsilon_i + \varepsilon_r)A \quad (2.5.9)$$

Where  $A$  is the cross-sectional area of the incident bar.

Regarding Newton's third law, the force on the left side of the specimen is:

$$\vec{F}_{ls} = -\vec{F}_{rl} = -E(\varepsilon_i + \varepsilon_r)A \quad (2.5.10)$$

At the different ends of a rod, the directions of forces that produces the same stress (compression or tension) are opposite. For example, at the bar's positive surface (right side), a compression stress (negative stress) is from a negative force. On the other hand, at the negative surface (left side) of the bar, a compression stress is from a force faces to positive direction. Therefore, it is found that the signs of force and stress are opposite at the negative surface and the signs are the same at the positive surface, also known as the theory of tensor. Therefore, the stress at the left surface of the specimen is:

$$\sigma_{ls} = \frac{-\vec{F}_{ls}}{A_s} = \frac{E(\varepsilon_i + \varepsilon_r)A}{A_s} \quad (2.5.11)$$

Here  $A_s$  is the cross-sectional area of the specimen.

In the experiment, because of high strain-rates, the strain at the left side of the specimen will not act simultaneously with the strain at the right surface of the specimen. In this model, the specimen will undergo compression stress twice. One is from the incident bar and the other is from the transmitted bar.

In the transmitted bar, the strain is  $\varepsilon_t$ . So, the stress acts on the left side of the bar  $\sigma_t = E \times \varepsilon_t$ . Because it is the left side of the transmitted bar, the applied force is:

$$\vec{F} = -\sigma_t \times A = -E \times \varepsilon_t \times A \quad (2.5.12)$$

Here  $A$  is the cross-sectional area of the transmitted bar. Since the two bars are identical in size, the cross-section areas of both bars are the same.

From Newton's third law, the reaction force on the right side of the specimen is:

$$\vec{F}_{rs} = -\vec{F} = E \times \varepsilon_t \times A \quad (2.5.13)$$

So, the stress at the right surface of the specimen is:

$$\sigma_{rs} = \frac{E \times \varepsilon_t \times A}{A_s} \quad (2.5.14)$$

The average stress on the whole specimen is:

$$\sigma_s = \frac{\sigma_{ls} + \sigma_{rs}}{2} = \frac{E(\varepsilon_i + \varepsilon_r + \varepsilon_t)A}{2A_s} \quad (2.5.15)$$

From this model,  $\sigma_{ls}$  and  $\sigma_{rs}$  happen at different times. Thus, this is an average stress about time.

When the specimen reaches force equilibrium, as shown in the Figure 2-14,  $\overrightarrow{F_{ls}} = -\overrightarrow{F_{rs}}$ .

From eq. 2.5.10 and eq. 2.5.13, total strain is:

$$\varepsilon_i + \varepsilon_r = \varepsilon_t \quad (2.5.16)$$

Plug eq. 2.5.16 into eq. 2.5.15 and eq. 2.5.7, it becomes:

$$\sigma_s = \frac{E\varepsilon_t A}{A_s} \quad (2.5.17)$$

$$\varepsilon_s = -\frac{2c}{L_s} \int_0^t \varepsilon_r dt \quad (2.5.18)$$

The strain rate is:

$$\dot{\varepsilon}_s = \frac{d\varepsilon_s}{dt} = -\frac{2c}{L_s} \varepsilon_r \quad (2.5.19)$$



Fig. 2-15: Specimen is in equilibrium

$\varepsilon_t$  and  $\varepsilon_r$  are only used to get the stress, strain and strain rate in eq. 2.5.17, eq. 2.5.18 and eq. 2.5.19. In other words, the reflected wave and transmitted wave is required to calculate all necessary values which is why this method is called the two-wave method.

The second part is the tensile test and is quite similar to the compression test. Figure 2-15 shows the normal tensile test. Like the compression test, the displacements of the two ends of the bars need to be determined. Therefore, the displacement of the right end of the incident bar is:

$$\bar{u}_1 = \int_0^t \bar{v}_1 dt = \int_0^t (\bar{v}_i + \bar{v}_r) dt = \int_0^t (-\bar{c}_i \varepsilon_i - \bar{c}_r \varepsilon_r) dt \quad (2.5.20)$$

The displacement of the left end of the transmitted bar is:

$$\bar{u}_2 = \int_0^t \bar{v}_t dt = \int_0^t -\bar{c}_t \varepsilon_t dt \quad (2.5.21)$$

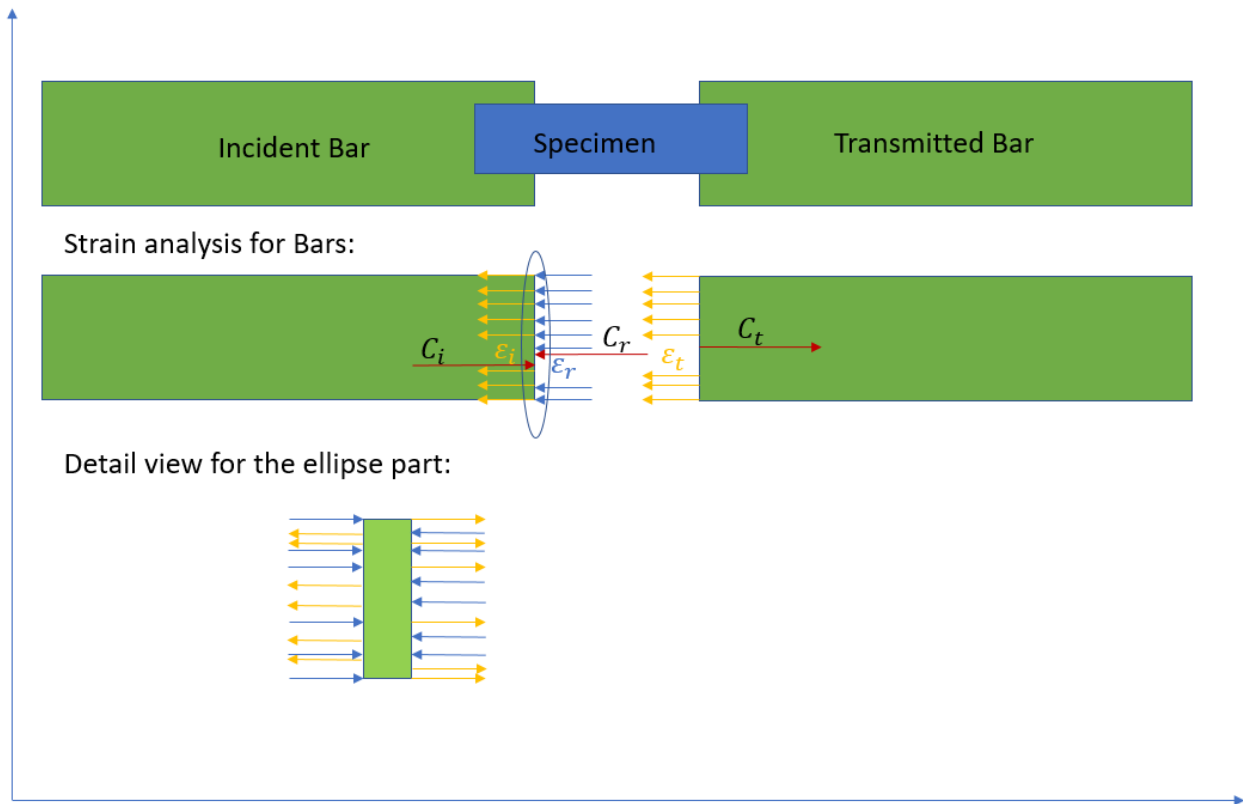


Fig. 2-16: A usual schematic of SHPTB

From the Figure 2-15, it is:

$$\vec{c}_i = -\vec{c}_r = \vec{c}_t = \vec{c} \quad (2.5.22)$$

Rewrite eq. 2.5.20 and eq. 2.5.21:

$$\vec{u}_1 = \vec{c} \int_0^t (-\epsilon_i + \epsilon_r) dt \quad (2.5.23)$$

$$\vec{u}_2 = \vec{c} \int_0^t -\epsilon_t dt \quad (2.5.24)$$

The deformation of the specimen is:

$$\vec{u}_2 - \vec{u}_1 = \vec{c} \int_0^t (\varepsilon_i - \varepsilon_r - \varepsilon_t) dt \quad (2.5.25)$$

