Static optimization of fuel cell plug-in hybrid electric vehicle

Sunday Julius Balogun

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This thesis focuses on the static optimization of a fuel cell plug-in hybrid electric vehicle. The vehicle is been powered by three (3) sources of electrical energy. These sources of electrical energy are: fuel cell, supercapacitor, and lithium-ion battery.

The main target of this thesis is to make good the performance of a fuel cell plug-in hybrid electric vehicle. This will be achieved by applying static optimization method on the dynamic equations of a moving hybrid vehicle.

The optimization model of this plug-in hybrid electric vehicle (PHEV) was formulated bases on multiple objectives. The objective parameters are: cost, volume, and mass. We were able to apply static optimization algorithm to find optimal solutions for both the objective values and decision variables of the multiple energy sources.

The optimization model formulated from the dynamic equations, objective specifications, and design constrains were found to be feasible, bounded, and optimizable by subjecting the primal optimization model to its equivalent dual optimization test.
Advanced vehicle simulator (ADVISOR) was used to stimulate vehicle performance of our design on a standard driving cycle. The results provide a better outcome than that of standard driving cycles.
STATIC OPTIMIZATION OF FUEL CELL PLUG-IN HYBRID ELECTRIC VEHICLE

BY

SUNDAY JULIUS BALOGUN
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A THESIS SUBMITTED TO THE GRADUATE SCHOOL
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Thesis Director:
Dr. Donald S. Zinger
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This thesis would not have been possible without the grace of Almighty God.

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DEDICATION

To Almighty God

My parents: Late Chief and Mrs. C. Balogun Falade

My wife Omobola

My Children: Kayode, Kolade, Ayodeji, and Oluwafemi

Whose support, patience, and love have made this thesis possible.
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Chapter 1

INTRODUCTION: HYBRID VEHICLE

Around the world, the awareness to the negative impact of pollution is higher than before. Today the clamor for reduction in global warming effects is gaining momentum. Pollution is of different forms, but pollution caused by toxic carbon in the air, is by far, most dangerous and potent.

Global warming is blamed for release of carbon dioxide into the air when fossil fuel in the earth are burned by excessive warming (higher temperature) environment. The release of carbon dioxide into the air can create smog and cause serious health problems. It is equally blamed for climate change and rise in the sea level, which in a short time can destroy the coastal cities in the world.

In addition, human beings and appliances used by them have been blamed for global warming phenomenon. Outside the earthly carbon resources, petroleum fuel used in the vehicles and industrial machines account for majority of the carbon dioxide in the atmosphere. To reverse the advert effect of carbon dioxide pollution, more attention is being focused on the use of renewable energy sources in our transportation industry.

Nevertheless, fuel (petrol) used in combustion engines still present a serious challenge because it is easily available. It has much specific energy than energy obtainable from battery. With technology limitation so far, no single alternative energy sources can compete favorable with petroleum fuel in Internal Combustion Engine (ICE).
By combining two or more alternative energy sources together, (Hybridization of energy sources), some of the advantages of ICE’s energy source can be reduced drastically if not eliminated. However, the act of combining different energy sources together has attracted greater research and interest. For example, the technological breakthrough in this field is making it possible for hybrid electric vehicle (HEV) to be providing a challenge to ICE.

For this research, fuel cell plug-in hybrid electric vehicle (FCPHEV) is being considered. The three sources to this type of hybrid electric vehicle are fuel cell, lithium-ion battery, and supercapacitor. This is because these components have the following advantages over ICE:

- More efficient, compact, and lighter than internal combustion engine (ICE).
- Fuel cells produce almost zero carbon emission, thereby, reducing carbon pollution
- Supercapacitor has more specific power density that petroleum been used in ICE.

In other to harness all these advantages over ICE, FCPHEV can bring together all the advantages of different electrical energy sources and minimize their disadvantages.

The ability to combine three sources of energy has opened the question, which strategy to use in optimizing all the power obtainable from these sources of energy.

This research is geared towards static optimization of fuel cell plug-in Hybrid electric vehicle using 3-sources of energy.
BACKGROUN AND CLASSIFICATION

A hybrid vehicle is the type of vehicle that uses two or more distinct types of power. It combines any two or more power sources. Depending on degree of hybridization, power could be delivered to the rest of powertrain by either combination of electric motor and an internal combustion engine, or an electric motor with a fuel cells with battery/and supercapacitor as energy storage.

The fundamental design of hybrid vehicles is based on two sources of energy: energy conversion source and energy accumulation source. Energy conversion source could be fuel cells, internal combustion engine (ICE) or any system that convert fuel (of various type) to energy. Energy accumulation source could be supercapacitor, battery, flywheel, or and energy storage system. Energy accumulation source can be a single energy storage or combination of more than one. So, with advancement in hybrid technology, we now have hybrid vehicles with more than two sources of energy. In this case, a least two of the energy sources must be energy accumulation sources and at most one energy conversion source.

Possible combinations include diesel/electric, gasoline/fly wheel, and fuel cell (FC)/battery. The combination of two power sources may support two separate propulsion systems. Thus, to be a true hybrid, the vehicle must have at least two modes of propulsion. []

A typical hybrid car is as shown fig1.1. [1]

The basic principle with hybrid vehicles is that different motors work better at different speeds; the electric motor is more efficient at producing torque, or turning power, and the combustion engine is better for maintaining high speed (better than typical electric motor).
Switching from one to the other at the proper time while speeding up could result in a better and more efficient vehicle.

The advantages of a hybrid vehicle over internal combustion engine vehicle are:

1) Reduced or zero carbon emission (depending on type of hybrid vehicle). This will in-turn results into reduction in global warming (caused mainly by carbon pollution)

2) Higher efficiency in fuel consumption.

3) Higher energy conversion rate. It is more efficient than IC vehicle

4) Regenerative braking system is possible with hybrid vehicles. Regenerative braking is when the kinetic energy at the wheel (s) is being converted into useful electrical energy that can be used by the vehicle or stored in energy storage component of the vehicle. Because of this recent breakthrough in hybrid electric vehicle (HEV) technologies, the specific power available to hybrid’s powertrain is competing favorable with IC vehicle.

5) Since the availability of fossil fuel is gradually getting to its extinction, and with recent renew interest in HEV by transport industries, it is a matter of time before HEV takes over from ICE.

6) Hybrid vehicle could provide electric motor drive/assist. The electric motor provides power to assist the engine in accelerating, passing, or climbing hill. This allows a smaller and more-efficient engine to be used. In some hybrids, the electric motor alone propels the vehicle at low speeds, where gasoline engines are least efficient.
7) Some hybrid could provide automatic start/stop. Automatically shuts off the engine when the vehicle comes to a stop and restarts it when the accelerator is pressed. This reduces wasted energy when idling.

![A typical hybrid vehicle](image)

From Fig. 1.1, the sources of power are internal combustion engine (IC) and electric motor. IC is used to drive the generator to produce electrical energy that could either be used to partially or fully drive the electric motor. In addition, battery in conjunction with electric motor could either be used to partially or fully drive the electric motor. The two energy sources could be aggregated in such a way that both can provide energy needed to propel the hybrid vehicle.

**Degree of hybridization**

As defined earlier, hybrid vehicle consists two or more power sources to drive the rest of the powertrain, which in turn will drive the vehicles’ wheel. We equally said, one of the power sources must be an electric motor.
The degree of hybridization provides the relationship between the various power sources. Degree of hybridization (H) is the ratio between the traction electric motor(s) and the total installed power (traction motor(s) and IC power).

That is,

\[ H = \frac{\text{Power of electric motor(s)}}{\text{Power of IC engine} + \text{Power of electric motor(s)}} \]

The Degree of Hybridization (DOH) has great effect on the fuel economy, emissions and performance of the Hybrid Electric Vehicle (HEV). The degree of hybridization could also indicate the classification of a hybrid vehicle. A micro and mild hybrid vehicle has the lowest degree of hybridization. While, plug-in full hybrid vehicle has the highest degree of hybridization.

**Classification of hybrid vehicle.**

Hybrid vehicle can be classified by its degree of hybridization and/or by type of powertrain design.

There are three types of powertrain designs by which hybrid vehicle can be classified:

1) Parallel hybrid.

2) Series hybrid.

3) Power-split or series-parallel hybrid.
Fig.1.2. [2] shows block diagram layout for a parallel hybrid vehicle. With this arrangement, both the 2 power sources are connected in parallel to a common point (Mechanical coupler). The vehicle wheels receive their power from ICE propulsion and electric motor propulsion. Both ICE and electric motor can individually drive hybrid vehicle, or both coupled up jointly to drive hybrid vehicle.

Regenerative braking system is heavily utilized in parallel hybrid. Both the generator and regenerative braking system could be used to charge the battery. It generally uses small ICE and small battery. This makes parallel hybrid vehicle to be compact. However, it has a complex control system. ICE is coupled to the final drive thereby does not allow ICE to operate at its optimal narrow region.
Series hybrid

As shown in Fig.1.3. [2], in a series hybrid system, the mechanical output is first converted into electricity using a generator. The converted electricity either charges the battery or can bypass the battery to propel the wheels via the motor and mechanical transmission. This arrangement makes series hybrid an ICE assisted Electric Vehicle (EV). ICE for Series hybrid vehicle is not coupled to the final drive directly. This arrangement allows ICE to operate at its optimal narrow region. It could take advantage of regenerative braking system. Does not need a complex transmission system. However, it needs a big electric motor and it is less efficient than parallel hybrid configuration.
**Power-split or series-parallel hybrid**

Series-parallel hybrid combine the features of both the series and parallel hybrid vehicles. However, this configuration needs additional electric machine and a complex control system. It is as shown in Fig.1.4. [2]

![Block diagram of power-split hybrid.](image)

Hybrid vehicles can also be classified by its degree of hybridization.

1) Micro hybrid
2) Mild hybrid
3) Full hybrid
4) Plug-in hybrid.
**Micro hybrid**

A micro hybrid vehicle does not use two sources of power in its operation. So, it is not a hybrid vehicle at all. It is mostly a conventional ICE vehicle with ability to start and go. At idling, the ICE shuts down only using battery to run the car. With a sensor unit, any higher power demand will activate the ICE to start working. It has no regenerative braking ability. It requires a bigger ICE.

**Mild hybrid**

As the name implies, it is a hybrid vehicle, with two power sources. However, the electrical powertrain cannot drive the wheels by itself. The major job of the electrical powertrain is to start the hybrid vehicle, to augment power the total power available to final drive system, and initiates/activates stop and go ability of the hybrid vehicle. Electric motor can also be used to provide regenerative braking ability to the hybrid vehicle. Both the battery system and electric motor are usually smaller than other hybrid cars.
**Full hybrid**

Full hybrid vehicle uses both the ICE powertrain and electrical powertrain to propel the final drivetrain of the vehicle. Motors are powerful enough to power a car on its lonesome for a limited range.

A full hybrid’s drivetrain works in three basic states: petrol and electric propulsion combined, electric motor/s only, or the petrol engine by itself. Regenerative braking, as well as siphoning off extra or excess energy during acceleration or cruising, is enough to keep the batteries charged.

With potential to increase energy storage system, the size, weight and cost of the drivetrain can be reduced.

**Plug-in hybrid vehicle.**

One of the limitations of full hybrid vehicle is that its electrical powertrain can only be used limited mileage coverage. The electrical powertrain capacity used to augment or provide full power needed to drive the wheel, in full hybrid, could be increased by either increasing the capacity of the battery or charging the battery at regular and predetermined distance.

Plug-in hybrid vehicle is equipped to able to charge the battery at dedicated charging stations or from grid at home.
Both full hybrid and series hybrid vehicles can be upgraded into plug-in hybrid vehicle. Because it can be charged constantly, plug-in hybrid vehicle can be drive with battery for longer period, hence increase in the total efficiency.

By all standard, the lowest degree of hybridization is found in micro and mild hybrid vehicles. The battery technology used in these vehicles is the lead-acid due to its cost-effectiveness and the fact that it would always be operating with the internal combustion engine. While, the vehicles with the highest degree of hybridization are full and plug-in. However, with increase in energy density required, modern full and plug-in hybrid vehicles use Lithium-ion as their battery. It provides both higher power density and specific energy than other types of batteries.

**HYBRID ELECTRIC VEHICLE**

As describe earlier, hybrid vehicle consists two or more energy sources. Hybrid electric vehicle also consist two or more power sources. One or more of these power sources must be an electrical powertrain. It may or not use ICE as one of the powertrains.

A conventional hybrid electric vehicle has ICE as one of its powertrains. However, unlike conventional ICE vehicle, the size and capacity of the ICE use in hybrid electric vehicle is smaller. The amount of gasoline required is notably less relative to a purely ICE vehicle. This is because the electric motor uses regenerative braking to capture energy and store it in the on-board batteries. This stored energy is then used to provide power to the electric motor.
Pure hybrid electric vehicle uses electrical powertrain as the major provider of propulsion power to the wheel. ICE is been used as a back-up to extend the range of the vehicle. The vehicle will continue to use the electric motor until battery levels reach a predetermined battery state of charge (SOC). At this SOC level, the vehicle enters a charge sustaining mode. The internal combustion engine, in most cases acting as a generator, then kicks in to supply power to the electric motor.

While in charge depleting mode, the main fuel source for PHEVs is electricity since they used stored energy that was sourced from the electricity grid. While PHEVs also take advantage of regenerative braking, the energy captured is not enough to sustain the electric motor as the main driving system.

Hybrid electric vehicles have different configurations. Some of these are:

- Battery/ICE configuration
- Capacitor/Battery/ICE configuration
- Battery/Fuel cells configuration
- Battery/Supercapacitor/Fuel cells configuration

Fig.1.5. [3] shows an example of hybrid electric vehicle.
Fuel cells are more efficient, compact, and lighter than internal combustion engine (ICE). Also, Fuel cells produce almost zero carbon emission. Because of these advantages, fuel cell has caught the interest of electric hybrid vehicle’s industries. Even with these advantages, fuel cell has low specific energy than ICE, low specific density than both battery and supercapacitor. However, it has higher specific energy than both battery and supercapacitor. Fig.1.6 shows the electrical characteristics of the three sources of energy.
By putting together advantages and disadvantages of fuel cells and aggregate this with the pros and cons of Supercapacitor and battery, a pure hybrid electric vehicle could be built. This thesis deals with fuel cells hybrid electric vehicle (FCPHEV).

In this design, three sources of energy are been used. These sources of energy are:

- Lithium-ion battery
- Supercapacitor
- Fuel cells

The output of these three sources energy is electricity which is used to power electric motor.

This design uses all the advantages of full and parallel hybrid’s configurations.

Fuel cells hybrid electric vehicle works like:
• Start-up: Both fuel cell and battery provide both the energy and power needed. Fuel-cell power does not get to the maximum at start-up (needs time to warm up) so battery is needed to augment both energy and power needed for start-up.

• Cruising: Both fuel cell and battery provide both the energy and power needed.

• Acceleration: During acceleration, more power is needed to overcome all opposing forces on a vehicle in motion, supercapacitor has more specific power than any of the two sources of energy. However, it has less specific energy than the two. The combination of all sources of energy and power shall be needed during acceleration.

