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Imu-Based Estimation of Body Posture in Commercial Fishing

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ABSTRACT

IMU-BASED ESTIMATION OF BODY POSTURE IN COMMERCIAL FISHING

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Northern Illinois University, 2021
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According to the U.S. Bureau of Labor Statistics (BLS), work-related musculoskeletal disorders (WMSDs) account for 33% of all occupational injuries and illnesses in the U.S. In addition, one of the most strenuous occupations in which musculoskeletal disorders are common is known to be commercial fishing. The first step to address this issue is to precisely measure the working postures of workers so that the relation between WMSDs and non-neutral working postures can be systematically investigated. Although the inertial measurement unit (IMU) has recently gained attention in posture measurement due to its portability, it is highly limited to the use as an accelerometer that is only suited to static 2-dimensional tilting-angle measurements. Although a gyroscope can measure 3-dimensional orientations even with dynamic (accelerated) motions, they have been avoided in the measurement of postures due to their intrinsic nature of a drift phenomenon, which could lead to a significant error over time. Therefore, in this thesis, two sensor fusion methods, in which all three sets of sensors (3-axis accelerometer, 3-axis gyroscope, 3-axis magnetometer) in an IMU are utilized, are explored to achieve the 3-dimensional orientation estimation with higher accuracy. The two methods of complementary filter and Kalman filter are implemented to estimate
the orientation of the torso and arm in two simulated tasks that are common in commercial fishing. The estimation accuracy of each filter in those tasks is verified using the reference data measured using a motion capture system consisting of multiple vision cameras.
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DEDICATION

To my parents, family and friends
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CHAPTER 1
INTRODUCTION

1.1 Motivation and Objective

Work-related musculoskeletal disorders (WMSDs) are a serious problem, accounting for 33% of all occupational injuries and illnesses that require days away from work in the United States [1]. For example, in the commercial fishing industry, highly physically demanding tasks expose fishermen to a number of WMSD risk factors such as repetitive motion, high force exertion, awkward body posture, and vibration [2]. Such an WMSD could bring significantly adverse consequences to the workers and their employers. Therefore, it is necessary to find ways to effectively identify and assess WMSD risks, which is the key to alleviating this problem.

Although numerous methods have been developed so far to discover the risk factors of WMSDs, self-report and observational methods are still dominantly used [3] [4]. However, the outcome of these methods can be biased and inaccurate. With advances in sensors and sensing technology, marker-based methods such as optical motion capture system with reflective markers and sensor-based methods such as inclinometers, gyroscopes or inertial measurement units (IMUs) have been used to collect human posture [5] [6]. One of the advantages associated with the sensor based methods is that it provides direct measurement of the human posture which would be unbiased and accurate. Collecting the body posture helps determine how much and how long the body deviates from the neutral posture for a specific work and whether it is healthy or causing MSD.
Although the optical motion capture system is highly accurate to estimate the body posture, it is limited to laboratory settings, requires expensive optical motion capture equipment, and attaching surface markers to the human body is time-consuming and tedious [7]. So, in this thesis, an inertial measurement unit (IMU) has been proposed to measure the body orientation in commercial fishing. The IMU is now made small and even wireless, which makes itself ideal for this particular application.

The main objective of this thesis is to estimate 3-dimensional orientations of body movements. The two methods of calculating body orientation using an IMU are discussed. The problems associated with each method are identified and the solutions are addressed. The result is verified by an optical motion capture system.

1.2 Literature Review

Professional fishing is a difficult job as the fishermen often work under harsh weather condition. Kucera et al.[8] studied the ergonomic risk factor of crab pot and gill net commercial fishermen of north Carolina. The objective of that research was to determine the association between low back pain (LBP) that limited or interrupted fishing work and ergonomic low back stress. The low back stress was measured by self-reported task and two ergonomic assessment methods of low back stress. From the study, the authors concluded that the tasks characterized by higher (unloading boat and sorting catch) and lower (running puller or net reel) ergonomic low back stress were associated with the occurrence of severe LBP.

For measuring non-neutral working posture, MEMS sensors such as an inclinometer and a gyroscope have been used. Douphrate et al.[9] used a triaxial accelerometer called the virtual corset to measure shoulder elevation and trunk inclination angle of dairy parlor worker. Tal Amasay et al.[10] showed that accelerometer-based estimation of shoulder elevation angle
works fine for static condition. But for dynamic motion, elevation angle provides error due to the presence of angular acceleration. Luinge et al. [5] developed a method for accurate measurement of the orientation of human body segments using an inertial measurement unit (IMU). Three-axis accelerometer and three-axis gyroscope have been used separately to measure the human body orientation. Kalman filter was used to fuse the measurement of each sensor. The fusion method was tested for movements of the pelvis, trunk and forearm. The authors showed that gyroscope based measurement suffered from the large integration errors. Elena Bergamini et al. [6] used a magnetic and an inertial measurement unit to estimate orientation of human body. The authors used complementary filter and kalman filter to fuse the measured data of gyroscope and accelerometer-magnetometer.

For estimating joint kinematics IMU system have been used widely. Zhou et al. [11] used inertial measurement unit to estimate the joint kinematics of the upper arm and shoulder. Jasiewicz et al. [12] used 9-axis IMU to measure the head movement for neck pain assessment. Favre et al. [13] used 2 IMU sensors to measure the 3-dimensional motion of knee joints. Based on a leg movement two inertial measurement units were aligned. On the combination of this alignment and a fusion algorithm, the three-dimensional knee joint angle is measured and compared with a magnetic motion capture system during walking. Bertuletti et al. [14] used inter foot distance step counter (IFOD) along with an IMU sensor to accurately detect the step motion. This method provides 99.8% accuracy for the instrumented foot step and 88.8% accuracy for the non-instrumented foot step during both rectilinear and curvilinear walks.
1.3 Outline

The rest of this thesis is organized as follows. The in-depth details of an inertial measurement unit, global and local frame of reference and the rotation representation method used for this thesis are discussed in Chapter 2. The explanation of the background theory for the orientation estimation by individual sensor of the IMU is presented in Chapter 3. The necessity of the sensor fusion and two methods of sensor fusions are discussed in Chapter 4. The experiments performed to verify the proposed algorithm and the results are discussed in Chapter 5, followed by concluding remarks in Chapter 6.
CHAPTER 2

BASIC THEORETICAL BACKGROUNDS

2.1 Inertial Measurement Unit

An inertial measurement unit (IMU) is considered as the sensing block of an attitude and heading reference system (AHRS) [15]. A typical IMU includes up to nine measurement axes: a three-axis accelerometer, a three-axis gyroscope, and a three-axis magnetometer. The accelerometer measures the body’s acceleration, the gyroscope measures its angular velocity and the magnetometer measures the earth’s magnetic field strength and direction at its location. Using all three sensor’s reading, it is possible to estimate its linear velocity and orientation in a 3D space relative to an established reference frame. Figure 2.1 is an example of the IMU, which is, in fact, the one used in this work.