The strain of the specimen is:

$$\varepsilon_s = \frac{\vec{u}_2 - \vec{u}_1}{L_s} = \frac{c}{L_s} \int_0^t (\varepsilon_i - \varepsilon_r - \varepsilon_t) dt \quad (2.5.26)$$

The strain rate of the specimen is:

$$\dot{\varepsilon}_s = \frac{d\varepsilon_s}{dt} = \frac{c}{L_s} (\varepsilon_i - \varepsilon_r - \varepsilon_t) \quad (2.5.27)$$

From eq. 2.5.26, it can be seen that the formula of the strain in the tensile test specimen is the same as the formula (eq.2.5.7) for the strain in the compression test. Usually, this formula is not used to calculate the tensile strain because in the compression test, the specimen is sandwiched between two bars. Therefore, the subtraction of the displacements of the two bars is the compressive deformation in the specimen. For the tensile test, due to the geometry of specimen and the way the specimen is connected with the bars, the real strain of the specimen is different compared to the strain calculated by this formula. The better method to get the strain and strain rate of the specimen is via the Digital Image Correlation (DIC). However, the stress formulas are still accurate and used for the tension tests since the forces are consistent all the time. The derivation of tensile stresses by the three-wave method as well as the strain, strain rate and stress via the two-wave method is the same as the compression test.

## **CHAPTER 3. THE PREPARATION FOR CONDUCTING THE EXPERIMENT**

### **3.1 Install the strain gauge**

The incident wave, reflected wave, and transmitted wave are inside the two bars. They are elastic waves, in other words, the elastic strain wave. That is why the strain gauges are required in this experiment. It is critical to install the strain gauge correctly. Here are the procedures and tips for bonding the strain gauges and soldering them on the bar. All chemical reagents and physical accessories are shown in Figure 3-1.

Surface preparation procedures

- a. Degrease the part of the bar where the strain gauges are going to be installed on with CSM-3 by using gauzes.
- b. Polish this area with a dry silicon carbide paper with a roughness of P400.
- c. Place one drop of M-PREP CONDITIONER A solution there and re-polish it with the P400 sandpaper.
- d. Use some gauze to remove the liquid on the surface.
- e. Make a mark on the bar. Usually, it is a circumferential circle that is used to make sure the strain gauge is parallel to the axial direction of the bar. If the bar is a steel bar, using a ballpoint pen; if it is an AL bar, using a pencil to draw the marker.
- f. Use the M-PREP CONDITIONER A and cotton swabs to get rid of the pencil or ballpoint pen ink. Technically, the slight scratch is still leaving on the bar.

- g. Clean the surface with gauze.
- h. Clean the whole area with M-PREP NEUTRALIZER 5A solution and cotton swabs.
- i. Wipe the area with a piece of new gauze again.

The preparation work of the strain gauge

- a. Clean a glass plate with M-PREP NEUTRALIZER 5A and gauzes. Wipe it only in one direction to make sure there is no new dirt.
- b. Pick up a strain gauge with a blunt tweezer and put it on the glass plate. The side of resistance wire is on the top.
- c. The type of tape used is PCT-2M. Because the first 2'' tape was exposed in the air all the time, it should be cut off. Use the following 2'' of long tape every time.
- d. Fold the tape at the free end a little bit to get a convenient non-sticky end.
- e. Tear off the tape and tape the strain gauge along with the longitudinal direction carefully. Avoid any air bubbles as much as possible. If there are any air bubbles between the strain gauge and tape, the tape should be removed and repeat steps d and e.

Transport the strain gauge from the glass plate and bond it on the bar

- a. Lift the whole tape and strain gauge at a small angle from the glass plate.
- b. Align the two corner triangles seen in the corners of the strain gauge along with the line drawn before. When the alignment is good, fix the strain gauge on the surface of the bar and press and push down the tape in one direction.
- c. Lift the tape up again but at a small angle until the whole strain gauge is exposed again. Because the strain gauge is bonded with tape, the bottom of strain gauge is exposed to the air right now.



- d. Place a small amount of 200 CATALYST with the small brush and paint it on the bottom of strain gauge.
- e. It will take at least 60 seconds to dry the 200 CATALYST.
- f. Place a drop of M-BOND 200 ADHESIVE underneath the strain gauge.
- g. Swipe the tape with gauze and then press the position of strain gauge immediately.
- h. Keep pressing the position of the strain gauge for two minutes and leave it there for another two minutes at least.
- i. Remove the tape.

#### Soldering process

- a. To eliminate the interference of the resistance of lead wire, there are three separate lead wires are used to connect the strain gauge with the amplifier. But there are only two soldering points on the strain gauge. So, two of the lead wires should be twisted together as one. For here, the black and white wire will be twisted together.
- b. Melt a small amount of lead wire using the solder pencil and drop a little of the molten lead on the BONDABLE TERMINALS CPF-75C work area.
- c. Place some molten lead onto the legs of the strain gauge.
- d. Curve the leg wires of the strain gauge in horizontal plane to make a dome shape to create a buffer for the wire.
- e. Bend the head of lead wire down to touch the tab on the BONDABLE TERMINALS CPF-75C and solder them together.
- f. Solder the lead wires for the amplifier on the tab also.
- g. Remove rosin with M-LINE ROSIN SOLVENT and dry it with gauze three times.
- h. Protect the soldering point with M-COAT A.

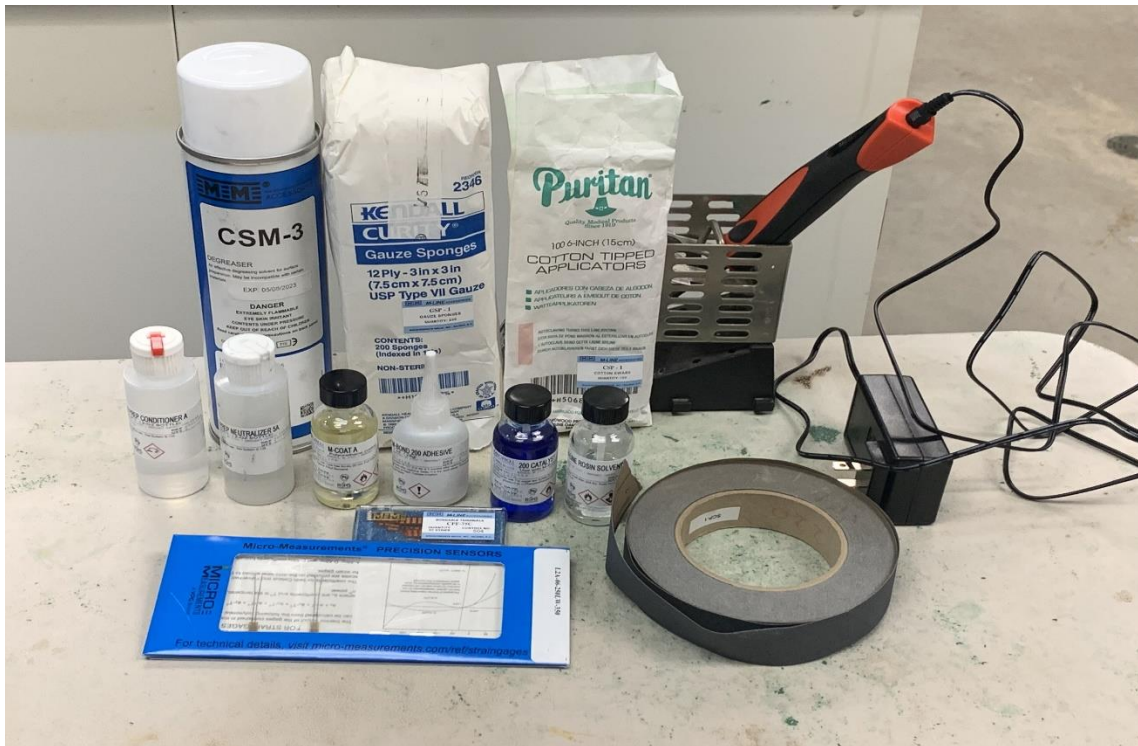


Fig. 3-1: Chemical reagent and accessories

### 3.2 Methods against Electric Magnetic Interference (EMI)

Protection layer for the wires

The lead wires between the strain gauges and the amplifiers are too thin to prevent the EMI in the environment. Therefore, it is necessary to make some protection foils for the wires. Based on the theory of Faraday's cage, a layer of aluminum foil is used to cover the outer skin of wires. A layer of tape is used to fix the aluminum foil and then this cage should be grounded. Because the lead wire is needed to be soldered with the BONDABLE TERMINALS on the bars and adaptors for the amplifier, there has to be some naked wires outside. However, the length of the naked wires should be shorter than a half inch.