• Supercapacitor has fast charging and discharging rate than both Lithium-ion and Fuel cell. This advantage is put into use in regenerative braking of hybrid electric vehicle. In this configuration, supercapacitor is been used to capture and save energy recovered.

• Fuel cells work continuously during driving cycle. It provides average energy needed during the driving cycle.

Plug-in configuration allows FCPHEV to be plugged into an electric outlet (at home) to recharge their batteries and as well, being charged on the move (dedicated charging stations on the road).

FCPHEV does save on fuel and reduces pollution
Components of fuel cell plug-in hybrid electric vehicle.

Below Fig.1.7 is the schematic box-diagram that show 3-components energy sources configuration being considered.

**Fig.1.7.** Powertrain of the 3-sources of energy
The 3-sources of energy can each form a powertrain to power the vehicle. This can be done as combination or as an individual.

The fuel cell is being powered by hydrogen. The battery is lithium-ion. The supercapacitor is of hybrid type (supercapacitor of asymmetric electrodes.)

Voltages generated by these sources of energy are connected to common DC Bus through their respective converters and conditioners. Output voltage of a fuel cell is usually very small, so its output needs to be enhanced. Connecting its output into the input of a boost converter, will increase the voltage to vehicle’s DC bus-bar level.

Bidirectional converters are connected to the output of both the battery and supercapacitor respectively. This allows bidirectional power flow between the components and DC bus-bar. This will allow both the supercapacitor and battery be used to supply power and capture/store power during regenerative braking. Bidirectional helps to recondition both power and voltage from these to the level required at DC bus-bar.

Due to design specifications for this thesis, one extra load on the DC Bus is being considered.

This extra load is:

- Energy dissipating load

Since all the loads around a bus constitute a close loop. This Thesis assumes that the summation of powers on DC Bus is zero. The total supply powers are equal to all the powers drawn from the DC Bus.

To accommodate for any excess voltage on the DC Bus, energy dissipating load is included in the design. This excess load could take the form of excessive generative action of the electric motor (excessive regenerative braking power) which the fully charged energy storage
components cannot handle. It is better to safely remove this excessive power before it causes damages to the system.
Chapter 2

BATTERY

A battery is an electrochemical device used primarily to store electrical energy. It consists of one or more cell(s). All the cell(s) in a battery are immersed in an electrolyte (which could be liquid or non-liquid).

In a battery that consist more than one cell, the cells are linked sequentially in series within the battery.

In its simplex form, a cell consists two plates (electrodes). Each pair of electrodes, in a cell, are of opposite polarity (-ve and +ve). The cells are physically connected in series. The negative (-ve) plate of one cell is connected to positive (+ve) plate of the adjacent cell. The two end plates after serial connection of the cells, are usually called the electrodes of a battery. These electrodes could be referred to as anode (+ve polarity) and cathode (-ve polarity) respectively. The polarity that is associated to either anode or cathode is determined by the direction of flow of electrons in a battery. During charging, electron flows from anode (+ve) to cathode (-ve) but during discharging, electron flows from cathode (-ve) to anode (+ve).

Fig. 2.1 [2] below shows both the external and internal structure of a typical lead-acid battery. A protective casing provides physical protection for other parts of the battery.
Fig. 2.1. Lead-acid battery. [2]

Each cell has a maximum voltage capacity (2.1 volts for lead acid battery). To have a higher voltage capacity, the cells are connected in series. A six-cell lead acid battery will be $6 \times 2.1 = 12.6$ volts lead acid battery.

More so, two or more battery can be connected to form a battery bank.

A bank of batteries, connected in series, as shown in Fig. 2.2, will have more voltage capacity than each battery. A bank that consist 2, 12volts lead acid battery, both connected in series, will have $2 \times 12 = 24$ volts capacity.
If more current is required from a battery bank, batteries can be connected in parallel. A bank that consists of 2, 10Ah lead acid batteries, with both connected in parallel, as in Fig.2.3, will have \(2 \times 10 = 20\text{Ah}\) capacity.

Fig. 2.2. A bank of 2, 12 volts batteries connected in series.

Fig. 2.3. A bank of 2, 12 volts batteries connected in parallel.
The connection could be mixed (series and parallel) as in Fig.2.4. This is a battery bank that consists 4, 12volts 10Ah, with two strings (each string containing 2 batteries in series and the two strings connected in parallel), will have 24 volts 20Ah capacity.

![Diagram](image)

Fig.2.4. A bank of 4, 12 volts batteries with a mixed connection.

Electrolyte is a conducting mineral that can be in liquid or paste form. Within a battery, there is a chemical reaction between the plates/electrodes and the electrolyte. This chemical reaction result in the movement of electrons. The direction of electron flow is dependent on either a battery is being charged or discharged.

When the electrodes of a battery are charged by chemical reaction within the battery, there will be a potential difference between the electrodes. This potential difference is referred to as the terminal voltage (nominal) of the battery.

Whenever, an external load is connected across a charged battery (with a potentially difference), chemical energy in the battery will be converted into an electrical energy. The direction of energy flow is determined by the load. Electrical energy is either been stored as chemical energy due to the chemical reaction inside the battery or being extracted from the
battery by the load on it. A battery is either absorbing energy, such as when it is being charged by an external load or releasing energy (discharging), as it operates a load.

The state and type of electrolyte in a battery are used to categorize a battery and define its type respectively. The state of the electrolyte is used to determine either a battery is wet (liquid) or dry (paste). Also, the type of electrolyte is used to differentiate different types of a battery. The physical construction of a battery could be used to determine either a battery a sealed valve type or non-sealed valve (flooded) type. A battery could be either rechargeable or non-rechargeable.

Generally, a flooded rechargeable battery (lead acid battery) is a battery that contain liquid electrolyte and are rechargeable. A rechargeable sealed valve battery contains no liquid electrolyte but are rechargeable. The electrolyte for SLA battery is either dry (paste or gel) or contain enough liquid that will not spill.

In other to be able to understand, study, and model a battery, some of its characteristics and environmental effect need to be discussed. Some of these are: State of charge (SOC), Depth of discharge (DOD), effect of temperatures on the battery, battery’s efficiency, and polarization effect in a battery.

Brief discussions on these are as follow.

**State of charge (SOC) and depth of discharge (DOD) of a battery**

The state of a battery is defined in two forms: state of charge (SOC) and depth of discharge (DOD).
SOC refers to how full a battery is charged.
DOD refers to how far down the battery has been drained [1].
DOD + SOC = 100% of a battery state [1].
DOD is an alternate method of indicating battery’s state of charge.

Temperature effect and efficiency of a battery

A battery has an internal resistance. The value of this internal resistant can determine the efficiency of a battery. The type of technology used in constructing a battery can greatly determine its internal resistance. Generally, Lithium-ion batteries have lower internal resistance than most other types of batteries.

The operating environment under which a battery is subjected to could greatly increase or decrease the internal resistance of a battery. The correlation between temperature, charge mobility, and internal resistant of a battery can best be analyze by Ohms law and Arrhenius equation.

Battery is an electrochemical device within which chemical reaction must take place before electrical energy can be stored or extracted. For a chemical reaction to take place, molecular/chemical energy above or equal to activation energy is needed. Activation energy is needed be overcome before reactants, can move together, overcome forces of repulsion, and start breaking bonds.
Arrhenius equation [5] [7]

Arrhenius equation [5] [7] defines the relationship between temperature, activation energy, and the rate of chemical reaction.

\[ K = A \cdot e^{-\frac{E_A}{R \cdot T}} \]  \hspace{1cm} (1)

Where

\( K \) = Rate of chemical reaction constant.
\( A \) = Pre-exponential factor. This is also call frequency factor which relate the frequency of collisions between molecules.
\( E_A \) = Activation energy. A constant representing the minimum energy needed for the reaction to occur.
\( R \) = Universal gas constant
\( T \) = Absolute temperature in degrees Kelvin

As can be seen from above equation, rate of chemical reaction constant increases exponentially when the temperature increases. Also, chemical reaction constant decreases exponentially when the activation energy increases. Different battery type has a unique rate of chemical reaction constant (K). Each manufacturer of a battery has different experimental values of “K” with their corresponding different temperatures. Also, each type of batteries has unique frequency factor (A), which can be provided its manufacturer.
Higher power batteries are important in many practical applications. Higher power output requires faster charge transfer reactions in the charge/discharge process. Lower activation energy correlates directly to faster charge or ion diffusion in a battery.

From.

\[ K = A \cdot e^{-\frac{E_A}{R \cdot T}} \] \[ (5) \] \[ (7) \]

By finding the natural log of above equation, i.e.

\[ \ln K = \ln(A \cdot e^{-\frac{E_A}{R \cdot T}}) \]

\[ \ln K = \ln A + \ln e^{-\frac{E_A}{R \cdot T}} \]

\[ \ln K = \ln A - \frac{E_A}{R} \cdot \frac{1}{T} \]

(2)

Comparing this to standard line equation, the slope of above equation = \(-\frac{E_A}{R}\)

By plotting natural log of various values of chemical rate constants (\(\ln K\)) to their corresponding inverse temperatures (1/T), as obtained from the manufacturers of each type of battery, activation energy for each type of battery can be obtained.

---

**Ohms law**

Ohms law states, voltage (\(V\)) across a battery is equal to products of rate of the charge carrier (current (\(I\))) and its internal resistant (\(R_i\)).

That is,
\( V = I \times R_i \) \hspace{1cm} (3)

\( R_i = \frac{V}{I} \) \hspace{1cm} But,

\( V = \frac{v_{cc}}{\mu \times l} \)

So,

\( R_i = \frac{v_{cc}}{l \times \mu} \times \frac{1}{\mu} \) \hspace{1cm} (4)

Where,

\( V = Voltage \ across \ the \ battery \)

\( R_i = Internal \ resistance \ of \ battery \)

\( \mu = Mobility \ of \ the \ charge \ constant \)

\( v_{cc} = Velocity \ of \ charge \ constant \)

\( l = Distance \ between \ the \ electrodes \ of \ a \ battery \)

From equation above, internal resistant of battery is inversely proportional to the mobility of the charge constant of the battery chemistry. Therefore, as the mobility of the chemical charged carrier increases, the internal resistance of the battery decreases.

It has been well documented in many literatures; an increase in temperature of chemical reaction in a battery will increase the mobility of the charge carries of the electrolyte. Since the internal resistant of battery is inversely proportional to the mobility of the charge constant, hence an increase in operating temperature of battery will result in the decrease of internal resistant of a battery.
From Arrhenius equation [5] [7], it has been established that chemical reactions internal to the battery is affected by its operating temperature. The hotter the battery, the faster chemical reactions will occur.

Also, from ohms law, it is shown that rate of chemical reaction of a battery and its mobility’s charge carries increases respectively with increase in temperature. This shows that high temperatures do increase the performance of a battery [7]: high temperature (heat), typically accelerates chemical activity within a battery. The performance of a battery is usually better with a slight high temperature than a cold one. Cold weather [8] tends to reduce the performance of a battery: reduces chemical activity of a battery. At colder temperatures, the battery’s ability to provide enough power to start and run a vehicle is diminished.

However, high temperature increases the rate of the unwanted chemical reactions; this will result in loss of battery life. The shelf life and charge retention of a battery depend on the self-discharge rate [7]. Self-discharge is the result of an unwanted chemical reaction in the cell [7]. High temperatures speed up internal corrosion within the cells and in turn reduce the life cycle of the battery. Temperature therefore affects both the shelf life and the cycle life as well as charge retention since they are all due to chemical reactions. Batteries function best at room temperature. Operating a battery at elevated temperatures improves performance [7] but prolonged exposure will shorten life.

From both Arrhenius equation and Ohms equation, it was shown that increase in operating temperature of a battery would reduce the internal resistance of battery. Conversely, cold operating temperature of a battery will increase the internal resistance of battery. Warm
battery is better than a cold one. Cold temperature increases the internal resistance and lowers the capacity of a battery. While, warm temperature decreases the internal resistance and increases the capacity of a battery. A battery [8] can provide 100 percent capacity at 27°C (80°F) and might typically deliver only 50 percent at –18°C (0°F). The momentary capacity-decrease differs with battery chemistry.

Consider Fig.2.5 being simple circuit representing static state of a typical battery.

With an open circuit voltage ($V_{OC}$), the efficiency of a battery is governed by:

$$V_{OC} = I_{BATT} \cdot R_i + V_{BATT}$$  \hspace{1cm} (5)

$$eff = \frac{V_{BATT}}{V_{OC}}$$

$$eff = \frac{V_{OC} - I_{BATT} \cdot R_i}{V_{OC}} \%$$  \hspace{1cm} (6)

Fig.2.5. Battery with open loop voltage ($V_{oc}$) and internal resistance ($R_i$)
From equation (6), the efficiency of battery decreases with increase in the internal resistance of the battery. Also, the efficiency of battery increases with decrease in the internal resistance of the battery.

Since internal resistant of a battery is indirectly proportional to both its internal and external temperature, efficiency of a battery increases with an increase in its operating temperature.

The terminal voltage of a typical lead-acid battery, as in Fig. 2.6 [7], will increase with an increase in temperature. However, an increase in the temperature, reduced the age of the battery.

The effect of temperature on battery vary considerately with the type of a battery. Lithium-ion battery can withstand temperature than most type of batteries. Temperature does not affect considerately the age of lithium-ion as lead-acid battery.
The total internal resistance of a battery is not limited to the ohmic internal resistance. Total internal resistance of a battery is a combination of ohmic internal resistance and polarization internal resistance. There two-types of battery’s polarization internal resistance: concentration polarization and electrochemical polarization resistance.

**Concentration polarization internal resistance**

Concentration polarization internal resistance of a battery is created because of concentration polarization effect in the battery. Concentration polarization effect is an isolating barrier [9] that exist between the electrodes of a battery and its electrolyte. It is the summation of all mechanical side effects that occur between the electrodes and electrolyte. This occurrence isolates electrode from the electrolyte thereby creating a resistance (polarization resistance) between electrodes and its surrounding (electrolyte). This resistance impedes the proper flow of current between electrodes and electrolyte. This in turn reduces potential difference across the terminals of the battery. This effect could increase the rate of self-discharging of a lithium-ion battery.
**Electrochemical polarization internal resistance**

Electrochemical polarization effect in a battery causes electrochemical polarization internal resistance. Electrodes of a lithium battery are made of porous materials. The porosity of the electrodes means that the chemical reaction between electrodes and electrolyte will not be the same across the surface area of the electrodes. This will result in electrical polarization within a battery. Polarization resistance accounts for polarization voltage drop in a battery. It manifests itself as discharging time progressed.

**Ohmic internal resistance**

Materials and components that obey Ohm's law are described as "ohmic" which means they produce the same value for resistance \( R = V/I \) regardless of the value of \( V \) or \( I \) which is applied. [10]

In a battery, ohmic resistance accounts for instantaneous potential drop when a resistive load is applied to a battery [11]. Sometimes, it is referred to as the dc internal resistance of a battery.
Types of battery.