Figure 2.1: An example of a portable, wireless IMU (LPMS B2, Life Performance Research Inc., Japan).
The orientation of a system could be determined from the measurement of two nonzero and noncollinear vectors: gravity (by the accelerometer) and the earth magnetic field vector (by the magnetometer). It could also be determined by integrating the angle rate obtained from the gyroscope. The recent development in the MEMS technology has made it possible to fabricate a cheap, compact, and low processing power IMU. Consequently, IMUs are being widely used in industry quality control, medical rehabilitation, robotics, navigation system, sports learning, augmented reality system and many other fields [16].

### 2.2 Rotation Representation by Rotation Matrix

In this thesis, the orientation of the IMU is defined as the orientation of the coordinate frame attached to the IMU (the body frame) with respect to a coordinate frame fixed to the world (the reference frame). The body frame attached to the IMU is shown in Fig. 2.1 with its origin $O_b$ at the center of the IMU and $Z$-axis ($Z_b$) out of the IMU plane. The orientations of this body frame, i.e., the IMU, are measured with respect to a fixed inertial frame called the world (reference) frame. In this thesis, the world frame of reference is defined using the ENU (East-North-Up) convention as shown in Fig. 2.2. In this convention, $X_n$ axis is toward the geodetic east, $Y_n$ is toward the geodetic north, and consequently, $Z_n$ is in the opposite direction of gravity.

The orientation configuration between the world and body coordinate frames will be represented by a rotation matrix in this thesis. The rotation matrix specifies the orientation by projecting each axis of the body frame onto the world frame [17] and it is given by
Figure 2.2: The world reference frame used in relation to the IMU. It is defined using the
ENU convention.

\[
R^n_b = \begin{bmatrix}
X_b \cdot X_n & Y_b \cdot X_n & Z_b \cdot X_n \\
X_b \cdot Y_n & Y_b \cdot Y_n & Z_b \cdot Y_n \\
X_b \cdot Z_n & Y_b \cdot Z_n & Z_b \cdot Z_n
\end{bmatrix}
\]

where \(X_b \cdot X_n\) represents the dot product of the two unit basis vectors.

For example, some basic rotation matrices, which are only about the principal axis are
given by

\[
R_x(\phi) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix},
R_y(\phi) = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix},
R_z(\psi) = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

The rotation matrix \(R^n_b\) can be used not only to represent the orientation of coordinate
frame \(O_bX_bY_bZ_b\) with respect to frame \(O_nX_nY_nZ_n\), but also to transform the coordinates of
a point from one frame to another. When a point(or a vector) is expressed in the frame
\(O_bX_bY_bZ_b\) as \(p^b\), it is expressed relative to the frame \(O_nX_nY_nZ_n\) by

\[
p^n = R^n_b p^b
\]
Two important properties of rotation matrix are:

- The inverse of a rotation matrix is equal to the transpose of it, i.e. $R^{-1} = R^T$.
- The determinant of a rotation matrix is equal to 1, i.e. $det(R) = 1$.

### 2.3 Parameterization of Rotations

Any arbitrary rotation can be represented using only three independent quantities. Among them two representations are used in this work: the Euler angle representation and the roll-pitch-yaw representation. \cite{17}. In these representations, the corresponding rotation matrix $R$ is described as a product of successive rotations taken in a specific order. In the case of the roll-pitch-yaw representation, it is with respect to the fixed frame of reference. These rotations define the roll, pitch, and yaw angles, which can be denoted as $\phi$, $\theta$, $\psi$ as shown in Fig. 2.3.

![Roll, pitch and yaw](image)

Figure 2.3: Roll, pitch and yaw. Counterclockwise is positive.
We specify the order of rotation as X-Y-Z, that is, first a rotation about $X_0$ by an angle $\phi$, then about the fixed $Y_0$ by an angle $\theta$, and finally about the fixed $Z_0$ by an angle $\psi$. The successive rotations are relative to the fixed frame, the correct composition law is to multiply the successive rotation matrices in the reverse order. The resulting rotation matrix $R^n_b$ which represents the rotation of the body frame with respect to the world frame is given by

$$R^n_b = R_z(\psi)R_y(\theta)R_x(\phi) = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}$$

where the shorthand notations $c_\theta$ and $s_\theta$ denote $\cos \theta$ and $\sin \theta$, respectively.

In the Euler angle representation, the corresponding rotation matrix $R$ is described as a product of successive rotations about the current frame taken in a specific order. While various orders of rotation is available in the Euler angle representation, we specify the order of rotation as X-Y-Z, therefore, as shown in Fig. 2.4, the first rotation is about $X_0$ by $\phi$, then $\theta$ about the current $Y_1$ axis, then finally $\psi$ around current $Z_2$ axis. For the Euler angle, the resulting rotation matrix $R^n_b$ will be
Figure 2.4: Euler angle representation. Counterclockwise is positive.

\[ R_n = R_x(\phi)R_y(\theta)R_z(\psi) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix} \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix} \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix} \]

To get the Euler angle back from this rotation matrix, we can do the following calculation:

\[ \theta = \text{atan2}(r_{13}, \sqrt{1 - r_{13}^2}) \] (2.3)

\[ \phi = \text{atan2}(-r_{23}, r_{33}) \] (2.4)

\[ \psi = \text{atan2}(-r_{12}, r_{11}) \] (2.5)

where \(\text{atan2}(y, x)\) represents the two-argument arctangent function and \(r_{13}\) is the first row and third column entry of the matrix (2.2). Similarly other entries are defined.
2.4 Similarity Transformation

Because the world reference frames used in the IMU and the motion capture system are different, the orientation results are different for the same motions. Therefore, to compare the IMU results with the motion capture system’s results, the similarity transformation is used.