Fig. 3-2: Protected lead wire for strain gauge

#### Ground connection

Since the signal collection system measures the electric signal, the system should be at the zero electric voltage position. In other words, all conduction in the system should be grounded. The conductors include the incident bar, the transmitted bar, the gun barrel, and the oscilloscope which are all grounded together.

For the incident bar, the transmitted bar and the gun barrel, a ground wire was twined on the bar and then taped.

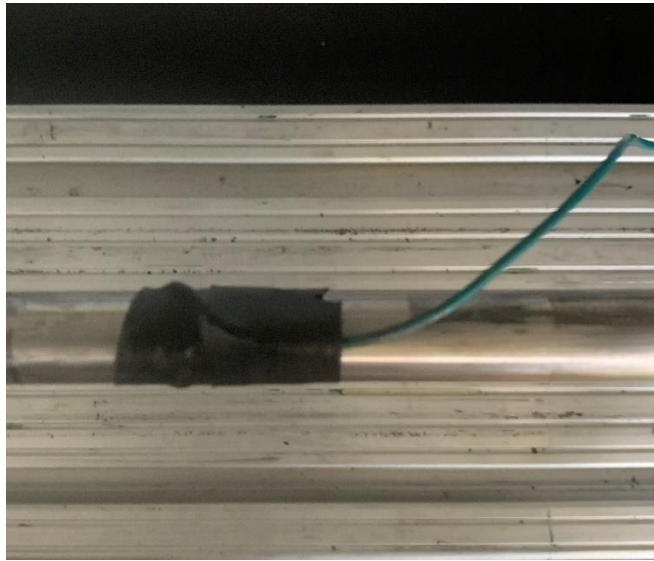


Fig. 3-3: The grounded wire on the bars

For the lead wires of the strain gauges, there is a Faraday's cage outside the wire. To make sure the Faraday's cage work well, it needs to be grounded. The method is that a wire with a clip head clamps a small part of the aluminum foil.



Fig. 3-4: The method to ground the Faraday Cage

There is a ground connector on the oscilloscope. The grounded method for that oscilloscope is that a wire with a clip head clamps the connector.



Fig. 3-5: The method for grounding the oscilloscope

In other words, grounding the system means the electric potential of all conductors should be the same. Thus, the method used here is that all lead wires are connected with a steel plate that lies on a bigger steel plate.



Fig. 3-6: The steel plate used as ground

### 3.3 The setting of the amplifier and the oscilloscope

The strain gauge measures the strain in the bars by changing its resistance passively but the oscilloscope can only analyze the voltage and current signal. So, the strain gauge is connected with an amplifier that has a Wheatstone bridge circuit inside that can transfer the variable of resistance of the strain gauge into a voltage signal. Another function of the amplifier is to enlarge the voltage signal. The connection between the lead wires of the strain gauges and the input adaptor of the amplifier is shown in Figure 3-7. The female connector J and H are connected together. Three lead wires of the strain gauges are connected with A, C and L respectively. As discussed before, two of these three wires are combined together, and they should be connected with A and C respectively. The setting of the interface of the amplifier is shown in Figure 3-8.





Fig. 3-7: The connection between the lead wires of the strain gauges and the input adaptor of the amplifier



Fig. 3-8: The interface of the amplifier

The collection mode of the oscilloscope for the experiments is by using the single trigger mode. The input mode for the oscilloscope is an analog signal input. Analog channels 1 and 2 are used. Each voltage division for channels 1 and 2 are 500 mv. The time represented by each square is 100 ms. The trigger mode is an edge trigger and it uses a rising slope. The trigger source is from channel 1 and the trigger level is 200 mv. The sample rate is 4 MSa/s. The meaning of these parameters and the way to change them can be founded in the user manual of the oscilloscope.

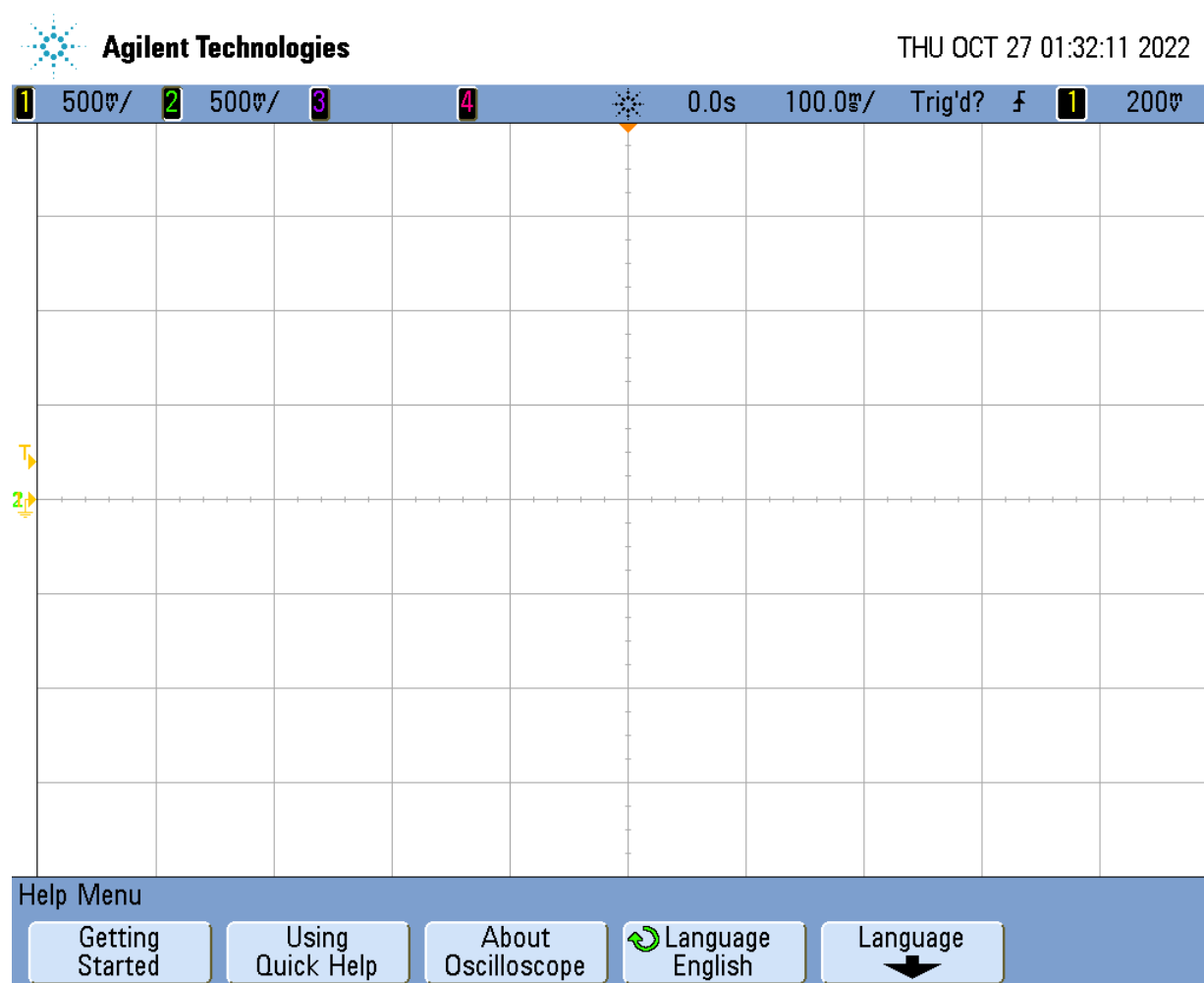


Fig. 3-9: The interface of the oscilloscope



## CHAPTER 4. MECHANISM OPTIMIZATION

### 4.1 Introduction of a general reflective SHTB

The reflective Hopkinson tension bar is determined by changing the connection method between the specimen and the loading rod based on the compression rod. Nicholas established a reflective tensile experiment (T, 1981). A compression ring with the same diameter as the experimental rod is added to the periphery of the tensile sample. When the compression wave starts to pass through the sample, the compression ring mainly bears the force. The compression wave is reflected from the end face of the transmission rod to form a tensile wave and then returns to the sample. Tensile loading is performed, so it is called a "reflective stretching". Figure 4-1 shows Nicholas's loading method. Figure 4-2 shows a schematic diagram of the reflective SHTB.