As stated above, depending on the construction, operational characteristic, and type of electrolyte used, a battery can be categorized into different types. Some of the types are [12]: nickel-cadmium, nickel metal hydride, lithium ion, and lead acid batteries. Even though, the thesis concentrates on lithium ion battery, brief descriptions of different types are discussed.

**Nickel-cadmium battery**

Nickel-cadmium (NiCd) battery uses nickel oxide as anode electrode, cadmium as cathode electrode, and potassium hydrate solution as its electrolyte. NiCd is rechargeable by forcing constant current at no constant voltage through its electrolyte. Other batteries are charge at constant current constant voltage. This different method of charging NiCd makes it very hard/complex to recharge it [].

Cadmium metal compound is highly toxic to the user and environment. Due to its toxic effect to the environment, this battery should be recycled rather than been discarded. NiCd is very durable under an extreme temperature (-40 to 70-degree cent) and it has long lifetime.

Compared to lead-acid battery, NiCd has deep cycle life, lighter, and small. NiCd needs a constant maintenance and due to toxicity, users and its immediate environment should be protected. Its application is very limited.
Nickel metal hydride

Nickel Metal Hydride (NiMH) battery uses nickel oxide as its anode electrode, hydrogen-absorbing alloy as its cathode electrode, and alkaline (potassium hydroxide) as its electrolyte.

NiMH has 30-40% more specific energy than NiCd. It is less toxic than NiCd hence more environment friendly than NiCd. It could also withstand a wide temperature range. However, it has limited service life than NiCd. It has high self-discharging rate than NiCd. It generates heat when fast charging.

Lead-acid battery

Lead acid is the most prevalently battery to date. It has found its usefulness in most battery applications. It cost less than most batteries. It is highly resilient to overcharging, trickle charging, and can withstand a lot of physical abuse.

Lead-acid battery has its positive electrode made of lead covered with lead dioxide, its negative electrode made of sponge lead, and its electrolyte is mostly Sulphuric acid (either in liquid or gel-sealed valve type). It is very bulky. It needs constant maintenance (unless sealed valve type). Its DOD (depth of discharge) is relative smaller than other types of batteries.
It is susceptible to sulfating effect at low SOC (state of charge) or at low electrolyte condition. This could result into permanent damage of the battery. The flooded type of lead-acid battery is susceptible to leakage which could result into environmental hazard. In application where weight could be a problem, other battery such as Lithium-ion batteries are preferable. Lead-acid is easily available, rugged, and cheaper than most other type of batteries. It is highly recyclable.

**Lithium ion battery**

Lithium-ion battery generally uses carbon has its anode, metal oxide (such as cobalt oxide) as cathode, and salt solution (that contains lithium-ions) as its electrolyte. When a load is connected across the electrodes, positive charged lithium ions are attracted to and move towards its cathode. This will make cathode more positive charged than anode. With cathode more positively charged than anode, electrons will move from the anode to the cathode. With the movement of electrons, electrical energy (as a form of work done by the electrons) will be extracted from the battery. Lithium-ion battery is rechargeable (the movement of its electrons are bidirectional).

Different combinations of materials can be used to construct lithium-ion battery’s electrodes. This makes more adaptable than other battery’s types. Lithium-ion battery has a deeper DOD than most batteries. This makes it to have more cycle life than other types of batteries.
Compared to other types of batteries, lithium-ion battery has high voltage and charge storage per unit mass and unit volume (lithium has a small size than most materials). Its cell’s voltage is high (up to 3.6 volts) than most battery. It combines high cycle life with lower external maintenance. It delivers the highest energy density than most battery’s type. It does this repeatedly (high cyclic life) with non-memory effect. It does not contain toxic cadmium material hence, it does not contaminate its immediate surrounding. However, Lithium could develop heat if not adequately protected.

Fig. 2.7 [14] shows different battery technologies. These are plots of gravimetric energy density (Wh/Kg) against volumetric energy density (Wh/m^3) –Fig.2.7a [14] and volumetric energy density against specific energy density-Fig.2.7b. [14]

![Fig.2.7a. Gravimetric Energy density vs. volumetric energy density][14]
From above figures, at the same specific energy density, volumetric energy density of a lithium batteries is higher than its equivalent lead-acid battery. Also, at the same volumetric energy density, the gravimetric energy density of lithium ion batteries is higher than its equivalent lead-acid battery.

Lithium-ion battery is very attractive and popular (in usage) than other types of battery in hybrid electric vehicle’s industry. It is gradually replacing previously used SLI (starting, light, and ignition) batteries.

To the SLI industry, lithium-ion provides the following advantages:
• Very small and lighter than other batteries.
• It provides high specific energy
• It has more cyclic life time and durable than most batteries.
• Combination of above advantages allow it to provide high fuel savings to hybrid electric vehicle (HEV)

As of now, lead-acid batteries are cheaper than lithium-acid but newer battery technology and its attractiveness to electric vehicle industry has gradually reduce the cost of lithium-ion battery.

**Advantages and disadvantages of a battery**

When compared to other sources or storage of energy, battery has higher density than both conventional capacitor and supercapacitor. It could have more power density than fuel cells. It is cheaper than most other types of energy source. It is widely available, and it is highly recyclable.

However, it has less power density than both conventional capacitor and supercapacitor. It very heavy when compared to other sources of energy.

Fig.2.8 [15] shows both the energy density and power density of 3-types of energy sources: fuel cells, supercapacitor, conventional capacitor, and a battery.
Fig. 2.8. Energy density vs. Power density. [15]

**Modelling of a battery**

In electrical engineering, model is a representation of an electrical system. It captures not all the attributes of a system, but rather only specific or relevant area of research. It is created for a certain purpose.
Modelling allows subsets of electrical system’s functionalities and characteristics to be analyzed and tested using computer software. This process could also allow exploitation of its optimum performance.

As stated in most literatures, there are three methods by which a battery can be model: experimental, electrochemical, and electric-circuit based.

In this thesis, electric-circuit based models shall be discussed. Electric circuit-based models for a battery can be used in representing electrical characteristics of the battery. Electrical based model electric circuit-based models can be useful to represent electrical characteristics of batteries.

Modelling of a battery shall be done in 2-phases. In the 1st phase, a detailed general model representing all types of batteries shall be considered and discussed. In the 2nd phase, a well-detailed model for lithium-ion battery shall be covered.

**Simple and common battery model**

For all the four major types of a battery discussed early, a common model can be developed. The construction of each battery is unique. Therefore, in other to model for each type, parameters and constants that are unique to each type only need to be changed in the general model. However, due to this constructional difference, a common dynamic battery model is not possible for all the type of batteries.
A common battery model for all types of batteries is shown in Fig. 2.9 [3]. In this figure, a controlled voltage source (E) of a battery is connected in series with its internal resistance (R_i). With this modelling type, many assumptions are made: internal resistance is assumed constant during both the charging and discharging cycles and parameters are based on the discharge characteristics of a battery. All constant parameter can be extracted from manufacture datasheets of a battery. In addition, the internal impedance of a battery could be obtained from its datasheet.

Fig. 2.9 constitute a voltage-current equivalent circuit model for a battery [3]. The circuit describe how terminal voltage (V_BATT) of a battery change with the changes in the battery current.

\[
E = E_0 - K \frac{Q}{Q - it} + A \exp(-B \cdot it) - \int_0^t it \, dt
\]

Fig. 2.9. Nonlinear battery model [3]
Where,

E = Open circuit voltage of a battery to be model (volts)

$E_c$ = Battery constant voltage (volts)

K = Polarization voltage constant (volts)

Q = Amount of available electrode active materials (Ah/m$^2$)

It = Discharge capacity (Ah)

B = Exponential zone time constant invert (Ah$^{-1}$)

A = Exponential voltage amplitude constant (volts)

$R_i$ = Internal resistant of the battery

I = Actual discharge current of a battery.

The relationship between these parameters are:

$$E = E_o - K \frac{Q}{Q-it} + A \exp(-B \cdot it) \ [3] \ (7)$$

But,

$$V_{BATT} = E - I_{BATT} \cdot R_i \ \text{Where } E=V_{OC} \ [8]$$

So,

$$V_{BATT} = E_o - K \frac{Q}{Q-it} + A \exp(-B \cdot it) - I_{BATT} \cdot R_i \ [9]$$

$$K = const \cdot X \ [10]$$

$$X = \frac{(E_{FULL}-E_{nom} + A \exp(-B \cdot Q_{nom}) - I_{BATT} R_i) - (Q_{FULL}-Q_{nom})}{Q_{nom}} \ [3] \ [11]$$
Where,

$E_{\text{FULL}} = E$ (when battery is fully charged)

$E_{\text{nom}} = \text{nominal battery voltage value}$

$Q_{\text{nom}} = \text{Amount of electric charges at nominal voltage of a battery.}$

$Q_{\text{FULL}} = \text{Amount of electric charges at full voltage of a battery.}$

const: This is a constant value. The value is battery type’s dependent.

\[ A = E_{\text{FULL}} - E_{\text{nom}} \quad (12) \]

Value of “A” for each type of battery is as shown in Table.2.1

B is battery type dependent and for each type of battery is as shown in the Table.2.1 below. It is a measure charge left in a battery at the end of exponential zone. Value of “B” for each battery type is as shown in Table.2.1.

And value of K for different types of battery is as shown in the Table.2.1.

When battery is fully charged,

\[ E_O = E_{\text{FULL}} + K + R_I * I_{BATT} - A \quad (13) \]

The voltage-current characteristics of a battery can be modeled with a lot of methods.

This thesis shall only consider Shepherd’s model [6].
Shepherd’s model takes into the consideration, the direct behavior of a battery parameters (\(V_{\text{BATT}}, \text{SOC}, V_{\text{OC}}, \text{and } R_i\)) and discharge current [6] [3]. This model is well accepted in battery industry. However, it is very complex in computation.

The combination of Fig.2.9 [3] and ideal discharge curve (as provided by battery manufacturers) of a battery, could yield almost the same result as shepherds’ model. Fig.2.10 [4] shows a typical ideal discharge curve of a battery.

<table>
<thead>
<tr>
<th>Table 2.1. Battery types parameter [16]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lead-acid Battery</strong></td>
</tr>
<tr>
<td>A=0.055*E</td>
</tr>
<tr>
<td>B=300/(0.08*Q)</td>
</tr>
<tr>
<td>K=1.1*X</td>
</tr>
</tbody>
</table>
**Discharge curve characteristics of battery**

A typical discharge curve, as in Fig.2.10 [4] shows battery’s voltage behavior when subjected to various constant discharge current. At a given c-rate, if a battery is subjected to a constant discharge current, the battery voltage tends to behave as shown in Fig.2.10. As seen in the Fig.2.10, the behavior of battery’s voltage under a constant discharge current could be classified into 3-sections: fully charged, exponential, nominal.

The 1\textsuperscript{st} section represents when battery is dropping exponentially from its fully charged value to the end of exponential zone.

The 2\textsuperscript{nd} section represents the charges that can be extracted at the end of exponential discharge of a battery until it reaches the end of nominal zone.

The 3\textsuperscript{rd} section represents the total discharge of the battery. This coincides to when voltage of the battery drops rapidly.

The shape of the curve could be determined by type of the battery, operating temperature, and c-rate of the battery.

A flat shaped curve could indicate the quality of a battery.
A comparative discharge curve for 4-types of battery is shown in Fig.2.11. This figure was obtained by plotting the curve for each of the battery by using above equations and Table.2.1 [16]. The experimental Table.2.1 [16] show the parameters of equations for each battery. The nominal voltage for each of the battery is 1.2 volts except lead-acid battery which is 12 volts.

For a better comparison, 10 batteries for each type (except lead-acid battery) was connected in series to get 12 volts nominal voltage for each type.
A typical Lithium-ion battery as shown in Fig.2.12 [7] and as discussed early, lithium-ion possesses better qualities when compared to other battery types: high specific energy, less weight, less susceptible to temperature, high cycle life, and high specific power. These characteristics make a Lithium-ion battery very attractive to electric vehicle industries.
Plug-in hybrid vehicle uses battery to supplement extra power during accelerations and for constant/continuous electric operations. These needs require a battery with high specific power and high specific energy respectively.

A battery with high specific energy can store large amount of electrical energy per its weight. The range that electric vehicle can cover is directly to its specific energy capacity. A battery with high specific power can charge and discharge stored power more quickly. This ability has a direct correlation to EV power burst and its regenerative capability.
Also, plug-in hybrid vehicle requires a battery with less weight. The weight of a battery affects could affect the efficiency of an electric vehicle. The more the weight, the less the efficiency. Conversely, the less the weight of the vehicle’s battery weight, the more the efficiency.

This thesis takes into consideration above stated characteristics in deciding to use Lithium-ion battery as a research focus.

Models

There are several methods to model a lithium-ion battery. Most of these models can be found in the literatures. This thesis concentrates on the electric-based modelling methods. Three basic methods are briefly discussed. One the three shall be discussed fully and model.

Ri model

This represents the simplex way to model a lithium-ion battery.

In the Fig.2.13 shown, this electric circuit consists of a lithium-ion battery whose open loop voltage \((V_{OC})\) is connected in series with its internal resistance \((R_i)\). The relationship between the terminal voltage \((V_{BATT})\) and the circuit is governed by:

\[
V_{BATT} = V_{OC} - I_{BATT} \times R_i
\]

Where,

\[
V_{BATT} = \text{Terminal voltage.}
\]

\[
V_{OC} = \text{Open loop voltage}
\]
$I_{BATT} = \text{Battery current}$

$R_i = \text{Internal resistance of lithium-ion battery}$

Both $V_{OC}$ and $R_i$ are functions of SOC and temperature.

This equation represents the charging and discharging equations for $R_i$ model of lithium-ion battery.

$I_{BATT}$ is positive during discharge and negative when charging.

This relationship does not show the dynamic and transitional behavior of a lithium-ion battery.

This cannot be used to adequately model a lithium-ion battery.

Fig. 2.13  $R_i$ model circuit diagram.
Thevenin / RC model

RC circuit usually present a dynamic behavior to an electric system. With an RC circuit, a dynamic behavior of lithium-ion battery can be incorporated to Rᵢ model. This model, a short period dynamic behavior of a lithium-ion battery can be analyzed. In this model, a parallel RC network is connected in series with both lithium-ion’s internal resistance and open loop voltage.

Fig.2.14 shows electric circuit schematic for Thevenin/RC model.

From Fig.2.14,

\[ i_c = i_{BATT} - i_R \]  \hspace{1cm} (14)

\[ C \frac{dv_c}{dt} = i_{BATT} - \frac{v_c}{R} \]

\[ \frac{dv_c}{dt} = \frac{i_{BATT}}{C} - \frac{v_c}{R \cdot C} \]  \hspace{1cm} (15)

\[ V_{BATT} = V_{OC} - I_{BATT} \cdot R_i - V_c \]  \hspace{1cm} (16)

Above relationship gives a detail analyze of lithium-ion battery for a short period of time. It has been found in most literatures that this relationship cannot be good representative of a lithium-ion behavior for long time.