Using the similarity transformation, a rotation matrix can be transformed from one frame to another. It is called similarity transformation [17]. If $A$ is a linear transformation (e.g., rotation matrix) in a frame $o_0x_0y_0z_0$ and $B$ is the representation of the same linear transformation in another frame $o_1x_1y_1z_1$, then $A$ and $B$ are related as [17]

$$B = (R_1^0)^{-1}AR_1^0$$

(2.6)

where $R_1^0$ is the coordinate transformation between frames $o_1x_1y_1z_1$ and $o_0x_0y_0z_0$. Later in Chapter 5, we use this similarity transformation to transform rotation matrices from the IMU frame to the motion capture system frame for comparisons of the estimated orientation angles.
CHAPTER 3
ESTIMATION OF ROTATIONS USING IMU

The IMU sensor can estimate body rotation in three axes by two different methods: by reading the gravitational acceleration and the earth magnetic vector and by integrating the rotation rate. The gravitational acceleration and the earth magnetic vector are sensed by the accelerometer and magnetometer, respectively. On the other hand, the gyroscope gives the reading of the rate of rotational angle. In this chapter, these two methods of calculating the body’s rotation are discussed in detail.

3.1 Estimation Using Accelerometer and Magnetometer

An accelerometer can only estimate tilt angles along the X and Y-axis (horizontal axis). But to get the rotation along the Z-axis (vertical axis), we need to additionally use the magnetometer.

3.1.1 Measuring Tilt (Roll, Pitch) using Accelerometer

In the roll-pitch-yaw representation of rotation, which is with respect to the fixed frame of reference, the roll ($\phi$, about X-axis) and the pitch ($\theta$, about Y-axis) can be measured
by the accelerometer. From Chapter 2.3, the rotation matrix with a roll, pitch, and yaw configuration is given by (2.1).

\[ R^n_b = R_z(\psi)R_y(\theta)R_x(\phi) \]

\[ = \begin{bmatrix}
    c_\theta c_\psi & s_\theta s_\phi c_\psi & c_\phi s_\theta + s_\theta s_\phi \\
    c_\theta s_\psi & s_\theta s_\phi + c_\phi c_\psi & c_\phi s_\theta s_\psi - c_\psi s_\phi \\
    -s_\theta & s_\phi c_\theta & c_\phi c_\theta
\end{bmatrix} \]

On the other hand, when the gravity vector, which is along the negative Z-axis of the world frame \((F_n)\), is expressed in the body frame \((F_b)\) of the IMU (Fig. 2.1), it is as a matter of fact the readings of the accelerometer. Therefore, we have a relationship between the coordinates \(P^n\) of the gravity vector in \(F_n\) and the acceleration readings \(P^b\).

\[ P^n = R^n_b P^b \quad (3.1) \]

where \(P^n = [0, 0, -g]^T\), \(P^b = [a_x, a_y, a_z]^T\) and \(g\) is the gravitational acceleration. \(R^n_b\) represents the orientation configuration of the body frame with respect to the world frame, as explained in Chapter 2.2.

Since \((R^n_b)^{-1} = (R^n_b)^T\), (3.1) can be rewritten as:

\[
\begin{bmatrix}
    a_x \\
    a_y \\
    a_z
\end{bmatrix} =
\begin{bmatrix}
    c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\
    s_\theta s_\phi c_\psi - c_\phi s_\psi & s_\theta s_\phi + c_\phi c_\psi & c_\phi s_\theta s_\psi - c_\psi s_\phi \\
    c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - c_\psi s_\phi & c_\phi c_\theta
\end{bmatrix}
\begin{bmatrix}
    0 \\
    0 \\
    -g
\end{bmatrix}
\]

which yields:

\[ a_x = g \sin \theta \quad (3.2) \]
\[ a_y = -g \cos \theta \sin \phi \] (3.3)
\[ a_z = -g \cos \theta \cos \phi \] (3.4)

From (3.3) and (3.4) we have

\[ a_y^2 + a_z^2 = g^2 \cos^2 \theta \]
\[ \Rightarrow g \cos \theta = \sqrt{a_y^2 + a_z^2} \] (3.5)

along with (3.2), we have

\[ \theta = \text{atan2}(a_x, \sqrt{a_y^2 + a_z^2}) \] (3.6)

where \text{atan2}(y, x) represents the two-argument arctangent function.

Since, \( g \cos \theta \) is always positive according to the (3.2), from (3.3) and (3.4), we have

\[ \phi = \text{atan2}(-a_y, -a_z) \] (3.7)

One important limitation of calculating roll and pitch using this method is that the range of pitch angle \( \theta \), is limited within \((-\frac{\pi}{2}, \frac{\pi}{2})\). It cannot even be +/- 90 degrees, otherwise the roll \( \phi \) will be undefined.

### 3.1.2 Heading (Yaw) Calculation Using Magnetometer

The Yaw is the angle that the body’s Y-axis makes with the geodetic north (Y-axis of the world frame of reference) on the horizontal plane. In other words, it is the rotation about the Z-axis of the body. Since the gravitational vector is along the negative Z-axis of the world reference frame, it is not possible to measure the yaw angle, which is a rotation
about Z-axis, only using an accelerometer. Therefore, we need to use a magnetometer that measures the Earth’s magnetic field vector, which is not aligned with the gravity vector. We assume the world reference frame is set up such that the Earth’s magnetic field \( B \) lies on the \( Y_0 - Z_0 \) plane (see Fig. 3.2). As a result, the horizontal component of the magnetic field vector \( B \) is aligned with \( Y_0 \) and the vertical component is with the negative \( Z_0 \). Therefore, when the magnetometer (IMU) is aligned with the world reference frame, the x-component of the magnetic vector will read zero.

As shown in Fig. 3.1, the angle between the earth’s magnetic field, \( B \) and its horizontal component is called the inclination angle, \( \delta \).