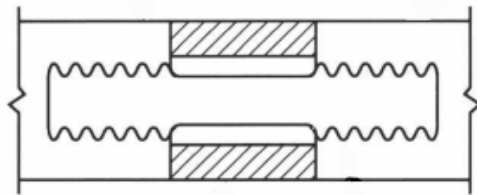


Fig. 4-1: Nicholas's loading method

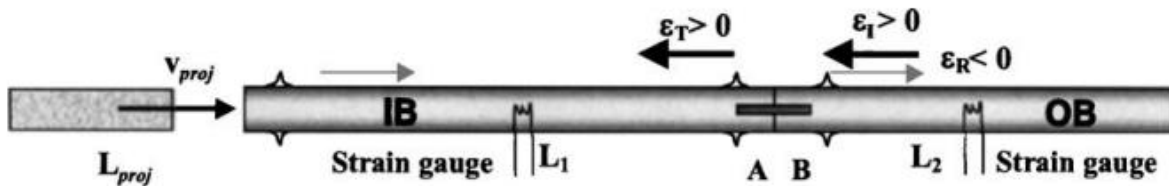


Fig. 4-2: A schematic diagram of the reflective SHTB

Take a typical reflective SHTB as an example. Figure 4-3 shows the wave propagation process in a typical reflective SHTB. The bullet hits the transmission rod in the axial direction with a certain velocity, causing a compressive stress wave to propagate in the rod. When the pressure pulse reaches the interface between the specimen and the pressure rod, it passes through the cross section formed by the pressure ring and the specimen in an ideally non-dissipative manner. Substantially, there are a few pulses  $\varepsilon_e$  that are reflected back into the transmitted bar. The cross-sectional area of the compression ring is designed to be more than 10 times larger than the cross-sectional area of the specimen, so it will withstand a major part of the compressive pulse, leaving the specimen with little or no compression and only elastic deformation. The compression pulse continues to travel through the sample, and when it reaches the free end of the incident rod, it propagates back in the form of a tensile wave. When the tensile wave reaches the sample, part of it passes through the sample to form a transmitted signal  $\varepsilon_t$ , and the other part is reflected to form a reflected signal  $\varepsilon_r$ . Since the pressure-bearing ring is not fixed on the pressure rod in any form, it can only bear compressive stress and cannot bear tensile stress, so the tensile pulses all act on the sample.

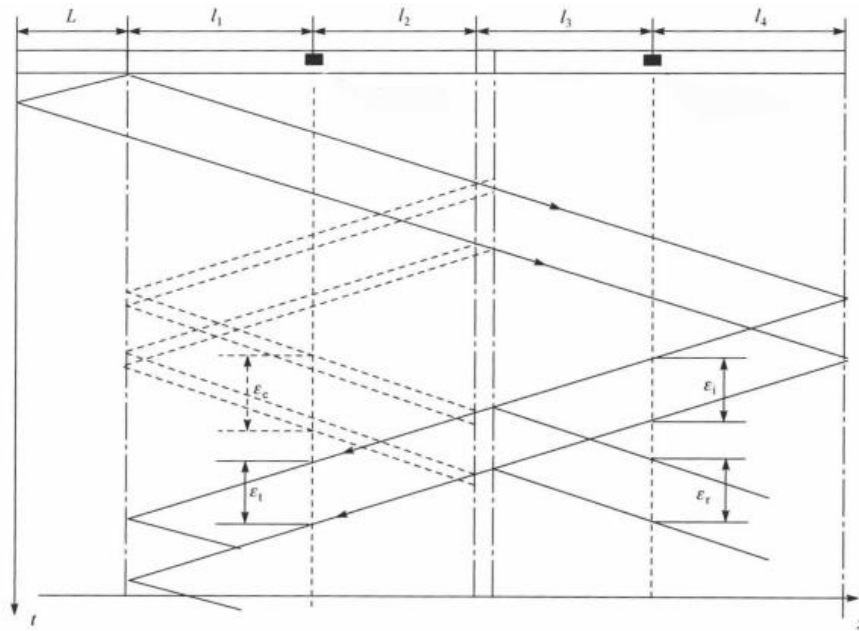


Fig. 4-3: The wave propagation process in a typical reflective SHTB

#### 4.2 The reflective SHTB in the NIU

Based on the theory and design, a type of reflective SHTB apparatus was designed. As Figure 4-2 shown, it mainly has three parts that are the projectile, the transmitted bar and the incident bar. Figure 4-4 shows the device in the NIU. There is a gas tank that is used to launch the projectile by high pressure air. The gun barrel is a hollow steel pipe used as the rail for the projectile. The outer diameter of gun barrel is 0.75 inch, and the inner diameter is 0.5 inch which is also the diameter of the projectile, the incident bar and the transmitted bar.

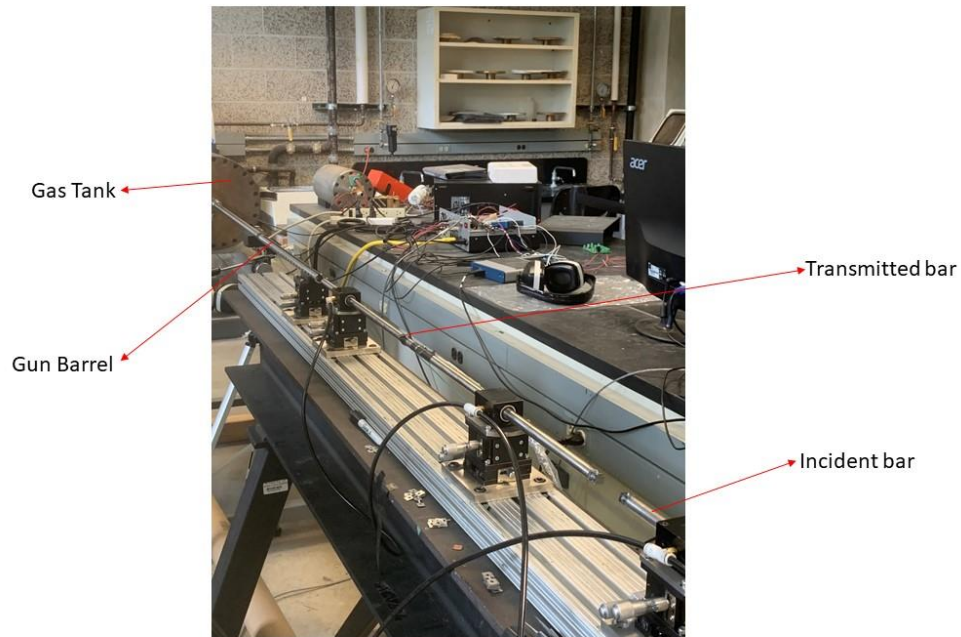


Fig. 4-4: The SHTB in NIU

Technically, the geometry of the specimen for the high strain rate tensile test should like the dog-bone. Figure 4-5 shows a general specimen for the tensile test. The specimen used is 0.1mm of thickness foil. Figure 4-6 is the plane drawing of the specimen. Those two holes on the specimen are used to align the sample with the bars and perform the tensile test.