It does not accurately and adequately represent the behavior of lithium-ion battery for a long time.
In order to properly analyze the dynamic behavior of lithium-ion battery, two types of polarization effect within a battery needs to be accounted for. They are concentration polarization and electrical polarization effects. These have been discussed early.

DP (double polarization) model accounts for these polarization effects within lithium-ion battery. The model also provides adequate representation for the dynamic behavior of lithium-ion battery.
DP, as in Fig.2.15 uses 2 resistances to model the resistance effect of polarization. $R_1$ is used to represent resistance due to concentration polarization and $R_2$ is used to represent resistant due to electrical polarization. $R_i$ represents the ohmic internal resistant of the lithium-ion battery. Capacitance $C_1$ and $C_2$ are used to represent dynamic behavior of a lithium-ion battery. $C_1$ and $C_2$ are also used to represent the capacitance due to concentration and electrical polarization respectively.

As can been seen in Fig.2.15, DP model is simply addition of extra RC circuit to the Thevenin’s model of Fig.2.14.

From Fig.2.15, the following space state equations can be deduced:

Considering the 1st RC circuit,

$$i_{c1} = i_{BATT} - i_{R1} \quad (17)$$

$$C_1 * \frac{d v_{c1}}{dt} = i_{BATT} - \frac{v_{c1}}{R_1}$$

$$\frac{d v_{c1}}{dt} = \frac{i_{BATT}}{c_1} - \frac{v_{c1}}{R_1*c_1} \quad (18)$$

Considering the 2nd RC circuit,

$$i_{c2} = i_{BATT} - i_{R2} \quad (19)$$

$$C_2 * \frac{d v_{c2}}{dt} = i_{BATT} - \frac{v_{c2}}{R_2}$$

$$\frac{d v_{c2}}{dt} = \frac{i_{BATT}}{c_2} - \frac{v_{c1}}{R_2*c_2} \quad (20)$$
In addition to simulating both electrochemical polarization and concentration polarization, DP model provides the best dynamic performance. It gives a more accurate SOC estimation for lithium-ion battery. DP model is closer to the real behavior of lithium-ion battery because it has smallest error compared to other types of models.

\[ V_{BATT} = V_{OC} - I_{BATT} \cdot R_i - V_{c1} - V_{c2} \]  \hspace{1cm} (21)

Fig. 2.15  DP model circuit diagram
Chapter 3

CAPACITORS AND SUPERCAPACITORS

Capacitor

The basic application of a capacitor is to store electric energy (W). The energy stored are in the electric field of a charged capacitor.

A capacitor’s cell basically consists two (2) electrodes with a solid dielectric materials sandwich between the electrodes.

The effect of a capacitor in an electric circuit is known as a capacitance (C). The capacitance of a capacitor is affected by: dielectric materials between the electrodes, distance between the electrodes, and the area of the electrodes.

A typical capacitor is shown in Fig.3.1. [17]
The schematic diagram and symbol for a capacitor is shown in Fig.3.2 [17] respectively.

![Schematic diagram and symbol for a conventional capacitor.][1]

The steady state general equation governing a capacitor is:

\[ C = \frac{A}{l} \times \varepsilon \]  

(1)

Where,

- \( C \) = Capacitance value of the capacitor (F)
- \( A \) = Area of electrodes/plate of the capacitor (m²)
- \( \varepsilon \) = Dielectric permeability constant of the electrode’s materials
- \( l \) = Distance between the electrodes (m)
When voltage (V) is applied across the electrodes of a capacitor, an electric field is formed across the dielectric material. This will result in an electric charge being formed on both electrodes. The energy (potential) used in forming these electric charges are said to have been stored in the capacitor.

Both electrodes will have electric charges (Q) formed evenly on them.

Positive electric charges will form on one electrode while negative charges will be formed on the second electrode.

The net result are charges being stored in a capacitor.

This relationship is governed by,

\[ V = E \times l \]  \hspace{1cm} (2)

But, \[ E = \frac{\sigma}{\varepsilon} \]

Then,

\[ V = \frac{\sigma}{\varepsilon} \times l \]

But, \[ \sigma = \frac{Q}{A} \]

Then,

\[ V = \frac{Q}{\varepsilon \times A} \times l \]  \hspace{1cm} (3)

From equation (1) above,

\[ C = \frac{A}{l} \times \varepsilon \]

Equating this to equation (3) and rearranging variables, then,

\[ Q = C \times V \]  \hspace{1cm} (4)
Then the basic relationship between applied voltage and the charges stored in a capacitor is governed by:

\[ Q = CV \quad (4) \]

\[ C = \frac{Q}{V} \quad (5) \]

Where,

- \( Q \) = Electric charges stored
- \( C \) = Capacitance of the capacitor
- \( V \) = Applied Voltage.
- \( A \) = Area of the electrodes
- \( E \) = Electric field
- \( \sigma \) = Constant charge density
- \( l \) = Lengt of seperator (Solid dielectric)
- \( \varepsilon \) = Dielectric constant

From above, the capacitance of a capacitor is directly proportional to the stored charges and inversely proportional to the applied voltage.

At a constant capacitance value, changes in electric charge \((dQ)\) in a capacitor is directly proportional to changes in applied voltage \((dV)\). i.e.

\[ dQ = C \cdot dV \quad (6) \]
Above is valid until the breakdown voltage of a capacitor is reached. Beyond breakdown voltage, the charges of nonlinear capacitor saturate, and its capacitance value decrease drastically.

To obtain a capacitor of higher voltage, two or more can be connected in series. Fig. 3.3 shows two capacitors connected in series.

![Fig. 3.3. 2-Capacitors connected in series.](image)

Current (I) through the 2-capacitors is constant. The electric charges (Q) across the capacitors is constant. Total voltage across the two capacitors is the addition of the voltage across each capacitor.

That is,

\[ V_{AB} = V_{C1} + V_{C2} \quad (7) \]

But from equation (5),

\[ C = \frac{Q}{V} \text{ i.e. } V = \frac{Q}{C} \]

And current through the 2 capacitors is the same \((I = \frac{Q}{t})\)

So, charges in both capacitors is the same.

Then,

\[ \frac{Q}{c} = Q \times \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \quad (8) \]
\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}
\]

\[
C = \frac{C_1 \times C_2}{C_1 + C_2}
\]  

(9)

Where,

\(C = \text{Total network capacitance}\)

\(C_1 = \text{Capacitance of 1st capacitor}\)

\(C_2 = \text{Capacitance of 2nd capacitor}\)

From equations (1) and (2), connecting capacitors in series will result into increasing the voltage across the capacitor network but a reduction in the total capacitance of the network. The reduction in capacitance value could result in reduction of the electric charges that can be stored in the network.

Care must be taken when connecting capacitors in series. This connection could result in unevenly distribution of voltage in the electric circuit. To prevent this unevenly voltage distribution, balancing resistor and/or diode, as in Fig.3.4 [19] can be incorporated into the network.
To obtain a capacitor that can handle large current (i.e. store more charges), 2 or more capacitors can be connected in parallel.

Fig.3.5 is a circuit of two capacitors connected in parallel.

Voltage across the 2-capacitors is the same.

That is.

\[ V = V_1 = V_2 \]  \hspace{1cm} (10)
But,

\[ I = I_1 + I_2 \]

And,

\[ I = \frac{Q}{t} \]

So,

\[ \frac{Q}{t} = \frac{Q_1}{t} + \frac{Q_2}{t} \]

Q = Q_1 + Q_2, from (4), Q = C*V

So,

\[ C*V = V*C_1 \quad V*C_2 \]

C = C_1 + C_2 \quad (11) \]

Connecting capacitors in parallel results to increase in the network capacitance, hence more charges could be stored in the network.

In other to be able to understand, study, and model a capacitor, some of its characteristics and environmental effect on it need to be discussed and analyzed. Some of these are: energy, power, maximum energy transfer, and voltage-current characteristics of a capacitor.

Brief discussions on these are as follow.
Energy and power

Among many applications of a capacitor in an electric system, is the use of a capacitor as an energy storage device.

This thesis is concern about how to use a capacitor as electric energy storage device and the use of stored energy in the capacitor as one of the sources of energy for a hybrid electric vehicle.

Equation below governs the relationship between the changes in work done (\(dW\)) by external power sources in moving small incremental charges (\(dQ\)) from the negative plate to positive plate of a capacitor. This energy is a form of a potential energy stored in the electric field within a charged capacitor.

The power in a linear capacitor is govern by:

\[
P = V \cdot I
\]

(12)

But,

\[
I = \frac{dQ}{dt}
\]

(13)

So,

\[
P = V \cdot \frac{dQ}{dt}
\]

(14)

However,

\[
dW = P \cdot dt
\]

(15)

From equation (14)

\[
dW = V \cdot \frac{dQ}{dt} \cdot dt
\]
\[ dW = V \cdot dQ \]

Integrating both sides,
\[ W = \int_0^Q V \cdot dQ \quad \text{but, } V = \frac{Q}{C} \]
So,
\[ W = \int_0^Q \frac{Q}{C} \cdot dQ \]
\[ W = \frac{1}{2} \cdot \frac{Q^2}{C} \quad \text{but, } Q = C \cdot V \]
So,
\[ W = \frac{1}{2} \cdot (C \cdot V)^2 \cdot \frac{1}{C} \]
\[ W = \frac{1}{2} \cdot C \cdot V^2 \quad \text{(16)} \]

Where,
\[ W = \text{Energy stored in the capacitor} \]
\[ P = \text{Power extracted from the capacitor} \]
\[ V = \text{Applied voltage} \]
\[ Q = \text{Electric charges in the capacitor.} \]
\[ C = \text{Capacitance of the capacitor} \]

An ideal capacitor stores and discharge energy without any loss. However, due to imperfection in a capacitor’s materials, a resistant called ESR (equivalent series resistance) could cause losses of energy during charging and discharging of a capacitor.
**Maximum energy**

Consider below, a general equation governing the relationship between dielectric electric strength and breakdown voltage of a capacitor:

\[ V_{BD} = E_{DS} \times d \]  \hspace{1cm} (17)

Where,

- \( V_{BD} \) = Breakdown voltage of a capacitor
- \( E_{DS} \) = Dielectric strength of a charged capacitor

\( d \) = *Distance between electrodes*

So,

\[ E_{DS} = \frac{V_{BD}}{d} \]  \hspace{1cm} (18)

Generally, the maximum energy that can be stored in a capacitor is determined by its dielectric strength. However, from equation (18) above, it could be seen that dielectric strength is direct proportional to the break down voltage of the capacitor. Hence, at a constant capacitance value, maximum energy stored in a capacitor is determined by its breakdown voltage. Because of this, applied voltage to a capacitor must not exceed the breakdown voltage unless an isolation circuit is connected across the capacitor.

**Voltage-current characteristics of a capacitor**

Consider a capacitor(C) with rated voltage \( V_R \) and internal resistance \( R_i \).
Let step Voltage with amplitude $V_{OC}$ be applied across the capacitor.

The charging voltage $V_C$ across the capacitor at any time ($t$) could be obtained by considering Fig.3.6, being an RC circuit. Circuit analysis Fig.3.6 is as follow.

Fig.3.6. Simple RC circuit.

From Fig.3.6, finding the Laplace of the elements and using voltage division method,

$$V_C(s) = \frac{\frac{1}{sC_1}V_{IN}(s)}{R_i + \frac{1}{sC_1}}$$

$$= \frac{V_{IN}(s)}{1 + sR_iC_1}$$

(19)

With $V_{IN}(s)$ a step input voltage, i.e.

$$V_{IN}(s) = \frac{V_{IN}(s)}{s}$$

(20)

Then,
\[ V_C(s) = \frac{1}{1 + sR_1C_1} \frac{V_{IN}(s)}{s} \]

By partial fractions,

\[ V_C(s) = \frac{V_{IN}(s)}{R_1C_1} \left( \frac{A}{s + \frac{1}{R_1C_1}} + \frac{B}{s} \right) \]

Where,

\[ A = -R_t * C_1 \]
\[ B = R_t * C_1 \]

So,

\[ V_C(s) = \frac{V_{IN}(s)}{R_1 * C_1} \left( \frac{-R_t * C_1}{s + \frac{1}{R_t * C_1}} + \frac{R_t * C_1}{s} \right) \]

\[ \tau = R_t * C_1, \text{ Time constant} \quad (21) \]

Then,

\[ V_C(s) = V_{IN}(s) \left( \frac{1}{s + \frac{1}{\tau}} \right) \]

Doing inverse Laplace,

\[ V_C(t) = V_{IN}(t) \left( 1 - e^{-\frac{t}{\tau}} \right) \quad (22) \]
Supercapacitor

Supercapacitor is an electrochemical high-density capacitor with virtually all the characteristics of a conventional capacitor and more. Supercapacitor can also be referred to as EDLC (electric double-layer capacitor) or an Ultracapacitor.

Unlike a conventional capacitor (electric field device), supercapacitor is an electrochemical high-density capacitor.

Like a conventional capacitor, the cell of a supercapacitor, basically consists two (2) electrodes but separated by a solid electrolyte material (solid material that contain ions). The electrodes are made of porous materials. A porous material are materials containing pores which allowed and/or filled (in case of a supercapacitor device) with ions from its electrolyte. All the electrical equations that govern the activities and effects of a conventional capacitor apply to a supercapacitor. But, unlike a conventional capacitor, whose charge saturates and decrease sharply at a certain voltage, the capacitance of a supercapacitor increases with the in-built supercapacitor’s voltage up to its maximum voltage rating.

The electrodes could be symmetry (both having the same capacitance value: Hybrid Supercapacitor) or asymmetry. The nominal electrical parameters of supercapacitor depend heavily on the type of material used for both the electrodes and its electrolyte. When the electrodes are polarized by an applied voltage, ions in the electrolyte membrane form electric double layers of opposite polarity to the electrode polarity. These double layers are analogous to two or more separate capacitors connected in series.
The gap between each electrode adjacent to the separator sides are extremely very small: in Nano meter. Hence, supercapacitor has extremely small value of internal resistance. Fig.3.7 [22] shows physical (internal and external) of typical supercapacitor.

Figure 3.8 [25] below shows the electric double layer formation and the effect of diffuse layer on the overall operation of a supercapacitor.

**Supercapacitor configuration and analysis**

A supercapacitor has its capacitance dependent on its in-built voltage. The structure of a supercapacitor when subjected to an applied voltage contains compact layer and diffuse layer. Fig.3.8 [25] shows this phenomenal.
The dimension of the diffuse layer is determined by the amount of concentration of electrolyte in the separator between the electrodes.

The compact layer is as a result of initial electrolytic characteristic of a supercapacitor. These two layers are structured as a serially connected layers in a charged supercapacitor. The 2-electrodes (plates) of a supercapacitor are made of porous large surface area materials. The gap ($l$) between each electrode and adjacent side of the separator is extremely very small (in Nano meter).

Considering the general equations governing capacitor and its internal resistant:
\[ C = \frac{A}{l} \varepsilon \]  \hspace{1cm} (23)

\[ R_i = \frac{l}{A} \rho \]  \hspace{1cm} (24)

Where,

\(A\) = Large surface area of a supercapacitor’s electrode

\(l\) = Small thickness (diameter) of the electrolyte’s molecule.