![Figure 3.1: Gravitational and magnetic field vector relative to the world frame of reference. The Earth’s magnetic field \( B \) lies on the \( Y_0 - Z_0 \) plane.](image)

Given an orientation configuration of the body (IMU), when \( M^n \) denotes the earth’s magnetic field vector expressed in the world frame while \( M^b \) is the same vector expressed in the IMU’s body frame that is basically the magnetometer reading,

\[
M^n = [0, \|B\| \cos \delta, -\|B\| \sin \delta]^T
\]
\( M^b = [M_x, M_y, M_z]^T \)

\( M^n = R^n_b M^b \)

where \( R^n_b \) is the rotation matrix between the world frame \( F_n \) and the body frame \( F_b \). Using the roll-pitch-yaw convention, \( R^n_b \) is given by \( R^n_b = R_z(\psi)R_y(\theta)R_x(\phi) \) from Chapter 2.3. Hence,

\[
M^n = R_z(\psi)R_y(\theta)R_x(\phi)M^b
\]

\[\implies R_z(-\psi)M^n = R_y(\theta)R_x(\phi)M^b\]

Thus,

\[
\begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 \\
||B|| \cos \delta \\
-||B|| \sin \delta
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}
\]

\[\implies ||B||
\begin{bmatrix}
\sin \psi \cos \delta \\
\cos \psi \cos \delta \\
-\sin \delta
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta \sin \phi & \sin \theta \cos \phi \\
0 & \cos \phi & -\sin \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi
\end{bmatrix}
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}
\] \quad (3.8)

From the first two rows of (3.8) we have

\[
||B|| \sin \psi \cos \delta = M_x \cos \theta + M_y \sin \theta \sin \phi + M_z \sin \theta \cos \phi
\]

\[
||B|| \cos \psi \cos \delta = M_y \cos \phi - M_z \sin \phi
\]

Since \( \cos \delta \) is always positive, the yaw angle is given by

\[
\psi = \text{atan2}(M_x \cos \theta + M_y \sin \theta \sin \phi + M_z \sin \theta \cos \phi, M_y \cos \phi - M_z \sin \phi) \quad (3.9)
\]
where $\phi$ and $\theta$ are the roll and pitch angles, computed in Section 3.1.1.

## 3.2 Gyroscope

We will explain how the orientation of the IMU can be estimated with the gyroscope only [24]. In doing so, the Euler angle representation, which is with respect to the current frame explained in Section 2.3 will be used with the Z-Y-X convention. The order of rotation to define the three angles $\phi$, $\theta$, and $\psi$ is shown in Fig. 3.2.

The first rotation is about Z by $\psi$, then $\theta$ about the current Y-axis, then finally around current X-axis by $\phi$.

![Figure 3.2: The definition of the Euler angles](image)

From the Euler angle representation, the rotation matrix is given by,

$$
R^i_b = R_z(\psi)R_y(\theta)R_x(\phi)
$$

$$
= \begin{bmatrix}
    c_\theta c_\psi & s_\theta s_\phi c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\
    c_\theta s_\psi & s_\theta s_\phi s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - c_\psi s_\phi \\
    -s_\theta & s_\phi c_\theta & c_\phi c_\theta
\end{bmatrix}
$$

and $R^i_b$ represents the rotation of the body frame relative to the world frame.
Now, let’s consider \( \omega_x, \omega_y, \) and \( \omega_z \) that are the projected components of the angular velocity \( \omega \) of the body onto frame \( F_b \). In other words, they are the angular velocity expressed in frame \( F_b \).

On the other hand, we can represent the euler angle rate, \( \dot{\psi}, \dot{\theta}, \dot{\phi} \) as the following vectors:
\[
\dot{\psi} \hat{z}_1, \dot{\theta} \hat{y}_2, \dot{\phi} \hat{x}_b
\]
where \( \hat{z}_1, \hat{y}_2, \hat{x}_b \) are the unit basis vectors shown in Fig. 3.2.

In fact, the addition of these vectors is the angular velocity of the object. Therefore, all we need to do is the coordinate change (that is, express them in the frame \( F_b \)) and add them, which would be basically the angular velocity expressed in frame \( F_b \), i.e., \( \omega_x \hat{x}_b + \omega_y \hat{y}_b + \omega_z \hat{z}_b \).

First in \( F_b \),
\[
\hat{p}_\phi^b = \dot{\phi} \hat{x}_b = \begin{bmatrix} \dot{\phi} & 0 & 0 \end{bmatrix}^T
\]
(3.10)

When \( \theta \hat{y}_2 \) is expressed in \( F_2 \), it is given by \( \hat{p}_\theta^2 = \begin{bmatrix} 0 & \dot{\theta} & 0 \end{bmatrix}^T \). Then using \( \hat{p}_\theta^2 = R_2^b \hat{p}_\theta^b \), the vector \( \dot{\theta} \hat{y}_2 \) in \( F_b \) is given by
\[
\hat{p}_\theta^b = (R_2^b)^T \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}
\]
(3.11)
where \( R_2^b = R_{x,\phi} \).

Similarly, with \( \hat{p}_x^1 = \dot{\psi} \hat{z}_1 = \begin{bmatrix} 0 & 0 & \dot{\psi} \end{bmatrix}^T \) and using \( \hat{p}_x^1 = R_1^b R_2^b \hat{p}_x^b \)
\[
\hat{p}_x^b = (R_2^b)^T (R_1^b)^T \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}
\]
(3.12)
where \( R_1^2 = R_{y,\theta} \).
Finally, by adding (3.10), (3.11) and (3.12), we have the angular velocity vector expressed in the body frame as

\[
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} = \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} + (R_{x,\phi})^T \begin{bmatrix}
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} + (R_{x,\phi})^T (R_{y,\theta})^T \begin{bmatrix}
0 \\
\dot{\psi}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} + \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix} \begin{bmatrix}
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} + \begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix} \begin{bmatrix}
0 \\
\dot{\psi}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\dot{\phi} - \sin \theta \dot{\psi} \\
\cos \phi \dot{\theta} + \sin \phi \cos \theta \dot{\psi} \\
-\sin \phi \dot{\theta} + \cos \phi \cos \theta \dot{\psi}
\end{bmatrix}
\]

Solving (3.13) for $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$ gives

\[
\dot{\phi} = \omega_x + (\omega_z \cos \phi + \omega_y \sin \phi) \tan \theta
\]  

(3.14)

\[
\dot{\theta} = \omega_y \cos \phi - \omega_z \sin \phi
\]  

(3.15)

\[
\dot{\psi} = (\omega_z \cos \phi + \omega_y \sin \phi) \sec \theta
\]  

(3.16)

where $\omega_x$, $\omega_y$, and $\omega_z$ are the projected components of the body’s angular velocity onto the body frame. Therefore, they are the gyroscope’s readings. Consequently, the angles can be obtained by numerically integrating (3.14) to (3.16). More specifically, the angles $\phi$, $\theta$, and $\psi$ in these equations are the previous step’s angle. At the first step, the roll $\phi$, pitch $\theta$, and yaw $\psi$ measured by the accelerometer and magnetometer are used. As we use the Z-Y-X convention with the Euler angle representation, the three quantities of $\phi$, $\theta$, and $\psi$ are the same as those roll, pitch and yaw angles obtained in Section 3.1.
CHAPTER 4
FILTERS FOR OPTIMAL RESULT

4.1 Complementary Filter

Estimation of the roll, pitch, and yaw angles using a gyroscope are not affected by the object’s dynamic conditions, e.g., in an accelerated motion, but even small measurement errors can accumulate over time due to its integration characteristics in the calculation of the orientation angles. Due to such a drift nature of the gyroscope, it’s not desirable to rely on the gyroscope for a long period of time. In contrast, the estimation using an accelerometer and magnetometer shows poor high-frequency accuracy but does not present any drift providing an excellent static performance [25]. Due to their complementary characteristics, the complementary filtering algorithm can be used to fuse the two sets of measurements to improve the accuracy of the orientation estimation [27].