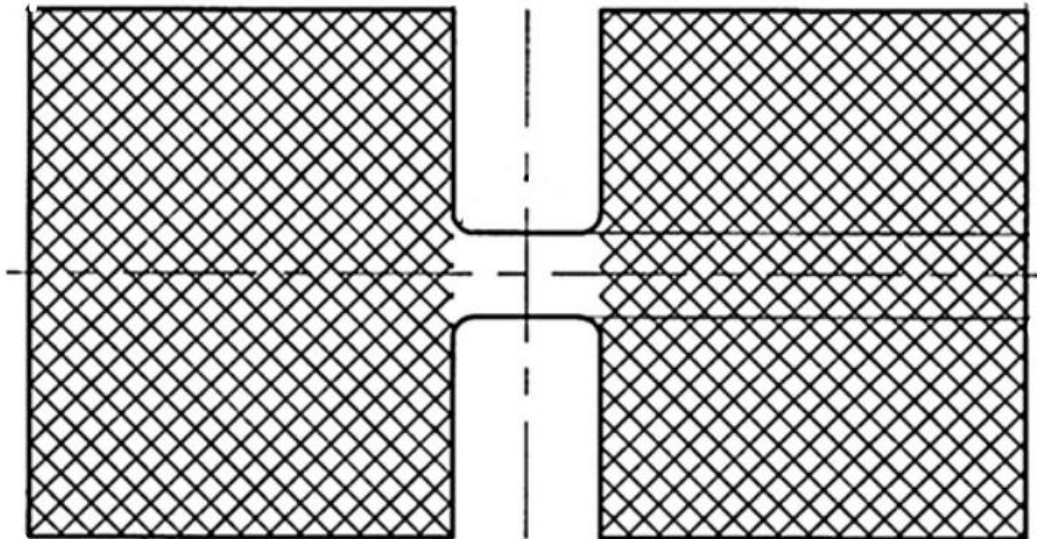


Fig. 4-5: A general drawing of the specimen for the tensile test (Peirs, Verleysen, Paepegem, & Degrieck, 2011)

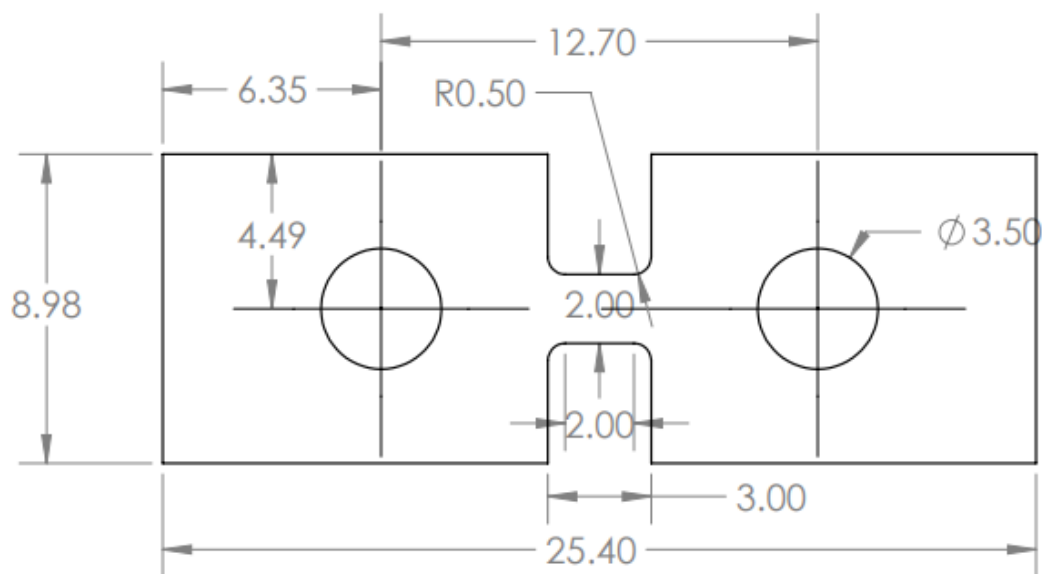


Fig. 4-6: The drawing of the specimen for SHTB in NIU

To assemble the specimens on the bars, the new bars with a special end were designed. Figure 4-7 is the drawing of the new bar. Figure 4-8 shows the assemble condition of samples and bars. In this design, there is no pressure ring to transport the stress wave. However, at the beginning of every experiment, the transmitted bar and the incident bar touch each other to propagate all elastic wave. As shown in the Figure 4-8, the specimen is only placed on the bars, there is no other constrains to fix it except two pins whose function is only alignment now. Therefore, the specimen will not influent the propagation of the first compression wave, and almost all compression wave will propagate through bars. However, the specimen will bear the influence of the tensile wave. When the compression wave is reflected back at the free end of the incident bar and becomes a tensile wave, this tensile wave will make the incident bar to move. However, at this moment, the transmitted bar has no movement. There is no other connections between the incident bar and the transmitted bar except the specimen. Therefore, the incident bar will pull the specimen by the pin on the bar, and the specimen will pull the transmitted bar by the pin on the bar correspondingly. That is how this tensile test process.

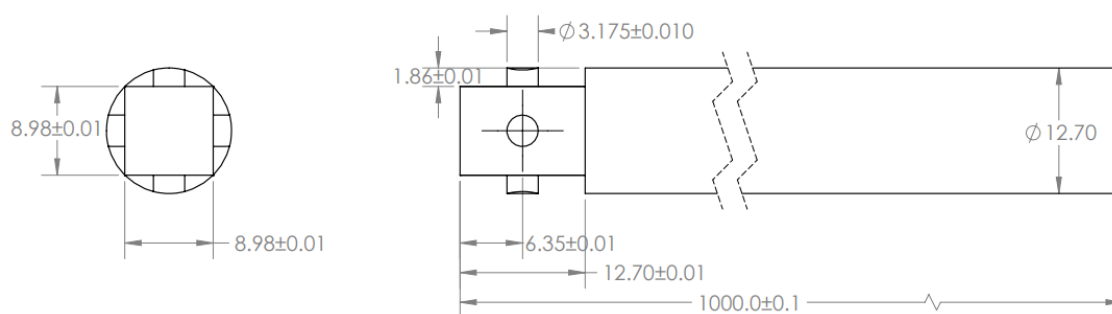


Fig. 4-7: The drawing of the new bar

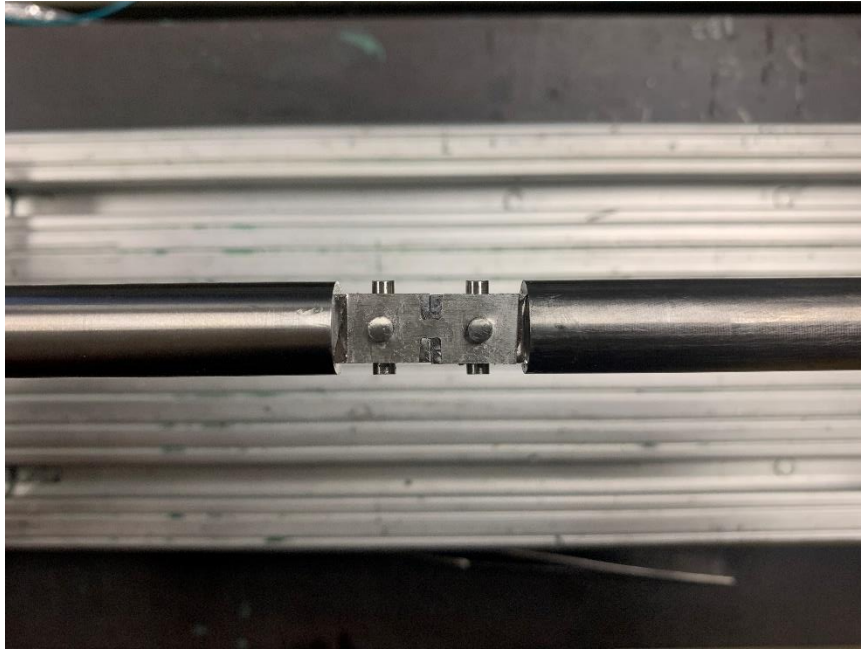


Fig. 4-8: The assemble condition of samples and bars

### 4.3 New fixtures for the gun barrel

In the past, one of the ends of the barrel was mounted on the rail with some clay. It was unsteady. Because of the huge energy produced by the collision between the projectile and incident bar, and high-pressure air, misalignment between the projectile and incident bar often happened. However, the good alignment for the experiment is critical. There will be less oscillation noise when there is a better alignment. So, some accessories were made to improve the alignment. Figure 4-9 shows the fixture for the end of the gun barrel. This fixture is supported by an adjustable platform. Therefore, when the gun barrel misalignments with the incident bar, they can be adjusted back alignment. The half-circle design of the fixture can prevent the movement of the gun barrel and makes the fixture reliable.