\(C\) = Capacitance of the capacitor

\(\varepsilon\) = Dielectric constant of the electrode material

\(\rho\) = Resistivity of both electrode and electrolyte

As can be seen from above equation (23), capacitance value of a capacitor is directly proportional to the surface area of its electrodes (plates) and indirect proportional to electrolyte’s molecule diameter. With a very large surface area \((A)\) and extremely small gap \((l)\), capacitance of a supercapacitor is very high. Its capacitance value is much high compared to conventional capacitor. The capacitance value is in farad as against micro/mill-farad of conventional capacitor.

Also, from above equation (24), the internal resistance of a supercapacitor is directly proportion to gap \((l)\) and inversely proportional to the surface area \((A)\) of the electrodes. With a very large surface area \((A)\) and extremely small gap \((l)\), internal resistance of a supercapacitor is very small. Its internal resistance is much smaller than conventional capacitors.

Supercapacitor is a high-density capacitor because its capacitance value is much higher, and its internal resistance is much lower than a conventional capacitor.
Since supercapacitor has very small internal resistance (ESR) and from:

\[ P = I \times V \quad (25) \]

But,

\[ I = \frac{V}{R_i} \]

So,

\[ P = \frac{V^2}{R_i} \quad (26) \]

Where,

\( R_i = \text{Equivalent to ESR (estimated series resistance of a supercapacitor)} \)

From equation (26), with small internal resistance, the power that can be extracted from a supercapacitor is very high. So also, is power that can be stored in a supercapacitor.

The Table.3.1 [22] below shows the comparative advantages and disadvantages of a supercapacitor when compared with both conventional capacitor and a battery.
Tab.3.1  Supercapacitor and battery compared. [22]

<table>
<thead>
<tr>
<th>Available Performance</th>
<th>Lead Acid Battery</th>
<th>Ultracapacitor</th>
<th>Conventional Capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge Time</td>
<td>1 to 5 hrs</td>
<td>0.3 to 30 s</td>
<td>10^{-3} to 10^{-6} s</td>
</tr>
<tr>
<td>Discharge Time</td>
<td>0.3 to 3 hrs</td>
<td>0.3 to 30 s</td>
<td>10^{-3} to 10^{-6} s</td>
</tr>
<tr>
<td>Energy (Wh/kg)</td>
<td>10 to 100</td>
<td>1 to 10</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>Cycle Life</td>
<td>1,000</td>
<td>&gt;500,000</td>
<td>&gt;500,000</td>
</tr>
<tr>
<td>Specific Power (W/kg)</td>
<td>&lt;1000</td>
<td>&lt;10,000</td>
<td>&lt;100,000</td>
</tr>
<tr>
<td>Charge/discharge</td>
<td>0.7 to 0.85</td>
<td>0.85 to 0.98</td>
<td>&gt;0.95</td>
</tr>
<tr>
<td>efficiency</td>
<td>Operating Temperature</td>
<td>-20 to 100 °C</td>
<td>-40 to 65 °C</td>
</tr>
</tbody>
</table>

The Tab.3.1. [22] shows that supercapacitor has a very low internal resistance, high capacitance value, high specific energy, and high specific power than conventional capacitor. The table also show, supercapacitor has, high specific power, low internal resistance, high longer lifetime (smaller rate of self-discharge), charging, and discharging rate than equivalent battery. However, it has low specific energy than battery. Due to its very high charging rate, it could be used to harvest, very rapidly, the regenerative power from braking system of a hybrid electric vehicle (HEV).

The following fig.3.9 shows the parameters performance of a supercapacitor when compared with conventional capacitor and lithium-ion batteries.
### Performance parameters of supercapacitors compared with electrolytic capacitors and lithium-ion batteries

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Aluminium electrolytic capacitors</th>
<th>Double-layer capacitors (memory backup)</th>
<th>Supercapacitors (high power)</th>
<th>Pseudocapacitors &amp; hybrid (Li-Ion) (long-term)</th>
<th>Lithium-ion batteries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature range, Celsius (°C)</td>
<td>-40 - +125°C</td>
<td>-40 - +70°C</td>
<td>-20 - +70°C</td>
<td>-20 - +70°C</td>
<td>-20 - +60°C</td>
</tr>
<tr>
<td>Maximum charge, Volts (V)</td>
<td>4 - 630 V</td>
<td>1.2 - 3.3 V</td>
<td>2.2 - 3.3 V</td>
<td>2.2 - 3.8 V</td>
<td>2.5 - 4.2 V</td>
</tr>
<tr>
<td>Recharge cycles, thousands (k)</td>
<td>unlimited</td>
<td>100 k - 1 000 k</td>
<td>100 k - 1 000 k</td>
<td>20 k - 100 k</td>
<td>0.5 k - 10 k</td>
</tr>
<tr>
<td>Capacitance, Farads (F)</td>
<td>≤ 2.7 F</td>
<td>0.1 - 470 F</td>
<td>100 - 12 000 F</td>
<td>300 - 3 300 F</td>
<td>—</td>
</tr>
<tr>
<td>Specific energy, milli-Watt hours per gram (mW·h/g)</td>
<td>0.01 - 0.3 mW·h/g</td>
<td>1.5 - 3.9 mW·h/g</td>
<td>4 - 9 mW·h/g</td>
<td>10 - 15 mW·h/g</td>
<td>100 - 265 mW·h/g</td>
</tr>
<tr>
<td>Specific power, Watts per gram (W/g)</td>
<td>&gt; 100 W/g</td>
<td>2 - 10 W/g</td>
<td>3 - 10 W/g</td>
<td>3 - 14 W/g</td>
<td>0.3 - 1.5 W/g</td>
</tr>
<tr>
<td>Self-discharge time at room temp.</td>
<td>short (days)</td>
<td>middle (weeks)</td>
<td>middle (weeks)</td>
<td>long (month)</td>
<td>long (month)</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>99%</td>
<td>95%</td>
<td>95%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>Working life at room temp., in years (y)</td>
<td>&gt; 20 y</td>
<td>5 - 10 y</td>
<td>5 - 10 y</td>
<td>5 - 10 y</td>
<td>3 - 5 y</td>
</tr>
</tbody>
</table>

Fig. 3.9. Performance parameters of supercapacitor and lithium-ion battery [25]

There are different types of supercapacitor. Fig. 3.20 [25] below shows the different types of supercapacitor.
Supercapacitors are classified by a lot of factors among these: the type of materials used in constructing the electrodes, the redox reactions in which electrodes are made, combination of the different types, the type of separator between electrodes, construction techniques, and newer technology discovery in capacitor industries. This can determine the capability, type of applications, ruggedness, and quality of a supercapacitor.

The class that a supercapacitor belongs could determine the usage.

Fig.3.21 [25] shows different classifications of a supercapacitor and its usage.
This thesis concentrates on the use of a supercapacitor as one of the sources in a hybrid electric vehicle.
Supercapacitor structure and representation

The electrodes of supercapacitor are made from porous materials.

A porous material exhibits a nonlinear structure which is analogous to nonlinear behavior of transmission line circuit.

As documented in electrical engineering textbooks, a short transmission line can be represented electrically as $n^{th}$ order RC ladder network as shown in Fig.3.22 [20] below.

![RC ladder network.

In its simplest form, a supercapacitor can be model as a 1st order short transmission line as in Fig.3.23 [20]
This model defines the structural configuration of a supercapacitor when subjected to applied voltage.

The general equivalent circuit for a superconductor when subjected to an applied voltage is as shown in Fig.3.24 [20]
This equivalent circuit shows an RC circuit equivalent to supercapacitor. The internal resistance (Ri) of a supercapacitor is connected in series with the voltage-controlled capacitance (C (v_c)) of the supercapacitor.

In Fig.3.24,

Rp = A shunt resistor representing leakage current in a supercapacitor
Ri = A series resistor representing the internal resistor of a supercapacitor.
C(v_c) = A voltage-controlled capacitance of a supercapacitor.

As can be seen from above circuit representation, supercapacitor is a nonlinear capacitor. The capacitance of supercapacitor is controlled by the in-built voltage in the capacitor.

Generally, the equation below, shows relationship between the in-built voltage and the capacitance of a supercapacitor:

\[ C(v_c) = C_0 + k_c \cdot v_c \]  \hspace{1cm} (27) [20]

Where,

\( C_0 \) = Initial linear electrostatic capacitance value of a supercapacitor.
\( k_c \) = Constant coefficient representing the effect of diffusion layer in a supercapacitor
\( v_c \) = Time varying in-built voltage in a supercapacitor
\( C(v_c) \) = Capacitance of supercapacitor as a function of its in-built voltage.

The compact layer capacitance is represented by \( C_0 \) while the effect of diffuse layer is represented by \( k_c \cdot v_c \).
From Fig.1.24 and equation (27) above, Fig.3.24 can further be simulated as in Fig.3.25 [20].

Fig.3.25. Supercapacitor circuit. [20]

Usually $R_p \gg \gg Ri$,

So, Fig.3.26 [20] represents the simplified circuit representation of a supercapacitor when subjected to an applied voltage ($V_T$).
Fig. 3.26. Simplified Supercapacitor circuit. [20]

In general Fig. 3.27 [20] is equivalent to above Fig. 3.26

Fig. 3.27  Supercapacitor circuit [20]

Where,

\[ C(v_c) = C_0 + k_c \cdot v_c \]  \hspace{1cm} (27) [20]

From Fig. 1.27 and equation (27)
\[ i_c = \frac{dQ}{dt} \]  

But,

\[ Q = C(v_c) \cdot v_c \] \hspace{1cm} (29)

Then,

\[ i_c = \frac{d[C(v_c) \cdot v_c]}{dt} \]

\[ = \frac{v_c \cdot dC(v_c) + C(v_c) \cdot dv_c}{dt} \]

\[ i_c = \frac{v_c \cdot dC(v_c)}{dv_c} \cdot \frac{dv_c}{dt} + \frac{C(v_c) \cdot dv_c}{dt} \]

\[ = \left[ \frac{v_c \cdot dC(v_c)}{dv_c} + C(v_c) \right] \cdot \frac{dv_c}{dt} \quad \text{but} \quad C(v_c) = C_0 + k_c \cdot v_c \]

Then,

\[ i_c = \left[ \frac{v_c \cdot d(C_0 + k_c \cdot v_c)}{dv_c} + C_0 + k_c \cdot v_c \right] \cdot \frac{dv_c}{dt} \]

\[ = \left[ v_c \cdot k_c + C_0 + k_c \cdot v_c \right] \cdot \frac{dv_c}{dt} \]

So,

\[ i_c = \left[ C_0 + 2 \cdot k_c \cdot v_c \right] \cdot \frac{dv_c}{dt} \] \hspace{1cm} (30)

And

\[ V_T(t) = R \cdot i_c + v_c \] \hspace{1cm} (31)

Where,

\[ C_0 = \text{Initial linear electrostatic capacitance value of a supercapacitor.} \]

\[ k_c = \text{Constant coefficient representing the effect of diffusion layer in a supercapacitor} \]
In order to be able to understand and analyze the usage of a supercapacitor in circuit or system, some of the characteristics of a supercapacitor need to be understood and discussed. Some of these characteristics are: energy, power, specific power, specific energy, life time, charging time and discharge profile.

**Energy**

Energy is the ability or capability to do work.

Supercapacitor stores electrical energy.

Unlike a battery, the storage is not because of chemical formation between the electrolyte and electrodes, rather, it does this in form of electric field between its 2-electrodes.

Energy stored in a supercapacitor, is directly proportional to its in-built voltage and its voltage-controlled capacitance.

The standard SI unit of energy is KWh (Kilowatts-hour).

The work done when power is extracted over a period of time from a capacitor is govern by:

\[ dW = P \times dt \]  

\[ (32) \ [22] \]
But,

\[ P = v_c \cdot \frac{dQ}{dt} \]  

So,

\[ dW = (v_c \cdot \frac{dQ}{dt}) \cdot dt \]

\[ dW = v_c \cdot dQ \]

But,

\[ Q = C(v) \cdot v_c \]

So,

\[ dW = V \cdot (C(v) \cdot v_c) \]

By using Leibniz notation of 2 function [9] i.e. \( d(UV) = U \cdot dV + V \cdot dU \)

Then,

\[ dW = v_c \cdot (v_c \cdot dC(v) + C(v) \cdot dv_c) \]

But, \( C(v) = C_0 + K_c \cdot v_c \)

Then,

\[ dW = v_c \cdot (v_c \cdot d(C_0 + K_c \cdot v_c) + (C_0 + K_c \cdot v_c) \cdot dv_c) \]

Integrating both sides,

\[ W = \int_{v_0}^{v} v_c \cdot (v_c \cdot d(C_0 + K_c \cdot v_c) + (C_0 + K_c \cdot v_c) \cdot dv_c) \]

\[ W = \int_{v_0}^{v} [0 + \frac{v_c^3}{3}K_c + \frac{v_c^2}{2}C_0 + \frac{v_c^3}{3}K_c] \]

\[ W = 2 \cdot \frac{v_0^3}{3}K_c + \frac{v_0^2}{2}C_0 \]
\[ W = \frac{V_c^2}{2} \left[ \frac{4}{3} \ast V_0 \ast K_C + C_0 \right] \]  

(34)

Where,

\( v_c \) = In-built voltage in a supercapacitor.

\( W \) = Energy stored

\( C(v) \) = Voltage-controlled capacitance of a supercapacitor

\( C_0 \) = Electrostatic capacitance of supercapacitor

\( v_0 \) = Initial in-built voltage in a supercapacitor.

\( k_c \) = Constant coefficient representing the effect of diffusion layer in a supercapacitor

Since the capacitance of a supercapacitor is much higher than the capacitance of a conventional capacitor, from equation above, it could be deduced that energy stored in a supercapacitor is much higher than the energy stored in a conventional capacitor.

**Specific energy and energy density.**

Specific energy is the ratio of energy stored/extracted from a supercapacitor to its mass/weight.

The SI unit of a specific energy is J/Kg (Joules per kilogram) or KW-h/Kg (kilowatts hour/kilogram)
Energy density is the ratio of energy stored/extracted from a supercapacitor to its area/volume.

The SI unit of an energy density is J/m² (Joules per meter square).

Consider equation (34)

\[ W = \frac{V_0^2}{2} \left[ \frac{4}{3} * V_0 * K_C + C_0 \right] \]

But

\[ C_0 = \frac{A}{l} \epsilon \]  \hspace{1cm} (35) [20]

So,

\[ W = \frac{V_0^2}{2} \left[ \frac{4}{3} * V_0 * K_C + \frac{A}{l} \epsilon \right] \]

Specific energy (SE) = \( \frac{W}{g} \)

\[ SE = \left( \frac{V_0^2}{2} \left[ \frac{4}{3} * V_0 * K_C + \frac{A}{l} \epsilon \right] \right)/g \]

\[ SE = \frac{V_0^2}{2} \left[ \frac{4}{3g} * V_0 * K_C + \frac{A}{l} \epsilon \right] \]

(37)

With,

\[ \frac{A}{l} \epsilon \gg \gg \frac{4}{3g} * V_0 * K_C \]

Then,

\[ SE \sim \frac{V_0^2}{2} \left[ \frac{A}{l} \epsilon \right] \]

(38)

Where,

\[ g = \text{Mass of a supercapacitor} \]
But \( A/g \) is the specific surface area of the electrodes a supercapacitor. With a constant voltage and mass, from equation (38), the specific energy of a supercapacitor is directly proportional to the specific surface area of the electrodes and thickness of molecule. A very higher surface area of the electrodes with small molecule’s thickness (diameter) will result into high specific energy of a supercapacitor.