A high pass filter is a filter that passes signals with a frequency higher than a selected cutoff frequency. Since the error from the gyroscope mainly has a low frequency, the high pass filter is suitable for its estimation of angle. In contrast, a low pass filter is a filter that allows to pass the signal with a frequency lower than the cutoff frequency. The estimates of the angles of the accelerometer and magnetometer are needed to pass through the low pass filter to eliminate high-frequency error. By combining both results, a better estimation of the roll/pitch/yaw angle can be achieved.

Let \( x_1 \) and \( x_2 \) be the measurements of a signal \( x \), deviated with high and low-frequency noise, respectively. When \( G(s) \) is defined to be a low-pass filter in the Laplace domain, then
\( \tilde{G}(s) \) is the corresponding high-pass filter such that \( G(s) + \tilde{G}(s) = 1 \) where \( s \) is the Laplace transform variable. The output of the complementary filter can be expressed as [26]

\[
\hat{x} = x_1 G(s) + x_2 \tilde{G}(s)
\]

where \( \hat{x} \) is the estimation of the actual signal \( x \) in the frequency domain.

The block diagram of this complementary filter applied to the orientation estimation is shown in Fig. 4.1.

![Block Diagram](image)

Figure 4.1: Block diagram of the complementary filter for orientation estimation in which the \( \int \) symbol block denotes integration.

Figure 4.2 shows the Laplace transform of the block diagram of the Fig. 4.2. In the Laplace domain, a low pass filter is represented by \( \frac{1}{\tau s + 1} \) where \( \tau \) is the filter’s time constant. The time constant \( \tau \) is the frequency at which the output signal power drops to half the value it has at low frequencies.
A high pass filter is given by \( \frac{\tau_s}{\tau_s + 1} \). While an integral is equal to the Laplace transform of the integrand multiplied by \( 1/s \). Figure 4.3 shows a further simplified form of this block diagram.

From Figs. 4.2 and 4.3, it can be seen that the accelerometer and magnetometer’s measurement of orientation and the gyroscope’s reading of angular velocity multiplied by \( \tau \) are passing through the low pass filter. The low pass filter in discrete-time is given by

\[
y_n = \alpha x_n + (1 - \alpha)y_{n-1} \tag{4.1}
\]

where \( \alpha = \frac{\Delta t}{\tau + \Delta t} \) is the weighting factor. \( x_n \) and \( y_n \) are the measured and the filtered values, respectively whereas \( y_{n-1} \) is the filtered value from the previous step. As shown in Fig. 4.3, the complementary filter is basically a low pass filter taking the sum of the accelerometer/magnetometer and the gyroscope measurements as input. Therefore, using (4.1) with \( \tau = \frac{\Delta t (1 - \alpha)}{\alpha} \), the complementary filter equation is given by

\[
\hat{x}_n = \alpha[x_a + x_g \tau] + (1 - \alpha)\hat{x}_{n-1}
\]

\[
= \alpha x_a + (1 - \alpha)[\hat{x}_{n-1} + x_g \Delta t]
\]
where $x_a$ denotes the roll/pitch/yaw measured by the accelerometer and magnetometer and $x_g$ denotes the Euler angle rate measured by the gyroscope. The value of $\alpha$ defines the weighting factor between the accelerometer/magnetometer and the gyroscope’s measurements which varies in the range of $(0,1)$.

### 4.2 Kalman Filter

The Kalman filter is an algorithm which uses a series of measurements (under noise and uncertainties) of unknown variables observed over time and estimate more accurate value of those unknown variables.

The Kalman filter algorithm consists of the process model and the measurement model. The process model is the mathematical model that describes the system’s dynamics. The equation of process model defining the evolution of the state from discrete time $k-1$ to $k$ is expressed as [28]:

$$x_k = Fx_{k-1} + Bu_{k-1} + \omega_{k-1}$$  \hspace{1cm} (4.2)
where $F$ is the state transition matrix applied to the previous state vector $x_{k-1}$, $B$ is the control-input matrix applied to the control vector $u_{k-1}$, and $\omega_{k-1}$ is the process noise vector that is assumed to be zero-mean Gaussian with the covariance $Q$, i.e., $\omega_{k-1} \sim N(0, Q)$.

On the other hand, the measurement model consists of the measured values of the unknown variables obtained directly (such as from a sensor). It contains error known as the measurement noise. The equation of measurement model at the current time step $k$ can be expressed as:

$$z_k = Hx_k + \nu_k$$

(4.3)

where $z_k$ is the measurement vector, $H$ is the measurement matrix, and $\nu_k$ is the measurement noise vector that is assumed to be zero-mean Gaussian with the covariance $R$, i.e., $\nu_k \sim N(0, R)$.

The steps of the Kalman filter algorithm that are maintained in this thesis are as follows [28]:

The state of the system is predicted by the following equation:

$$x_k = F\hat{x}_{k-1} + Bu_{k-1}$$

(4.4)
The state vector $\hat{x}_{k-1}$ is defined as

$$
\hat{x}_{k-1} = \begin{bmatrix}
\phi_{k-1} \\
\dot{\phi}_{k-1} \\
\phi_{b,k-1} \\
\theta_{k-1} \\
\dot{\theta}_{k-1} \\
\dot{\theta}_{b,k-1} \\
\psi_{k-1} \\
\dot{\psi}_{k-1} \\
\psi_{b,k-1}
\end{bmatrix}
$$

The outputs of the filter are the roll, pitch, and yaw angles ($\phi$, $\theta$, $\psi$), the gyroscope’s rates ($\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$) and biases ($\dot{\phi}_b$, $\dot{\theta}_b$, $\dot{\psi}_b$). The transition matrix $F$ is the matrix that relates the state at the previous time step $k-1$ to the current step $k$. In this thesis, it is defined as:

$$
F = \begin{bmatrix}
1 & \Delta t & -\Delta t & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \Delta t & -\Delta t & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta t & -\Delta t \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$
When the sensor is at rest, the output of the gyroscope should be ideally 0. But, in practice, the raw data from the gyroscope sensor shows constant bias of a certain value at resting position. It is related to the mechanical structure of the gyroscope MEMS. Including the biases in the state vector gives the ability to track the bias and model the drift in the filter to minimize error and create a more accurate solution [30]. The control matrix, B in (4.4) is defined as 0 since we don’t have any control input vector $u_k$ which is constant and remains unchanged throughout the course of the experiment.