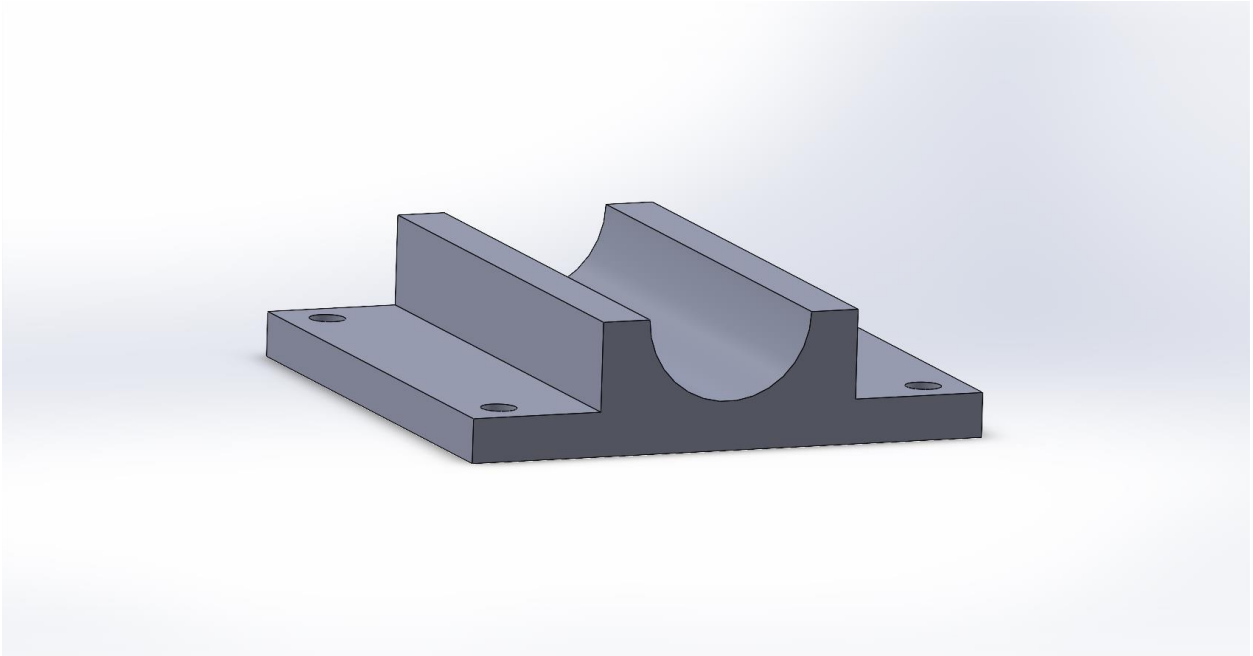


Fig. 4-9: The fixture for the end of the gun barrel



## CHAPTER 5. DATA ANALYSIS

### 5.1 Eliminate the interference effect noise

During the experiment, the ideal touch condition between the incident bar and the transmitted bar is impossible. Thus, the interface effect will appear when the compression wave propagates from the incident bar to the transmitted bar. That interface effect noise  $\varepsilon_e$  is inevitable. Figure 5-1 shows a typical interference by interface effect in the reflective tensile test (Lu, Chen, Lin, Zhao, & Zhang, 2013).

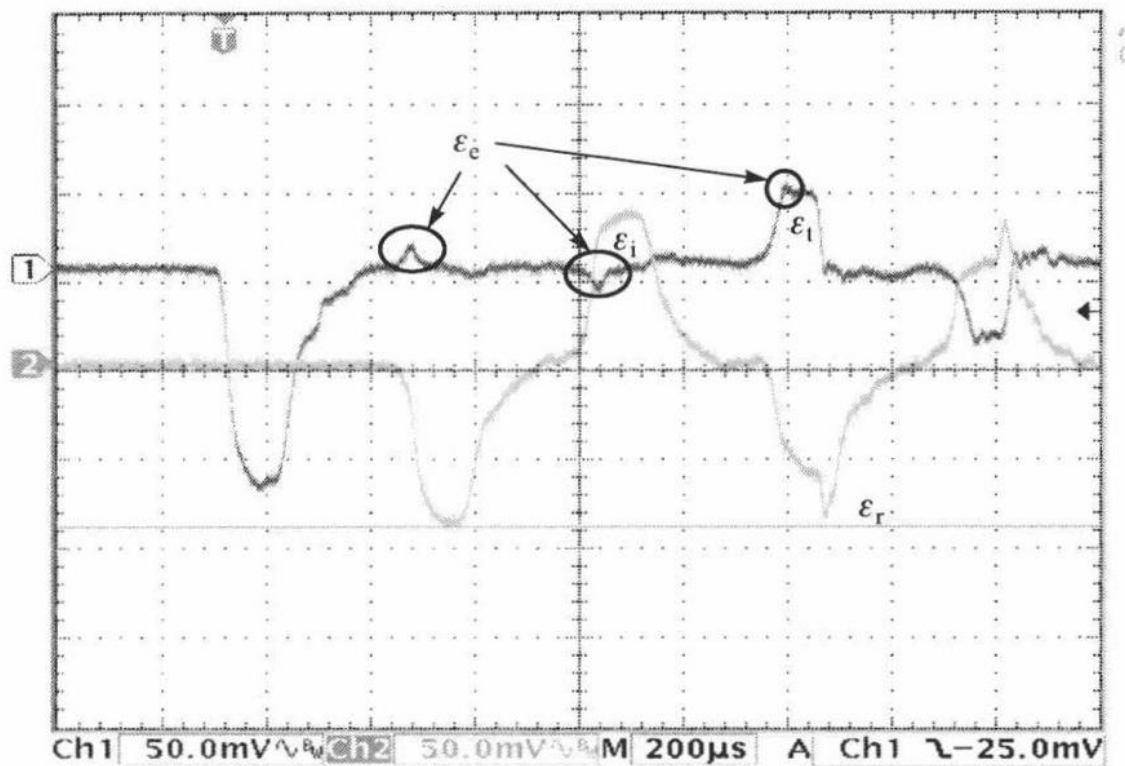


Fig. 5-1: A typical interference by interface effect in the reflective tensile test

The noise  $\varepsilon_e$  influences the transmitted wave mainly. Here are steps to amend it:

1. Calculate the speed inside the bars. Find the start time points of the first compression waves in the two bars respectively, and then calculate the time difference  $\Delta t$ . So, the speed of wave  $c_0$  in the bar is:  $c_0 = \frac{L}{\Delta t}$ . Where L is the length of the bar. In the experiment, the incident bar and the transmitted bar have the same geometry and physical properties. During the  $\Delta t$ , the elastic wave passes through the second half of the incident bar and the first half of the transmitted bar. So, the distance of the elastic wave traveling equals the total length of a bar.
2. Collect the interference  $\varepsilon_e$ . If the start point of the first compression wave in the incident bar is used as the zero point, the start time point of interference is  $t_1 = \frac{2L}{c_0}$ . The sampling duration equals the duration of the incident wave.
3. Collect the transmitted wave  $\varepsilon_t$ . Find the start point of transmitted wave  $t_2 = \frac{3L}{c_0}$ . the sampling width equals the width of the incident wave.
4. Amend the transmitted wave. Add  $\varepsilon_e$  and  $\varepsilon_t$  together, the revised transmitted wave  $\varepsilon'_t$  is got.