Due to the large surface area of electrodes in supercapacitor than the conventional capacitor and small molecule’s thickness (diameter) than dielectric length in conventional capacitor, supercapacitor has a higher specific energy than conventional capacitor. However, supercapacitor has less specific energy than both petrol fuel and an equivalent battery.

With new technology in supercapacitor industry, efforts are being made in improving drastically the specific energy of a capacitor.

Today porous Carbon material are been used as electrodes of supercapacitor. These carbon materials have higher specific surface area to the magnitude of between 500 to 2000 \( \text{m}^2/\text{g} \).

The combination of porous electric materials (which has high electric conductivity) and high specific surface area of newly developed supercapacitor, is closing the specific energy gap between supercapacitor and battery.

Also, Graphene based electrode, are very encouraging. Graphene supercapacitor made from hydrogen-annealed graphene material which has a high porous structure (small ESR about 4.26 ohms) and up to 2675 \( \text{m}^2/\text{g} \) specific surface area are performing much better than equivalent battery in terms of equaling and even having more specific energy. [21]
**Power**

Power is the rate of change of energy stored or extracted from a supercapacitor.

The SI unit is in watts.

Power extracted from a battery is govern by

\[ P = I \cdot V \]

\[ = \frac{V^2}{R_i} \]

Where,

\[ R_i = \text{ESR (estimated series resistance of a supercapacitor)} \]

Voltage is the in-built voltage in a supercapacitor.

The ESR of a supercapacitor is very small, hence the power that could be stored/extracted from supercapacitor is very higher.

During discharging period, power is been extracted from a supercapacitor. Also, during charging period, energy/time (power) is been stored in a supercapacitor.

Supercapacitor can be charged/discharged much quickly than conventional capacitor and equivalent battery. This makes supercapacitor, a very good, energy source in electric vehicle industry. It could be used to provide much power to a vehicle and is been used to harvest power from regenerative system in an electric vehicle.
**Maximum power**

The maximum power (or matched power) of a supercapacitor is the maximum rate of change of energy that can be extracted from a supercapacitor.

From electrical engineering literatures, consider Fig.1328.

\[ C = \text{Capacitance of a supercapacitor} \]
\[ V_C = \text{In-built voltage across supercapacitor} \]
\[ R_i = \text{Internal resistance of the supercapacitor} \]
\[ R_o = \text{Equivalent load resistance of the external load} \]
\[ I = \text{Current flowing through the load} \]

From Fig.
\[ I = \frac{V_C}{R_i + R_O} \]  
\[(39)\]

So,

\[ V_i = \frac{R_i * V_C}{R_i + R_O} \]  
Voltage across \( R_i \)

But,

\[ P_i = I * V_i \]

So,

\[ P_i = \left( \frac{V_C}{R_i + R_O} \right) * \left( \frac{R_i * V_C}{R_i + R_O} \right) \]

For maximum power transfer from source to the load.

\[ R_i = R_O \]  
\[(40)\]

So,

\[ P_{iMAX} = \frac{V_C^2 * R_i}{(2 * R_i)^2} \]

\[ P_{iMAX} = \frac{V_C^2}{4 * R_i} \]  
\[(41)\]

From equation (41), it could be seen the maximum that can be delivered to a load at a constant capacitor’s voltage is determined by the internal resistance of the supercapacitor.

With a very small internal resistance supercapacitor, large power could be extracted from it.
However, according to international electro technical commission (IEC), the useable or efficient power of a supercapacitor is

\[ P_{IMAX} = \frac{V_C^2}{\theta + R_i} \]  

(42) [20]

**Specific power**

The specific power of a supercapacitor is the amount of power that can be extracted from it per its mass. The unit of specific power is W/Kg (watts per kilogram)

\[ S_P = \frac{V_C^2}{R_i \cdot g} \]  

(43)

The maximum specific power \( (S_{PMAX}) \) that can be extracted from a supercapacitor is:

\[ S_{PMAX} = \frac{V_C^2}{4 \cdot R_i \cdot g} \]  

(44)

Where,

\[ S_P = \text{Specific power of a supercapacitor} \]

\[ g = \text{Mass of the supercapacitor} \]

\[ R_i = \text{Internal resistance of a supercapacitor} \]

\[ S_{PMAX} = \text{Maximum specific power} \]

With a very small internal resistance, supercapacitor has a very high specific power than its equivalent conventional capacitor and a battery.
Fig.3.29 [22] below, compares power density vs energy density of a supercapacitor and different types of battery.

Power Density vs Energy Density

Fig.3.29. Power density vs Energy density of supercapacitor and batteries [22]

From Fig.3.29, it could be seen that supercapacitor has higher power density than all the types of battery.

However, for now, it has low energy density than the batteries.
Fig.3.30 shows the power density and energy density of supercapacitor when compare with other sources of energy namely: fuel cell and battery.

From the fig.3.30, supercapacitor has higher power density than both fuel cell and batteries. It does however, has less energy density than both fuel cell and batteries.
Life time.

Supercapacitor as an electrochemical high-density capacitor, unlike a battery, does not rely on chemical changes within it, hence its lifetime is mostly depended on the rate of evaporation of electrode’s materials/minerals. The rate of evaporation is a function of supercapacitor’s temperature, load current, cycle frequency, and applied voltage.

The temperature under which a supercapacitor operates could affect the life time of the capacitor. Operating a supercapacitor at high extreme temperature will reduce its life time. But, increase temperature may improve its performance. Efficiency of a supercapacitor might be enhanced with slight increase in its operating temperature.

Conversely, operating supercapacitor at a low temperature will increases the internal resistant of capacitor hence it decreases the power can be extracted from a supercapacitor. However, operating a supercapacitor at a moderately low temperature might increase the life time of the supercapacitor.

The maximum temperature rating of a supercapacitor must not be exceeded. Fig.3.31 [22] shows how temperature can affect both the capacitance and internal resistance of a supercapacitor.
Fig. 3.31. Supercapacitor’s performance with temperature. [22]

It can be seen from Fig. 3.31, that capacitance of a capacitor does not substantially change with changes in temperature. However, the internal resistance of a supercapacitor does change with changes in temperature, at a high temperature, the internal resistance of a supercapacitor decreases but, increases at a lower temperature.

With adequate temperature control system, a supercapacitor network, could last between 10-15 years. This a great advantage over a battery which has a less life time than a supercapacitor.

Fig. 3.32 [25], shows the effect of temperature on both the life time and voltage output of a supercapacitor.
It can be seen from Fig.3.32 that at a higher temperature, the voltage output from a supercapacitor is high as compared to lower voltage output at low temperature. It is also seen that operating a supercapacitor at higher temperature could reduce the life time of supercapacitor.

**Estimation of the life time of a supercapacitor**

Consider the equation (45) below. This could be used to determine the life time of a supercapacitor for any temperature.

\[ L_t = L_g \ast 2^{(T_m - T_p)/10} \]  \hspace{1cm} (45) [25]

Where,
Lₜ = Life time to be determined.

Lₘ = Specified life time for a supercapacitor by the manufacturer.

Tₘ = Maximum temperature of a supercapacitor

Tₚ = Temperature which supercapacitor will operate.

With above equation, the lifetime at any temperature can be determined for a supercapacitor.

Also, it is well documented that the lifetime of a supercapacitor depends on the applied voltage. The rate by which gas is develop in the mineral electrodes depends on the applied voltage. With smaller the voltage, less gas will be developed at the electrodes. Hence less evaporation gas and longer lifetime of a supercapacitor.

**Charging time and discharge profile**

In hybrid electric vehicle, during the boost time, a large amount of power is needed in short period of time. Knowing the power needed, the time it will take to fully extract the power from a supercapacitor can be calculated using the following governing equation.

Energy that can be stored or extracted from a supercapacitor is governed by:

\[ W = \frac{1}{2} C(v_c) v_c^2 \]

But,

\[ W = P \cdot t \]

By equating equations (46) and (47) and rearranging the variables,
If $v_c = V_{max} - V_{min}$

Then,

$$t = \frac{1}{2 \pi P} C(v_c) * v_c^2$$  \hspace{1cm} (48)

From equation (49), knowing the power needed, the time it will take battery discharge from maximum charge to minimum charge could be determined.

Also, the constant current that will be required to during regenerative process (harvesting power from vehicle from brake) could be determined by:

From equation (29)

$$Q = C(v_c) * v_c$$

During extracting power from the brake –ve current is needed (charging of supercapacitor).

-ve $Q = - C(v_c) * v_c$

But,

-ve $Q = - t \cdot I$

Then,

$$- I \cdot t = - C(v_c) * v_c$$

$$t = - \frac{C(v_c) \cdot v_c}{I}$$

$$v_c = V_{max} - V_{min}$$

Then,
\[ t = C(v_c) \cdot \frac{V_{\min} - V_{\max}}{I} \quad (50) \]

So, from equation (50), knowing the constant current that will be needed to charge supercapacitor from minimum value to maximum value can be estimated.

From equation (50), maximum discharged time, when minimum voltage tends to zero is:

\[ t_{\text{max}} = |t| = C(v_c) \cdot \frac{V_{\max}}{I} \]

Where, \( v_c = V_{\max} \)

So,

\[ t_{\text{max}} = \frac{1}{I} (C_0 \cdot v_c + k_c \cdot v_c^2) \quad (51) \]

Where,

\[ t_{\text{max}} = \text{Maximum discharge time.} \]

**Modeling of a supercapacitor**

Simple model for supercapacitor when subjected to instantaneous applied voltage could be deduced from Fig and equation previously discussed.

From equations (30) and (31), the state space equations for supercapacitor model been presented are as follow:

\[ \frac{dv_c}{dt} = \frac{i_c}{C_0 + 2 \cdot k_c \cdot v_c} \]

And

\[ V_T(t) = R \cdot i_c + v_c \]
These state space equations can be modeled as in Fig.3.33 [20]

Fig.3.33. Nonlinear model of a supercapacitor. [20]

Fig.3.33 is nonlinear simple model for a supercapacitor [20]. Simulation of this model could be very tasking because of nonlinearity of this model.

Both the circuit and the state space equations can be linearized by using both Taylor series expansion method and known linearization techniques for nonlinear state space equations.

Taylor series expansion for any series is:

\[
f(x) = f(x_0) + \left. \frac{df}{dx} \right|_{x_0} \Delta x + \left. \frac{d^2f}{dx^2} \right|_{x_0} \frac{\Delta x^2}{2} + \ldots + \left. \frac{1}{n!} \frac{d^n f}{dx^n} \right|_{v_{co}} \frac{\Delta x^n}{n!}
\]

(52) [16]

Where,

\[
\Delta x = X - x
\]

(53) [16]
Considering the higher order terms to be HOT and using only the lower order terms, then,

$$f(x) \approx f(x_0) + \frac{df}{dx} \bigg|_{x_0} \Delta x$$  \hspace{1cm} (54)

That is,

$$\frac{df}{dx} \bigg|_{x_0} \Delta x = f(x) - f(x_0)$$  \hspace{1cm} (55)

But,

$$\Delta f(x) = f(x) - f(x_0)$$  \hspace{1cm} (56)

Then,

$$\Delta f(x) = \frac{df}{dx} \bigg|_{x_0} \Delta x$$  \hspace{1cm} (57)

This is a linear equation.

Where,

$$\Delta x = \text{Small increment of variable } x.$$  \hspace{1cm} (58)

$$\Delta x = \text{Which is a perturb value of } x.$$  \hspace{1cm} (58)

In our equations (30) and (31), $df$ is a function of two variables ($i_c$ and $v_c$),

That is,

$$v_c = f(v_c, i_c)$$  \hspace{1cm} (60)

Whose output is also a function of two variables ($i_c$ and $v_c$),

$$v_T = g(v_c, i_c)$$  \hspace{1cm} (61)

Where,

$$v_c = \text{State variable.}$$

$$i_c = \text{Input}$$
\( v_T = \text{Output.} \)

So, Tylor series expansion for our function with two variables \((i_c \text{ and } v_c)\),

\[
\Delta f(v_c, i_c) = \frac{df}{d(v_c, i_c)} \bigg|_{v_c0, i_c0} \Delta(v_c, i_c)
\]  
(62)

Are,

\[
\Delta f(i_c) = \frac{df}{d(i_c)} \bigg|_{i_c0} \Delta i_c
\]  
(63)

\[
\Delta f(v_c) = \frac{df}{d(v_c)} \bigg|_{v_c0} \Delta v_c
\]  
(64)

And,

\( \Delta v_c = \Delta v_c \)  
(65)

\( \Delta i_c = \Delta i_c \)  
(66)

\( \Delta v_T = \Delta v_T \)  
(67)

The state space linearized solutions to equations (30) and (31) are of:

\[
\tilde{v}_c = A*\tilde{v}_c + B*\tilde{i}_c
\]  
(68) [16]
\[
\v = C\hat{v}_c + E\hat{i}_c 
\]

(69) [16]

Here,

From equations (63), (64), (68), and (69)

\[
A = \frac{df}{dv_c}\bigg|_{v_{co}} = -2k_c i_c \left( \frac{1}{c_0 + 2k_c v_c} \right)^2 
\]

\[
B = \frac{df}{di_c}\bigg|_{i_{co}} = \frac{1}{c_0 + 2k_c v_c} 
\]

\[
C = \frac{dg}{dv_c}\bigg|_{v_{co}} = 1 
\]

\[
E = \frac{dg}{di_c}\bigg|_{i_{co}} = R 
\]

So,

\[
\hat{v}_c = -2k_c i_c \left( \frac{1}{c_0 + 2k_c v_c} \right)^2 \hat{v}_c + \frac{1}{c_0 + 2k_c v_c} \hat{i}_c 
\]

(70)

\[
\hat{v}_T = \hat{v}_c + R\hat{i}_c 
\]

(71)

By applying Laplace transform to equations (70) and (71),

\[
s * v_c(s) = \frac{1}{c_0 + 2k_c v_c} i_c(s) - \left( \frac{2k_c i_c}{c_0 + 2k_c v_c} \right)^2 * v_c(s) 
\]

(72)
\[ v_T(s) = i_c(s)R + v_c(s) \]  \hspace{1cm} (71)

By rearranging equation (72), the new linear equations for supercapacitor model are:

\[ v_c(s) = \frac{C_0 + 2k_cV_c}{s(C_0 + 2k_cV_c)^2 + 2i_cR + k_c} \times i_c(s) \]  \hspace{1cm} (73)

Let,

\[ CP(s) = \frac{C_0 + 2k_cV_c}{s(C_0 + 2k_cV_c)^2 + 2i_cR + k_c} \]

So,

Equation (73) becomes

\[ v_c(s) = CP(S) \times i_c(s) \]  \hspace{1cm} (74)

\[ v_T(s) = i_c(s)R + v_c(s) \]  \hspace{1cm} (75)

The linear model of a supercapacitor is then as shown in Fig. 1.34. [4]

Fig. 3.34. Linear model of a supercapacitor. [20]
Fuel Cell is an electrochemical device/system that convert hydrogen content of fuel to electricity cleanly and efficiently [52]. For this research, hydrogen is the fuel of the fuel cell been considered.