The equation for the prediction of the error covariance matrix is given by

$$P_{k|k-1} = FP_{k-1}F^T + Q$$

(4.5)

where $P_{k|k-1}$ is the predicted error covariance matrix which will be updated at the end of the cycle. The error covariance matrix denotes the uncertainty associated with the state prediction.

The equation for the Kalman filter gain is given by

$$K = (P_{k|k-1}H^T)(HP_{k|k-1}H^T + R)^{-1}$$

(4.6)

The kalman filter gain serves as a minimizing factor for error covariance. From (4.6), it can be said that if the measurement error covariance $R$ is less, the Kalman filter gain will increase and the filter will put more weight on the measurement. Conversely, if the state error covariance $P$ starts approaching zero, the Kalman gain also starts decreasing, implying that the error in prediction is less, so the prediction of the state can be trusted.
H is the measurement matrix which transforms the predicted state into a vector so that the difference between the measurement and predicted state can be taken and kalman gain can be applied. H matrix is defined as

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Finally, the equation for state estimation is given by:

\[
\hat{x}_k = x_k + K(z_k - Hx_k)
\]  \hspace{1cm} (4.7)

where \(z_k\) is the state measurement vector which consists of roll, pitch, and yaw angle measured by accelerometer (\(\phi, \theta, \psi\)) and the Euler angle rate measured by the gyroscope (\(\dot{\phi}, \dot{\theta}, \dot{\psi}\)). The state measurement vector is defined as

\[
z_k = \begin{bmatrix}
\phi_{acc} \\
\dot{\phi}_{gyr} \\
\theta_{acc} \\
\dot{\theta}_{gyr} \\
\psi_{acc} \\
\dot{\psi}_{gyr}
\end{bmatrix}
\]
The equation for the updated error covariance matrix is given by:

\[ P_k = (I - KH)P_{k|k-1} \]  

(4.8)

where \( I \) is the identity matrix.

### 4.2.1 Calculation of Process and Measurement Noise Covariance Matrix

From (4.5) and (4.6) we can see that it is necessary to calculate the process and measurement noise covariance matrices \( Q \) and \( R \) to calculate the Kalman filter gain \( K \). To calculate measurement noise covariance matrix \( R \), first the estimated mean value of a set of roll, pitch, and yaw calculated by the accelerometer and magnetometer that contains no rotations is calculated. The mean value of the roll, pitch, and yaw angles is denoted as \( \mu \), which is estimated as [30]

\[ \mu = \frac{1}{N} \sum_{i=1}^{N} n_i \]  

(4.9)

where \( N \) represents total count of angle (roll/pitch/yaw) and \( n_i \) represents the value of the angle in each count.

Once the mean of the roll, pitch, and yaw angles is calculated, it can be used to calculate the variances in the roll, pitch, and yaw measurements obtained from the accelerometer and magnetometer. For example, the variance in the roll measurements can be estimated as:

\[ \sigma_r^2 = \frac{1}{N} \sum_{i=1}^{N} (n_{r_i} - \mu_r)^2 \]  

(4.10)
where $\sigma_r^2$ represents the variance in roll measurement, $n_{ri}$ represents the value of the roll in each count, and $\mu_r$ is the mean in the roll measurement calculated by the (4.9).

The measurement noise matrix can now be created using the variances for the roll, pitch, and yaw, which will be represented by $\sigma_r^2$, $\sigma_p^2$, $\sigma_y^2$ respectively. The variances are placed into the diagonal elements of the measurement noise matrix denoted by $R$. The roll, pitch, and yaw are mutually independent, and therefore the off diagonal elements will remain zero. Similarly, the variances in the measurement of the Euler angle rates in all three axes by the gyroscope are calculated.

$$ R = \begin{bmatrix}
\sigma_r^2 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_\phi^2 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_p^2 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_\theta^2 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_y^2 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_\psi^2
\end{bmatrix} \quad (4.11) $$

The process noise covariance matrix, $Q$ should be defined based on the uncertainty that we expect in the state equations. Typically there are some form of uncertainty in the state equations due to modeling errors, measurement errors, discretization, approximations in the derivation, etc. If we are very confident in state equations, $Q$ will be close to zero whereas if state modeling is not very close to the true value, the $Q$ value should be large [29]. The value of $Q$ is calculated by trial and error.
Experiments were conducted to measure the torso and arm orientation of the human body with an IMU in a lab equipped with an optical motion capture (hereafter MoCap) system. The orientation estimation results obtained from the IMU are verified using the MoCap system.

5.1 Experimental Setup and Procedure

The IMU chosen for the experiment was a 9-axis IMU with bluetooth technology (LPMS-B2, LP-reserach). It consists of a 3-axis gyroscope, a 3-axis accelerometer and a 3-axis magnetometer. It communicates with a host computer via a bluetooth classic 2.1 or low energy 4.1 connection. The sensor’s maximum data transmission rate is 400 Hz while lower transmission rate is also available.

A MoCap (Optirack, NaturalPoint, Inc) was used to separately measure the body orientation to be compared with the IMU estimation results. The software platform that was associated with Optitrack is called Motive. Motive allows the user to calibrate and configure the system. It also provides interfaces for both capturing and processing 3D data. The system consists of eight OptiTrack Flex-3 cameras, with resolution 640 × 480 and maximum frame rate 100 FPS for each camera.

For the experiment, these steps were followed:
1. Camera calibration: Calibration is an essential step for optical motion capture systems. In the calibration process, the system computes the position and orientation of each camera and the amount of distortions in captured images. This information is used to build a 3D capture volume (the area within which the motion capture data will be acquired) in Motive. This is accomplished by observing 2D images from multiple cameras and associating them with the position of the known calibration markers through triangulation process.

2. Setting the world reference frame of the MoCap: Using a calibration square as shown in Fig.5.1, MoCap’s the world x-axis was set to the south and z-axis to the west. Consequently, the y-axis is set upward.