## 5.2 The procedures to use MATLAB to analyze data and the result

There is an open-source MATLAB code online to analyze the data of SHPB experiment (Francis, Whittington, Lawrimore, & al., 2017). Here are procedures to use it to analyze the data of the tensile test:

1. Open the code 'SHPB\_Analysis\_Tool.m' in MATLAB 2020 or newer MATLAB.
2. Click the Run in the Editor Tab, and then the 'SHPB\_Analysis\_Tool.fig' will be opened.
3. Make an Excel file that only includes the data of the transmitted bar and the incident bar.  
In other words, there are only two columns of data in this file.
4. The rest of procedures are all finished in the interface of the 'SHPB\_Analysis\_Tool.fig'.  
Next step is importing data. Click File -> Load -> Gauge Voltage -> Grouped -> Excel -> Automatic, and then choose the Excel File in the next window.
5. In Dataset, please choose your file again. Because it can accommodate many data files at the same time.
6. Fill up the sample geometry tab. It includes the length, diameter, and the cross-section area of the specimen.
7. Create the bar about the experiment. In this step, the tab of the bar, Gauge Factor, Sampling Frequency, Modulus, Density, Wave speed, Diameter of the bar, the distance between the strain gauge and the specimen are needed. Based on the elastic wave theory, the Modulus, Density, and Wave speed are only needed any two of them, the third one can be calculated using the other two.
8. Rough Crop the wave in the Voltage Signal Editing tab. This step is to crop the portion of the signal that will be used.
9. Null the incident bar and the transmitted bar. Because of the EMI and noise signal, the start point of each signal is not at zero potential voltage. However, the calculation should be at zero potential voltage.

10. In wave clipping, select the right incident wave, and then click the ‘align to the incident’ wave button to select the corresponding reflected wave and the transmitted wave. There should be the ‘Global Optimization Tool’ extension in your MATLAB to use the ‘align to the incident’ function. In other case, three waves can be determined manually by selecting the proper ‘pulse width’ and the right position of three waves. These functions are at the bottom of the ‘wave clipping’ interface.
11. When every step above is done, click apply; and then click ‘plot/Display’ tab on the top.
12. Many kinds of plots can be outputted, like engineering strain-engineering stress (3 wave), true strain-true stress (1 wave), and strain-strain rate (1 wave) and so on.

By using this MATLAB code, the dynamic properties of the material can be got. In this experiment, twelve layers of stainless steel 304 that have 0.1 mm of thickness were used. Figure 5-2 is the result of this experiment.

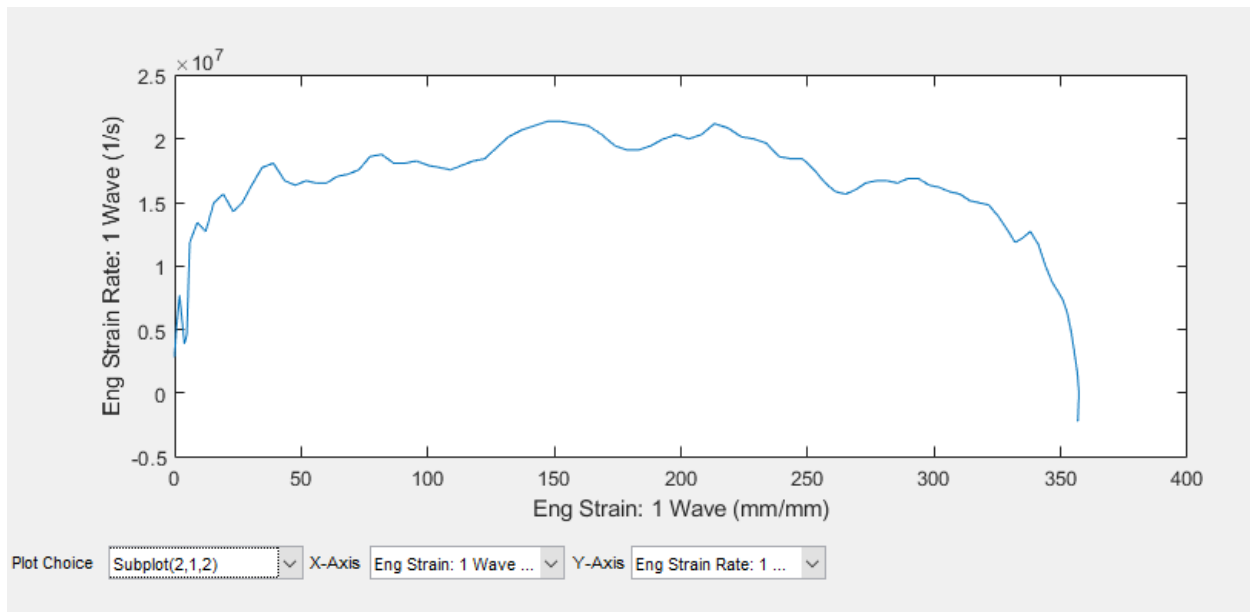


Fig. 5-2: The strain-stress curve of specimen

## **CHAPTER 6. CONCLUSION**

This report explains and derives formulas of the one-dimension wave propagation theory and its application in the split Hopkinson bar experiment. It introduces the wave condition in the bars and how to use three waves from the bars to calculate the dynamic properties of the specimen in detail. It also summarizes the formula to transfer the voltage signal to strain signal in the bars. So, it almost introduces all basic theories about the SHPB. In chapter 3, it introduces a lot of preparation job, including the installment of strain gauges, the usage of oscilloscope and the amplifier. It shows the new generation SHTB apparatus in NIU. It is a reflective SHTB. The new samples, new bars and new implementation are introduced. An open-source MATLAB code is referenced in the data processing job. The function of the code is huge and it is convenient. The procedures how it was used is explained in detail.

Due to the precision of the apparatus, the data of the stainless steel 304 with 0.1 mm thickness can't be got by this device yet. However, a sample with thicker height is good to use for this machine. And useful signal was analyzed with MATLAB code. Although the accuracy of result is not perfect, the feasibility is proved. Therefore, the future work is that improve the precision of the system and try some ways to test the 0.1mm thick specimen.

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## APPENDIX A

The 1-D wave propagation equation is:

$$c^2 \times \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad (\text{A-1})$$

This equation describes the relationship between the displacements of particles ( $u$ ) in the system and the position of particle ( $x$ ), the time ( $t$ ). In other words, the  $u$  is a binary function  $u=f(x, t)$ .

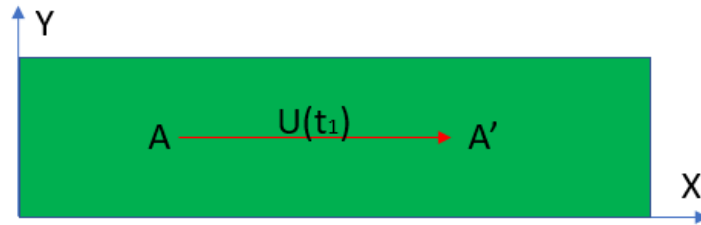


Fig. A-1: A bar

Figure A-1 shows a normal bar with a moveable particle A. A particle whose original position is A, and after time  $t_1$ , this particle moves to position A'. It can be written mathematically as:  $U[A, t_1] = \vec{A}' - \vec{A}$ . The  $\vec{A}$  and  $\vec{A}'$  represent the original coordinates of the particle and the new coordinate of the particle, respectively. Therefore,  $U$  represents the displacement of particles.

However, this displacement is in a coordinate system that is relatively static to the bar. For example, if a bar is moving right at a uniform speed  $\vec{v}$  relative to ground. At the same time, the



particle A after time  $t_1$  moves to the A' position. Then, the distance that particle A moves relative to ground is  $\vec{u} + \vec{v} \times t$ . Relative to the bar, the A particle only moves  $\vec{u}$ .