In its simplest form as shown Fig.4.1 [52], fuel cell consists of two electrodes: Anode (-ve) and Cathode (+ve). An electrolyte is sandwiched between these electrodes. Fuel (Hydrogen) is fed into the Anode and Air (Oxygen) is fed into the cathode. Catalyst (Platinum) at Anode side is used to split the hydrogen molecules into electrons and protons. The protons so produced passed through the electrolyte into the Cathode. The electrons are blocked by the electrolyte, but it will pass through an externally connected electrical circuit and return to the cathode. The passage of electrons through the electrical circuit will result into electricity. Returned electrons at the cathode will chemically be combined with protons and oxygen to form water and heat.

With hydrogen as the fuel, the end products of this process are electricity, water, and heat. This is close to zero carbon emission.
A single fuel cell produces less than 1 volt of DC electricity [53]. Because of the lower amount of voltage obtainable from a cell of fuel cell, cells are usually stacked together to produce higher value of voltage and power. Also, to increase the amount of current obtainable from a cell of fuel cell, cells are combined in parallel to produce high current value. However, the value of power generated depends on cell size, cell numbers, operating temperature, pressure of hydrogen and air fed into the electrodes.

Beside fuel cell unit, there are other equipments that are needed to enhance the performance of a fuel cell unit. Some of the equipments can include:

- **Humidifier**: The electrolyte been used in most of fuel cell do not work when they are dry hence, a humidifier is needed to make air intake is not dry. This increase the efficiency of a fuel cell unit.

- **Compressor**: The efficiency of a fuel cell unit is greatly enhanced by increasing the pressure of the reactant gases. Compressor is then been used to increase the pressure within fuel cell unit.

- **Depending on the type of fuel use in a fuel cell, the fuel needs to be converted to a useable one. For the fuel cell (PEM) we are considering, hydrogen is the fuel that PEM uses. To use any other hydrocarbon in PEM, this hydrocarbon material needs to be converted into hydrogen. A fuel processor like a reformer unit is needed.**

Eventhough, fuel cells are electrochemical equipments, they work by been fed constantly with hydrogen and oxygen. Other electrochemical unit like battery has its chemicals content already present in the battery. Fuel cells can produce electricity continuously for as long as fuel and oxygen are supplied.
Fuel cells provide the following advantages and disadvantages over other energy conversion systems:

- Operates quietly when compared to ICE.
- Do not run down (compared to Battery and Supercapacitors. It produces electricity if hydrogen and air are supplied to the electrodes.
- Higher specific energy than battery and supercapacitor.
• Longer lifespan than battery.
• High power density than battery
• Its fuel has less weight/volume when compare to other fuel oil types
• Higher durability.
• Higher efficiencies (compared to internal combustion engine- ICE). It converts chemical energy into electrical energy efficiently (about 60% compare to 33% for ICE) [53].
• Produces near zero carbon emission. Hence no air pollution (compared to ICE).

However,
• Fuel (Hydrogen) very expensive than petrol in ICE.
• Hydrogen has low energy density than petrol in ICE.

Fuel Cell are classified by the type of Electrolyte they use.

There are basically 7 classifications of fuel cell.

They are:

i) Polymer Electrolyte Membrane (PEM).
ii) Direct Methanol (DMFCs).
iii) Alkaline (AFCs).
iv) Phosphoric Acid (PAFCs).
v) Molten Carbonate (MCFCs).
vi) Solid Oxide (SOFCs).

vii) Reversible (RFCs).

In this research, PEM will be considered.
A brief discussion will be made on other types of fuel cell.

**Alkali fuel cells (AFCs)**

Alkali fuel cells operate on compressed hydrogen and oxygen at the anode and cathode respectively. The old type of AFCs uses solution of potassium hydroxide (KOH) in water as their electrolyte. The newer type of AFCs uses polymer membrane as its electrolyte. These newer fuel cells are closely related to conventional PEM fuel cells, except that they use an alkaline membrane instead of an acid membrane. The high performance of AFCs is due to the rate at which electro-chemical reactions take place in the cell. Efficiency is about 70 percent, and operating temperature is 300 to 400 degrees F. Cell output ranges from 300 watts (W) to 5 kilowatts (kW) [54].

The initial susceptibility of carbon monoxide (CO\textsubscript{2}) poison in AFCs has been reduce by changing from liquid form of electrolyte to solid membrane. Alkaline membrane fuel cells (AMFCs) address these concerns and have lower susceptibility to CO\textsubscript{2} poisoning than liquid-electrolyte AFCs do. Since the lower level poison effect of CO\textsubscript{2} still affects AMFs performance, AMFCs still lag in performance and durability than PEMFCs. AMFs are widely used in the U.S. space program to produce electrical energy and water on-board spacecraft. AMFCs are being considered for applications in the W to kW scale.

Challenges for AMFCs include tolerance to carbon dioxide, membrane conductivity and durability, higher temperature operation, water management, power density, and anode
electrocatalysis [54]. Also, the materials (pure hydrogen and platinum electrode catalysts) needed for its operation are presently very expensive.

**Molten Carbonate fuel cells (MCFCs)**

Molten carbonate fuel cells (MCFCs) use high-temperature compounds of salt mixture of molten carbonates (CO$_3$) as their electrolyte. They operate at high temperatures up to 1,200°F. Operating at temperature, allows MCFCs to use non-precious metals as catalysts at the anode and cathode, thereby, reducing costs. Efficiency ranges from 60 to 80 percent [54]. High temperature limits damage from carbon monoxide "poisoning" of the cell and waste heat can be recycled to make additional electricity. But the high temperature also limits the materials and safe uses of MCFCs. Also, carbonate ions from the electrolyte are used up in the reactions, making it necessary to inject carbon dioxide to compensate [53].

**Phosphoric Acid fuel cell (PAFCs)**

Phosphoric Acid fuel cells (PAFCs) use phosphoric acid as the electrolyte. PAFCs are less susceptible to carbon monoxide (CO$_2$) poisoning. Carbon monoxide binds to the platinum catalyst at the anode, decreasing the fuel cell's efficiency. Efficiency ranges from 40 to 80 percent, and operating temperature is 300 to 400 degrees F. PAFCs tolerate a carbon
monoxide concentration of about 1.5 percent, which broadens the choice of fuels they can use. If gasoline is used, the sulfur must be removed. Platinum electrode-catalysts are needed, and internal parts must be able to withstand the corrosive acid.

Operating PAFC only by itself, its efficiency is only slightly more than that of combustion-based power plants, which typically operate at around 33% efficiency. At the same weight and volume, PAFCs are less powerful than other fuel cells. Also, they are typically large and heavy than other fuel cells and more expensive.

**Solid Oxide fuel cells (SOFCs)**

Solid oxide fuel cells use a hard, non-porous ceramic compound of metal oxides (chemically, O₂) as electrolyte. They are operating at temperatures of about 1,800 degrees F. At such high temperatures a reformer is not required to extract hydrogen from the fuel, and waste heat can be recycled to make additional electricity. Because they operate at high temperature, it allows SOFCs to use non-precious metals as catalysts at the anode and cathode, thereby, reducing costs. They are less susceptible to both carbon monoxide (CO₂) poisoning and Sulphur. This property allows SOFCs to use natural gas, biogas, and gases made from coal. Efficiency ranges from 60 to 80 percent. Cells output is up to 100 kW.

However, high-temperature operation has disadvantages. It results in a slow startup and requires significant thermal shielding to retain heat and protect personnel, which may be acceptable for utility applications but not for transportation. Also, high temperature limits
applications of SOFC units and they tend to be rather large. While solid electrolytes cannot leak, they can crack.

**Direct Methanol (DMFCs).**

Direct Methanol (DMFCs) are powered by mixture of water and pure methanol such ethanol. Unlike other fuel cells, DMFCs use methanol directly at their anode [53]. Methanol has a higher energy density than hydrogen. Because methanol is liquid, it is easier to transport and supply to different locations using presently available public and private transport systems. DMFCs are often used to provide power for small applications.

**Reversible fuel cells**

Reversible fuel cells produce electricity from hydrogen and oxygen and generate heat and water as byproducts, just like other fuel cells. However, reversible fuel cell systems can also use electricity from solar power, wind power, or other sources to split water into oxygen and hydrogen fuel through a process called electrolysis. Reversible fuel cells can provide power when needed, but during times of high-power production from other technologies (such as when high winds lead to an excess of available wind power), reversible fuel cells can store the excess energy in the form of hydrogen. This energy storage capability could be a key enabler for intermittent renewable energy technologies. [53]
Proton Exchange Membrane fuel cells (PEMFCs)

Proton Exchange Membrane fuel cells work with a solid polymer electrolyte membrane: a thin, permeable sheet. Because of this, it is also known as polymer electrolyte membrane fuel cells. They also use porous carbon electrodes containing a platinum or platinum alloy catalyst. They need only hydrogen, oxygen from the air, and water to operate. They are typically fueled with pure hydrogen supplied from storage tanks. PEM fuel cells operate at relatively low temperatures, around 176°F. Low-temperature operation allows them to start quickly (less warm-up time) and results in less wear on system components, resulting in better durability. Efficiency is about 40 to 50 percent. Cell outputs generally range from 50 to 250 kW. The solid, flexible electrolyte will not leak or crack [54].

It requires expensive platinum be used to separate the hydrogen's electrons and protons. This increases system cost. The platinum catalyst is also extremely sensitive to carbon monoxide poisoning.

PEM fuel cells are used primarily for transportation applications and some stationary applications. Due to their fast startup time and favorable power-to-weight ratio, PEM fuel cells are particularly suitable for use in hybrid electric vehicle.

Fig.4.2. [52] shows a typical PEMFCs and its mode of operation. Electrolyte is solid, so it does not evaporate easily as compared to battery.
Fig. 4.2. Simple proton exchange membrane fuel cells’ operations. [52]
Chapter 5

SWITCHED-MODE DC-DC CONVERTER

A converter (step-down or/and step-up converter) is a DC-to-DC power electronics system which steps down or/and up voltage (steps down or/and up current) from its input (sources) to power a load (output). It is a voltage/energy regulation power electronics circuit/system. It is simple in operation, flexible in design, efficient in performance, and generally durable.

The converters described in this thesis are referred to as switched-mode dc-dc converters. A switched-mode dc-dc converter is a better alternative to linear dc-dc converter. It is more efficient than linear dc-dc converter that operates like a linear variable resistor (its transistor operates in the linear region of transistor’s voltage-current characteristic curve). Linear dc-dc converters are more susceptible to power loss hence less efficient than switched-mode converters [48].

DC-DC Converter can be classified as a non-isolated, isolated, multi-input, bidirectional dc to dc converter
Non-isolated converter

A converter system is said to be non-isolated if it is a transformer less converter. In this type of system, a transformer is not used to isolate any part of the system. With no transformer, it makes the system to weigh less. This is one of the major advantages of non-isolated converters.

A non-isolated converter, in its simplex form, consists of an energy storage element (inductor(s) and capacitor(s)) and active electric switching component(s) (transistor(s) and diode(s)).

Depending on the technique used (switched capacitor, switched inductor, voltage multiplier, and/or multi-input), a dc-dc converter can be classified as:

- Buck converter
- Boost converter
- Buck Boost converter
- Bidirectional converter

Buck Converter

A DC to DC converter steps down voltage from its input (source) to power an output load [46]. It basically consists of input source (voltage port) and an output current port linked to the
load to be powered. With the stepping down of voltage from the source, current is then enhanced at the output port.

In a buck converter system, energy is being transferred from the input voltage port to the output current port. This a one-way energy transfer. This is one of the disadvantages of a conventional unidirectional dc to dc converter.

A simple designed buck converter is shown in Fig.5.1. [48]
Fig. 5.1a. shows a simple designed buck converter while Fig. 5.1b and Fig. 5.1c. show mode of operation when pulse input to the transistor is “1” and when input to transistor is “0” respectively. Fig. 5.1d. are the waveform of different parameters (pulse input q to the transistor, diode voltage \( v_d \), inductor voltage \( v_L \), inductor ripple current \( i_{\text{ripple}} \), inductor current \( i_L \), and input current \( i_{in} \) of the system. [48]

Transistor in the circuit acts like a switch. It operates in either the saturation region (ON-mode) or cut-off region (OFF-mode) of transistor’s characteristic curve.

**Analysis of the circuit.**

In analyzing Fig5.1a, both state space and averaging theorems shall be used to obtain equations of the system when the transistor is on and when it is off.

When the transistor is ON, it acts like close switch.

So,

\[
\frac{di_L}{dt} L = V_S - V_O \quad (1)
\]

\[
\frac{di_L}{dt} = \frac{V_S-V_O}{L} \quad (2)
\]

Applying weight theorem at ON state,

\[
DT \cdot \frac{di_L}{dt} = \left(\frac{V_S-V_O}{L}\right) \cdot DT \quad (3)
\]

When the transistor is OFF, it acts like open switch.

So,

\[
\frac{di_L}{dt} L = -V_O \quad (4)
\]
\[
\frac{di_L}{dt} = \frac{-V_O}{L} \tag{5}
\]

Applying weight theorem at OFF state

\[(1 - D)T \frac{di_L}{dt} = \left(\frac{-V_O}{L}\right) * (1 - D)T \tag{6}\]

Applying weighted sum to equations (3) and (6)

\[(1 - D)T \frac{di_L}{dt} + DT \frac{di_L}{dt} = \left(\frac{V_S - V_O}{L}\right) * DT + \left(\frac{-V_O}{L}\right) * (1 - D)T\]

By rearranging terms,

\[T \frac{di_L}{dt} = DT * \left(\frac{V_S - V_O/D}{L}\right) \tag{7}\]

By averaging equation (7)

\[\frac{di_L}{dt} = D * \left(\frac{V_S - V_O/D}{L}\right)\]

As

\[\frac{di_L}{dt} \rightarrow 0 \quad \text{(At steady state)}\]

\[D * \left(\frac{V_S - V_O/D}{L}\right) \approx 0\]

So,

\[V_O = V_S * D \tag{8}\]

Where,

D = Duty cycle of the system.
**Boost converter**

This a type of DC to DC converter that steps up voltage from its input (source) to power an output load. It consists of input source (current port) and a higher voltage output port linked to the load. With the stepping up of voltage from the source (lower voltage but higher current input), current is then reduced at the output port but with higher voltage. In a boost converter system energy is being transferred from the input current port to the output voltage port.

This also a one-way energy transfer.