![Figure 5.1: Calibration square axis convention](image)

3. Setting the body frame of reference of the Motive: In the MoCap system, a collection of three or more markers maintaining a constant distance to each other can be set as a rigid body. We used a marker plate on which four markers are mounted to define a rigid body (see Fig.5.2). To set the body frame of reference in the MoCap, the marker plate is first placed on the floor at the origin of the world frame, aligning the desired
local x and z axes with those of the world frame. Then, using a software function of the Motive, the body frame of reference is set relative to the rigid body (marker plate).

4. Attaching the IMU onto the marker plate: The IMU was attached at the center of the marker plate in such a way that both systems local x-axes were aligned with each other as shown in Fig.5.2. Because the ways each local frame is attached to the marker plate and the IMU are different, the y and z axes of the IMU is $90^\circ$ apart from those of the marker plate, as shown in Fig.5.2.

![Image of MoCap marker plate and IMU](image)

Figure 5.2: The MoCap marker plate and the IMU attached on it. The MoCap’s body frame (attached to the marker plate) is shown in red and the IMU’s in black.

5. Subject Preparation: The subject was asked to wear a special black jacket to cover any reflective objects on the subject’s body. The marker plate with the IMU on a belt was worn around the subject’s body or arm for the Motive to track the markers.

6. Experimental Procedure:

- The belt with IMU and marker plate is worn around the backbone of the subject.
- The subject stands straight in the neutral posture and the recording of the IMU and MoCap data starts.
- Then the subject bends forward.
• Then the subject holds straight position again.

• The previous two steps are repeated for a given time.

• Stop the recordings of both system.

• The belt is now needed to attach to the upper arm of the subject. Another recording session starts.

• The subject now mimics the pot unloading action of fishermen from the neutral hand position.

• Again, the previous step repeats for a given time.

• Stop the recordings in both system.

Figures 5.3 and 5.4 show the torso bending experiment and arm rotation experiment, respectively.

7. Data Collection and Post-processing: As the subject bends forward and move the arm, the reading of each sensor of the IMU change. The IMU’s readings are stored as a csv
file in the host computer via bluetooth communication. A Matlab code was written to read the csv file and calculate the torso bending and shoulder joint rotation.

As the Optitrack cameras track the four markers from different angles, these views are then used to construct the orientation of the rigid body in 3D after being recorded. This orientation data could be faulty due to the tracking error which is inevitable due to the nature of marker-based motion capture systems. The tracking errors, specifically the label errors is corrected by reassigning proper labels to markers during the post-process. After that, a csv file can be obtained from the Motive containing the orientation data of the rigid body. Motive supplies the orientation data in the form of Euler angle.

5.2 Similarity transformation of the IMU and Motive’s frame of reference

Let’s suppose, $G$ and $F$ represent the local frames of reference of the IMU and vision system respectively and $R_{2}^{1}$ is the torso or arm motion experienced by the IMU in its local frame. Then, according to the similarity transformation,

\[
(R_{2}^{1})' = (R_{F}^{G})^{-1}R_{2}^{1}R_{F}^{G}
\]  

(5.1)

where $R_{F}^{G}$ is the coordinate transformation between frames $F$ and $G$, $R_{2}^{1}$ is the matrix representation of the torso or the arm motion in the IMU’s local frame and $(R_{2}^{1})'$ is the same motion in the vision system’s local frame.
Examining the local coordinate frames shown in Fig. 5.2, the coordinate transformation matrix of the vision system with respect to that of IMU is given by

\[
R^G_E = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & -1 \\
0 & 1 & 0
\end{bmatrix}
\]

In similarity transformation, the torso and arm motion were calculated with respect to the initial orientation of the local frame for both IMU and Motive. So, \(R^1_2\) in (5.1) represents motion of the torso or arm with respect to the initial orientation of the local frame of the IMU. \(R^1_2\) is calculated as

\[
R^1_2 = (R^0_1)^{-1}R^0_2
\]

where \(R^0_1\) is the matrix representation of initial orientation of the local frame attached on the body with respect to the global frame and \(R^0_2\) is the matrix representation of torso/arm motion with respect to the global frame. The following steps are maintained for MoCap system to get the rotation with respect to the initial local frame:

1. Read the Euler angles obtained from the Motive.
2. Construct a rotation matrix from the Euler angles using (2.2). This rotation matrix is \(R^0_2\) which is the matrix representation of torso/arm motion with respect to the global frame of the MoCap system.
3. Get \(R^0_1\) which is the matrix representation of the initial orientation of the local frame attached to the body with respect to the global frame of the MoCap system. The Euler angles necessary for this matrix are obtained from the first reading of the Motive when the marker plate is attached to the body and the subject is in neutral posture.
4. Get \(R^1_2\) from the relation \(R^1_2 = (R^0_1)^{-1}R^0_2\).
5. Get the Euler angles from the $R_2^1$ matrix using (2.3) to (2.5).

The following steps are maintained for IMU to get the rotation with respect to the initial orientation of the attached local frame and express the rotation to the vision system’s frame of reference:

1. Get the roll, pitch, and yaw angle obtained from the complementary/Kalman filters.

2. Since roll, pitch, and yaw rotation representation is with respect to the fixed frame of reference, construct a rotation matrix from the roll, pitch, and yaw using (2.1). This rotation matrix is $R_2^0$ which is the matrix representation of torso/arm motion with respect to the global frame of the IMU system.

3. Get $R_1^0$ which is the initial orientation of the attached local frame with respect to the global frame of the IMU system. The roll, pitch, and yaw angle necessary for this matrix are obtained from the first reading of the IMU when the IMU is attached to the body and the subject is in neutral posture.

4. Get $R_2^1$ from the relation $R_2^1 = (R_1^0)^{-1} R_2^0$.

5. Now, using (5.1), convert the rotation experienced in IMU’s local frame to MoCap system’s local frame. This is $(R_2^1)'$.

6. Get the Euler angles (with respect to the current frame of reference) from the $(R_2^1)'$ matrix using (2.3) to (2.5) since we want to represent the rotation in vision system’s frame of reference.

5.3 Coefficient values for Complementary and Kalman filter

The coefficient $\alpha$ of the complementary filter has been set by trial and error.
The measurement noise matrix, $R$ was calculated using the variances (according to the procedure mentioned in Section 4.2.1) for the roll, pitch, and yaw calculated by the accelerometer and magnetometer and the variance in the Euler angle rate calculated by the gyroscope. The variances are placed into the diagonal elements of the measurement noise matrix denoted by $R$. The roll, pitch, and yaw and the Euler angle rate are mutually independent, and therefore the off-diagonal elements will remain zero.