Therefore, particles have movements relative to the system, and the movements is a function of time. In other words, it is a disturbance to the system. Similarly, a wave is also a disturbance to its carrier. Therefore, eq. A-1 is a wave equation. For a solid object, the term  $c = \sqrt{\frac{E}{\rho}}$  in the eq. A-1, which E is the Young's Modulus of solid, and  $\rho$  is the density of the solid. The procedures below explain the physical meaning of c.

The general solution for eq. A-1 is:

$$u = f(x - c \times t) \quad (\text{A-2})$$

Where f is a function of  $x - c \times t$ .

The calculation to prove this solution is right will be shown. The first derivation of u respect to x is:

$$\frac{\partial u}{\partial x} = f'(x - c \times t) \quad (\text{A-3})$$

The second derivation of u respect to x is:

$$\frac{\partial^2 u}{\partial x^2} = f''(x - c \times t) \quad (\text{A-4})$$

The first derivation of u respect to t is:

$$\frac{\partial u}{\partial t} = -c \times f'(x - c \times t) \quad (\text{A-5})$$

The second derivation of  $u$  respect to  $x$  is:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \times f''(x - c \times t) \quad (\text{A-6})$$

From eq. A-4 and eq. A-6, it has  $c^2 \times \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$  and this is the eq. A-1. So, the general solution of this equation is  $u = f(x - c \times t)$ .

In eq. A-2,  $x$  is the particle's position in the system,  $u$  is the displacement of particle, and  $t$  is the time. For example, there is a wave  $u = f(x - c \times t)$  inside one bar, it is shown in Figure A-2. At the beginning,  $t=0$ , a point whose coordinate is  $x_0$  moves a distance  $u_0$  to the position whose coordinate is  $x'_0$  due to the wave/disturbance. Transferring this movement into a mathematic expression is  $u_0 = f(x_0 - c \times 0) = f(x_0)$ . There is another one particle whose coordinate is  $x_1$ , and  $x_1 = x_0 + c \times \Delta t$ . So, the displacement of particle  $x_1$  after  $\Delta t$  time from beginning is  $u_1 = f(x_1 - c \times \Delta t) = f(x_0 + c \times \Delta t - c \times \Delta t) = f(x_0) = u_0$ . This means that after  $\Delta t$  time, the particle  $x_1$  repeats the movement of particle  $x_0$  at the beginning. In other words, the disturbance happened at particle  $x_0$  initially passes through a distance  $c \times \Delta t$  during  $\Delta t$  time. So, the speed of the propagation of this disturbance or wave is  $c$  and it has its units in velocity. In an elastic-plastic solid,  $c$  represents the speed of elastic wave propagation.

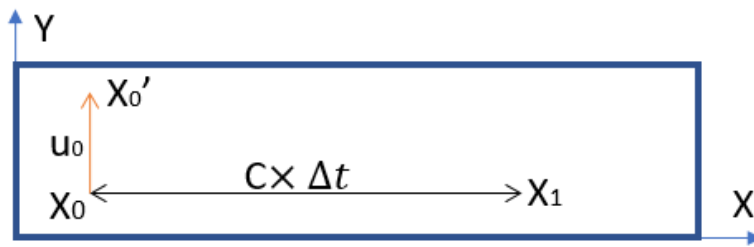


Fig. A-2: A wave in a rod

## APPENDIX B

The derivation of Eq. 2.4.4 is shown here. The Wheatstone Bridge Circuit is shown again in figure B-1.

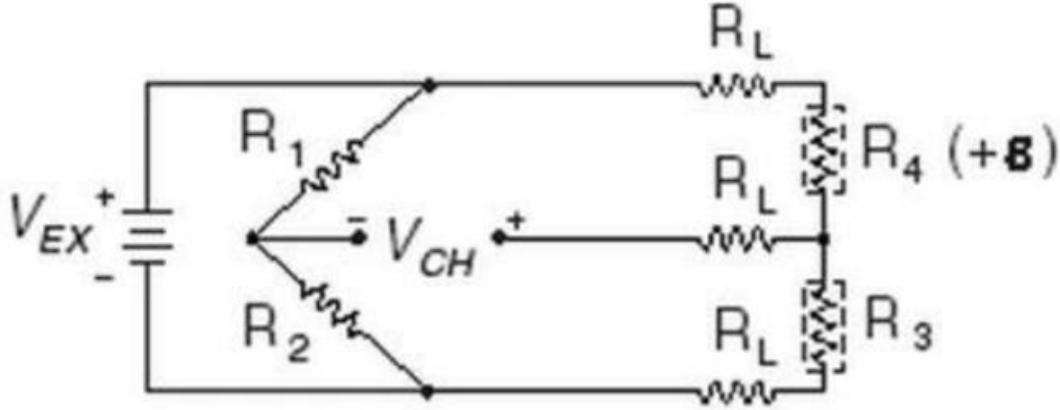


Fig.B-1: Three-wires Wheatstone Bridge Circuit

In eq. 2.4.6, it has two terms:  $V_{ch}(strained)$  and  $V_{ch}(unstrained)$ . From the circuit figure, we can get  $V_{ch}(strained)$  and  $V_{ch}(unstrained)$ :

$$V_{ch}(strained) = V_{ex} \times \left( \frac{R_3 + R_L}{R_4 + \Delta R + R_3 + 2R_L} - \frac{R_2}{R_1 + R_2} \right) \quad (B.3)$$

$$V_{ch}(unstrained) = V_{ex} \times \left( \frac{R_3 + R_L}{R_4 + R_L + R_3 + R_L} - \frac{R_2}{R_1 + R_2} \right) \quad (B.4)$$

Rewrite eq. 2.4.6:

$$V_r = \frac{R_3 + R_L}{R_4 + \Delta R + R_L + R_3 + R_L} - \frac{R_3 + R_L}{R_4 + R_L + R_3 + R_L} \quad (B.5)$$

Use  $R_5 = R_4 + R_L + R_3 + R_L = 2 \times (R_3 + R_L) = 2 \times (R_4 + R_L)$ . From theory of Wheatstone Bridge:  $R_3 = R_4 = R_g$ . So, rewrite eq. B.3:

$$V_r = \frac{-\Delta R}{2 \times (R_5 + \Delta R)} \quad (B.6)$$

Plug eq. B.4 into the eq.2.4.4:

$$\varepsilon = \frac{\frac{2 \times \Delta R}{R_5 + \Delta R}}{\frac{\Delta R}{R_g \varepsilon} + 2 \times \frac{\Delta R}{R_g \varepsilon} \times \left[ \frac{-\Delta R}{2 \times (R_5 + \Delta R)} \right]} \times \left( 1 + \frac{R_L}{R_g} \right) \quad (\text{B.7})$$

$$\varepsilon = \frac{2 \times \Delta R}{\frac{\Delta R}{R_g \varepsilon} \times (R_5 + \Delta R) - \frac{\Delta R^2}{R_g \varepsilon}} \times \left( 1 + \frac{R_L}{R_g} \right) \quad (\text{B.8})$$

$$\varepsilon = \frac{2 \times \Delta R \times R_g \varepsilon}{\Delta R \times (R_5 + \Delta R) - \Delta R^2} \times \left( 1 + \frac{R_L}{R_g} \right) \quad (\text{B.9})$$

Cancel the  $\Delta R$ , it has:

$$\varepsilon = \frac{2 \times R_g \varepsilon}{R_5} \times \left( 1 + \frac{R_L}{R_g} \right) \quad (\text{B.10})$$

It has  $R_5 = R_4 + R_L + R_3 + R_L = 2 \times (R_3 + R_L) = 2 \times (R_4 + R_L)$  and  $R_3 = R_4 = R_g$ . So, rewrite eq. B.8:

$$\varepsilon = \frac{R_g \varepsilon}{R_g + R_L} \times \left( 1 + \frac{R_L}{R_g} \right) \quad (\text{B.11})$$

$$\varepsilon = \frac{R_g \varepsilon}{R_g + R_L} + \frac{R_L \varepsilon}{R_g + R_L} = \frac{(R_g + R_L) \varepsilon}{R_g + R_L} = \varepsilon \quad (\text{B.12})$$