The error covariance matrix, $P$ was initialized using the variances for the roll, pitch, and yaw calculated by the gyroscope, the variances in the Euler angle rate, and the variances in the gyroscope’s bias. The gyroscope bias was initialized by calculating the bias from the sensor on a flat table for a few seconds.

The initial state $x_0$ consists of the first measurements and gyroscope’s bias.

The process noise covariance matrix $Q$ models the uncertainty of the state matrix. The diagonal elements of $9 \times 9$ matrix have been set to model this uncertainty and help correct the precision errors in the state matrix and gyro bias.

### 5.4 Torso Bending Result

The orientation of torso was estimated by IMU in 4 different methods: by accelerometer/magnetometer only, by gyroscope only, by complementary filter and by Kalman filter. The results of accelerometer and gyroscope estimations and complementary and Kalman filter along with the MoCap are shown in Fig. 5.5 and in Fig. 5.6, respectively. Note that similarity transformation is done on the complementary and Kalman filter to represent the results in the vision system’s frame of reference. From Fig. 5.5, as expected, it can be seen that the accelerometer’s estimation is noisy but the gyroscope’s estimation creates a smooth curve. As torso bending is one-dimensional motion, we have obtained motion along one axis
(x-axis) only (see Fig. 5.6). There are small motions observed in the y-axis and z-axis at the time of bending (7-9 seconds, 10-12 seconds, 13-15 seconds), too. The result of torso bending matches well with the MoCap system with a small error that will be discussed next.

![Roll vs Time](image)

![Pitch vs Time](image)

![Yaw vs Time](image)

**Figure 5.5: Torso bending estimation by accelerometer and gyroscope**

The root mean square error (RMSE) has been used as a standard statistical metric to measure model performance. It is the standard deviation of the residuals (prediction errors). Residuals are a measure of how far from the regression line data points are; RMSE is a measure of how spread out these residuals are. In other words, it tells how concentrated the data is around the line of best fit. Thus lower RMSE value indicates a better fit. On the
other hand, higher RMSE value indicates that estimation is widely spread out around the actual value and is not a good fit. Root mean square error is commonly used in climatology, forecasting, and regression analysis to verify experimental results. For \( n \) samples of model errors \( (e_i, i = 1, 2, \ldots, n) \), the RMSE is calculated as

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (e_i)^2}
\]
The root mean squared error between the MoCap and IMU using complementary filter for torso bending is 0.5153, 1.2543, 1.8078 in x, y and z axis, respectively. For Kalman filter it is 1.1941, 1.7283 and 1.8488 in x, y, and z axis.

5.5 Arm Motion Result

Similarly, the results of accelerometer and gyroscope estimations and complementary and Kalman filter along with the MoCap for the arm motion are shown in Fig. 5.7 and in Fig. 5.8, respectively. As arm motion is 3-dimensional motion, we obtained rotations in all three axes. The Gyroscope’s smoothness in orientation estimation and accelerometer’s noisy estimation can be observed from the Fig. 5.7. Since the arm rotation experiment is done for longer time (60 seconds), gyroscope’s drift can be observed in there. From Fig. 5.8, it can be said that the Kalman filter’s result matches well with the vision system’s result.

For arm motion, the root mean squared error between Motive and IMU with complementary filter is 6.7085, 3.2376, 11.2619 in x, y and z axis, respectively. For the Kalman filter it is 6.6245, 2.2355 and 11.6463 in x, y, and z axis.

5.6 Discussions

There might be multiple reasons for the errors in estimated orientation. One source of error could be the improper alignment of the local x-axis of the IMU and the local x-axis of the marker plate. A better alignment method such as using a vision system may need to be considered.

Another source of error is the usage of a magnetometer to measure the orientation. Any ferromagnetic material interferes with the earth’s magnetic field vector which makes the
orientation result faulty although it may not be possible to obtain a ferromagnetic-free environment for the experiment. Additionally, the yaw angle measured by the magnetometer requires the value of roll and pitch angle measured by the accelerometer. In dynamic conditions, the acceleration field obtained from the accelerometers contains kinematic acceleration besides gravitational acceleration. So, in dynamic conditions, the errors in the roll and pitch measurement combine in the yaw angle and create larger errors in the yaw angle.

The data transmission rate of the IMU and the frame rates of the motion capture system were set to 100 Hz. Both systems should start and end at the same time. Any unsynchronized measurements will cause a mismatch in the result.
The complementary and the Kalman filters both are used to obtain optimal orientation. The complementary filter requires the tuning of only one parameter $\alpha$ but the Kalman filter requires the initial state vector, measurement covariance matrix $R$, process covariance matrix $Q$ as well as the initial error covariance matrix $P$ are needed to be well-tuned. Thus the difficulty of the Kalman filter to achieve a more optimal result increases. But the Kalman filter is an algorithm that consistently tries to find the statistically most optimal value. Since the state vector also contains the rate of bias, we can predict the bias rate from the Kalman filter.
The limitation of this study is the usages of the Euler angle and roll, pitch, and yaw angle to represent the rotation matrix. These representations suffer from the gimbal lock. Gimbal lock is the loss of one rotational degree of freedom due to the alignment of two rotation axes. Due to the gimbal lock, the pitch cannot reach 90 degrees either in the positive or negative direction. This problem could be eliminated by representing the orientation by Quaternion.
CHAPTER 6
CONCLUSION

In this thesis, two methods of measuring the orientation of an object using an IMU were examined. The first method uses both the accelerometer and the magnetometer of the IMU to read the components of the gravitational acceleration and the Earth magnetic field projected onto the accelerometer and magnetometer axes. This method is less accurate when the object is not stationary. On the contrary, the second method that uses only the gyroscope of the IMU unavoidably shows drift in the estimation of orientation over time, although it is robust to dynamic conditions. As a way to combine the advantages of each method, two sensor-fusion techniques were used: Complementary filter and Kalman filter. These two filters were demonstrated with two simulated tasks in commercial fishing where the orientation estimation methods can be useful for musculoskeletal disorder studies. The accuracies of the two filters were addressed in terms of RMS (root mean square) error deviated from the direct measurements using an optical motion capture system. In the estimation process, the orientation is represented by three independent angles. Since the world reference frames (as well as the body frames) are different between the IMU and the motion capture system, to do the error analysis, a similarity transformation has been established and successfully used in this thesis. Although it is difficult to generalize due to its specific application used in this thesis, the complementary filter method provides better accuracy than the Kalman filter, based on the experimental results obtained in this work.
REFERENCES