Fig. 5.2. [48] shows a simple boost converter with its operational waveform.
Fig 5.2. Simple boost converter with its operational waveforms. [48]

Where $V_O$ is the output voltage across the load.
$V_{in}$ is the input voltage at the current port

Where “→” is direction of energy/power flow.

Also, Fig.5.2a can be analyzed by applying both state space and averaging theorems.

The equations of the system when the transistor is on and when it is off are:

When the transistor is ON, it acts like a close switch.

\[ \frac{di}{dt}L = V_S \quad (9) \]
\[ \frac{di}{dt} = \frac{V_S}{L} \quad (10) \]

Applying weight theorem at ON state,

\[ DT \cdot \frac{di}{dt} = \frac{V_S}{L} \cdot DT \quad (11) \]

When the transistor is OFF, it acts like a close switch.

Therefore,

\[ \frac{di}{dt}L = V_S - V_O \quad (12) \]
\[ \frac{di}{dt} = \frac{V_S-V_O}{L} \quad (13) \]

Applying weight theorem at OFF state,

\[ (1 - D) \cdot T \cdot \frac{di}{dt} = \left( \frac{V_S-V_O}{L} \right) \cdot (1 - D)T \quad (14) \]

Applying weighted sum to equations (11) and (14)
\[ DT \cdot \frac{dI_L}{dt} + (1 - D) \cdot T \cdot \frac{dI_L}{dt} = \frac{V_S}{L} \cdot DT + \left(\frac{V_S - V_O}{L}\right) \cdot (1 - D)T \]  

(15)

By averaging and rearranging equation (15)

\[ \frac{dI_L}{dt} = \frac{V_S}{L} - \left(\frac{V_O}{L}\right) \cdot (1 - D)T \]

As

\[ \frac{dI_L}{dt} \to 0 \quad \text{(At steady state)} \]

\[ \frac{V_S}{L} - \left(\frac{V_O}{L}\right) \cdot (1 - D)T \approx 0 \]

Therefore,

\[ V_O = \frac{V_S}{1-D} \]  

(16)

Where,

\[ D = \text{Duty cycle of the system.} \]

\[ T = \text{Period.} \]

**Buck Boost converter**

This is a type of a dc-dc converter that combines the functions and characteristic of both buck and boost converter.

These two functions can only be done one at a time.

It is also unidirectional dc-dc converter.

The directional of the energy flow is determined by:

- Direction of the current in inductor
• The value of the duty cycle from the pulse wave modulator into the input of the switching device.

Simple Buck Boost converter is as shown in Fig.5.3. [48]

![Simple Buck Boost converter with its operational waveforms](image)

When the transistor is ON, it acts like close switch

\[
\frac{di_L}{dt} L = V_S \quad (17)
\]

\[
\frac{di_L}{dt} = \frac{V_S}{L} \quad (18)
\]
Applying weight theorem at ON state

\[ DT \cdot \frac{di_L}{dt} = \frac{V_S}{L} \cdot DT \]  \hspace{1cm} (19)

When the transistor is OFF, it acts like close switch

\[ \frac{di_L}{dt} L = V_O \]  \hspace{1cm} (20)

\[ \frac{di_L}{dt} = \frac{V_O}{L} \]  \hspace{1cm} (21)

Applying weight theorem at OFF state

\[ (1 - D)T \cdot \frac{di_L}{dt} = \frac{V_O}{L} \cdot (1 - D)T \]  \hspace{1cm} (22)

Applying weighted sum to equations (19) and (22)

\[ DT \cdot \frac{di_L}{dt} + (1 - D) \cdot T \cdot \frac{di_L}{dt} = \frac{V_O}{L} \cdot (1 - D)T + \frac{V_S}{L} \cdot DT \]  \hspace{1cm} (23)

By averaging and rearranging equation (23)

\[ \frac{di_L}{dt} = V_O + V_S \cdot \frac{D}{1 - D} \]

As

\[ \frac{di_L}{dt} \to 0 \]  \hspace{1cm} (At steady state)

\[ V_O + V_S \cdot \frac{D}{1 - D} \approx 0 \]

Therefore,

\[ V_O = -V_S \cdot \frac{D}{1 - D} \]  \hspace{1cm} (24)
From equation (24), it can be seen that the value of the duty cycle will determine either if buck-boost converter is operating as a buck converter or as a boost converter. At value \( D > 0.5 \), the output voltage will be larger than input voltage hence, it is operating as a boost converter. However, when \( D < 0.5 \), the output is less than input hence, it is operating as a buck converter.

A unidirectional DC-DC converter has the following disadvantages.

- Power/energy flows is one direction
- Could enter into discontinuous mode
- Discontinuous mode could stress and needs higher rating for the passive components.
- Discontinuous mode could result to system noise and EMI (Electromagnetic interference)

**Bidirectional DC TO DC converter**

By simply replacing the diode in switching unit of both buck and boost early discussed with a controllable switch, a bidirectional converter is formed. Diodes do not allow the flowing of current in both directions.

To be simply put, a bidirectional dc-dc converter allows energy/power to flow in both direction of a dc to dc converter system. It does this by using two (2) or more switching elements. These controllable switching elements could be a Metal Oxide Semiconductor Field
Effect Transistor (MOSFET), or Bipolar Junction Transistor (BJT) or Thyristor (family of silicon-controlled rectifier (SCR)).

A simple designed bidirectional transistor could be as shown in Fig.5.4. [48]

Simplify schematic diagram of Fig.5.4a. is shown in Fig.5.4b

This simple design consists of 2 MOSFET, 2 Diodes, an inductor, a capacitor, and an inverter. The inverter is used to generate a complimentary pulse signal from pulse width modulator (PWM). The 2 complimentary pulse signals (q and q') are feed into the input of the 2 transistors. When q=1, q' = (1-q) = 0

These complimentary pulse signals will force only one of the transistors to operate one at a time.

This type of bidirectional converter (Fig.5. 4a) is called buck boost bidirectional converter.
The direction of the current in the inductor, determines the direction of power/energy flow. Also, it determines if the converter is operating in buck or boost bidirectional mode.

Left MOSFET and diode with inductor are components of buck converter

Right MOSFET and diode with inductor are components of boost converter.

With either direction of \( i_L \) (negative (flowing in the opposite direction shown) or positive), by applying a pulse signal to the transistor will make either the left transistor (buck mode) or the right transistor (boost mode).

As can be seen above, the direction of the energy flow is either to the right or to the left i.e. bidirectional.

Bidirectional dc to dc bidirectional has the following advantages over unidirectional converter.

- Eliminate discontinuous condition in our system
- Reduces drastically noise and EMI in our system.
- Easily adaptable.

Bidirectional DC-DC converter finds a lot of use in:

- Alternative power source industries
- Electric vehicle industries
- Uninterruptible systems
- DC and AC motor control systems
Multi-input bidirectional DC-DC converters

A multi-input bidirectional dc to dc converter is broadly define as a bidirectional dc to dc converter that has multiple inputs and possibly one or more outputs [50].

A multi-input bidirectional converter allows power to flow both direction of the converter.

The topology of a multi-input bidirectional converter depends on how the converter is configured.

With advent of bidirectional converters, a lot of topologies/configuration are available. Some are:

• Multi-input Buck-Boost bidirectional converter.
• Multi-input Buck Buck-Boost bidirectional converter
• Multi-input Boost Buck-Boost bidirectional converter
• Multi-input Buck-Buck bidirectional converter

As can be seen, each topology is defined by how the converter is configured. These different topologies make a multi input bidirectional dc to dc converter, a versatile and multi-functional electric device/system.

Multi-input bidirectional converter can be applied in various electrical applications. This could include:

• Plug in electric, hybrid electric. Such as a typical drive trains or in the inputs of the air compressor devices of the electric vehicles. One such application, in the drive train of
the combined energy storage unit of the hybrid electric vehicle. The block diagram of this case is shown in Fig. 5.5 [49] below.

- Fuel cell vehicles where two or more power sources are used. As shown in Fig. 6.6 [50] below.

**Fig. 5.5.** Simple control of drive train of an electric vehicle. [49]

Analysis of fig. 5:

This system consists of:

B: Battery

UC: Ultracapacitor / Supercapacitor
The bidirectional integrated buck boost – buck boost converter is taken as the double input dc-dc converter and is linked with an integrated inverter and induction motor drive. During the speed ramp up, the supercapacitor is discharged. A sensor acts when the voltage of the supercapacitor goes below the minimum voltage and then the battery takes over. When the speed is constant, the necessary power is supplied alone by the battery. During the speed ramp down all the regenerated energy is used to charge the supercapacitor.

Fig. 5.6. Simple design configuration for both fuel cell and supercapacitor energy sourced electric car. [50]

The load can be the drivetrain of a hybrid electric vehicle.
**Inverter.**

An inverter is a power processing electronic device or system. Its basic function is to convert direct current (DC) to alternating current (AC).

Inverter does the opposite function of a rectifier. A rectifier converts alternating current to direct current.

The inverter does not generally produce any power; the power is provided by the DC/AC source (depending on the use) [56].

Due to new technology and greater interest in inverters, a bidirectional inverter that combines the functions of both unidirectional inverter and rectifier is now available.

There are many different types of inverter. Inverter can be classified according:

- Inverter’s output waveform. An inverter can produce a square wave, modified sine wave, pulsed sine wave, pulse width modulated wave (PWM) or sine wave depending on circuit design [57].
- Type of application is been used for.
- Its composition. It could be a combination of electrical and mechanical. It could be electrical only. In this thesis, only electronics type is being described.
- Its electrical and mechanical characteristics.

The input voltage, output voltage and frequency, and overall power handling depend on the design of the specific device or circuitry [56].

Generally, a sine wave output waveform from an inverter is desirable because many electrical products are engineered to work best with a sine wave AC power source. An inverter that has good smooth sinusoidal output does not generate distortions in the operation of other power electronics components/devices been used in conjunction with the inverter.
A none-pure sinusoidal inverter usually operates with less efficiently owing to the harmonics associated with a modified sine wave and produce a humming noise during operation [56]. This also affects the efficiency of the system as a whole. Therefore, pure sine wave inverters may provide significantly higher efficiency than modified sine wave inverters.

However, most AC motors will run on non-too-smooth sinusoidal inverters with an efficiency reduction. However, they may be quite noisy [59].

A power inverter will often have an overall power rating expressed in watts or kilowatts. This describes the power that will be available to the device the inverter is driving and, indirectly,

**Bidirectional inverter**

Bidirectional inverter combines the functions of both unidirectional inverter and rectifier. Its design could be such that it combines the functions of an inverter, a rectifier, and a converter (either buck, boost, or both). These different devices are house as a single unit. These are the devices that are used by in both EVs and hybrids to manage their electric drive systems. Along with a built-in charge controller, this unit supplies current to the battery pack for recharging during regenerative braking and it also provides electricity to the motor/generator for vehicle propulsion [60]. Due to its level of utilization and inherent heat that can develop during operation, adequate cooling and ventilation are provided to keeping the components operational. For this reason, Bidirectional inverter installed in hybrid vehicles has its dedicated cooling systems, complete with pumps and radiators that are entirely independent from the engine's cooling system [61].
The inverter in a bi-directional inverter accepts the high voltage from the traction battery and converts it typically in the three phase AC voltage suitable for electric traction motor. During the deceleration (regenerative breaking period) the motor absorbs the torque and provide the AC voltage to the inverter [62]. The inverter controls the amount of the energy generated by the motor and consequently the intensity of the braking. The harvested energy is applied back to the HV battery as a charge. The regenerative breaking improves significantly the vehicle efficiency and in turn the distance the vehicle can travel on a given battery charge [62].
Chapter 6

VEHICLE FORCES AND POWER EQUATIONS

Vehicle forces

For a vehicle to move/accelerate, its propulsion power must be equal or greater than the total opposing power created by opposing forces acting on the vehicle. The power or force delivered by the propulsion unit helps to overcome these opposing forces and its inertia. [33]. Considering fig.6.1 [33], forces acting on a vehicle in a motion can be classified into:

- Aerodynamic drag/force ($F_w$). This is the opposing force caused by the air around a moving vehicle.
- Rolling force ($F_r$). This a resistance caused by the friction between the vehicle’s tires and the travelling hard surfaces.
- Grading resistant ($F_g$) caused because of a vehicle climbing up-hill.
- Inertia or accelerating force ($F_a$). This must be overcome before a vehicle can accelerate. It is an opposing force caused by total mass of a moving vehicle and its moving parts.
The total opposing forces ($F_{OPP}$) on a typical vehicle is:

$$F_{OPP} = F_w + F_r + F_g + F_a$$  \hspace{1cm} (6.1)

Fig. 6.1. Forces acting on vehicle in motion.[33]

From the literatures, the equations governing these forces are briefly discussed here.

**Rolling force ($F_r$)**

Rolling force ($F_r$) is a resistance caused by the friction between the vehicle’s tires and the travelling hard surfaces.
\[ F_r = M \times g \times f_r \times \cos(\theta) \]  \hspace{1cm} (6.2)

Where,

- \( M = \text{Mass of vehicle} \) \hspace{1cm} Kg.
- \( g = \text{Acceleration due to gravity} \) \hspace{1cm} m \times s^{-2}
- \( \theta = \text{Road angle} \) \hspace{1cm} rad.
- \( f_r = \text{Rolling resistant coefficient} \)

**Aerodynamic drag/force (F_w)**

Aerodynamic drag/force (F_w) is an opposing force caused by the air around a moving vehicle. Aerodynamic drag is directly proportional to surrounding air density, frontal area of the vehicle, drag coefficient, and square of the vehicle’s velocity. Governing equation is:

\[ F_w = \frac{1}{2} \times \rho \times A_f \times C_D \times V^2 \]  \hspace{1cm} (6.3)

Where,

- \( \rho = \text{Air density} \) \hspace{1cm} Kg \times m^{-3}
- \( A_f = \text{Vehicle frontal area} \) \hspace{1cm} m^2
- \( C_D = \text{Drag coefficient} \)
- \( V = \text{Vehicle speed} \) \hspace{1cm} m \times s^{-1}
Grading resistant ($F_g$)

Grading resistant ($F_g$) is caused by vehicle climbing up-hill. This force is directly proportional the mass of the vehicle, gravitational force on the vehicle, and angle of inclination of the vehicle. Relationship between grading resistant and these three factors is:

$$F_g = M \times g \times \sin(\theta) \quad (6.4)$$

Where,

$M = \text{Mass of vehicle} \quad \text{Kg.}$

$g = \text{Acceleration due to gravity} \quad m \times s^{-2}$

$\theta = \text{Road angle} \quad \text{rad.}$

Inertia or accelerating force ($F_a$)

Inertia or accelerating force ($F_a$) is directly proportional to vehicle effective mass, moment of inertia of the moving parts, rolling radius of the wheels, and acceleration of the vehicle. The relationship is governed by following:

$$F_a = \left( M + \sum \frac{I_r}{r_r^2} \right) \times a \quad (6.5a)$$

$$M_{eff} = \left( M + \sum \frac{I_r}{r_r^2} \right) \text{ is referred as the effective mass of the vehicle}$$

So,

$$F_a = M_{eff} \times a$$
\[ F_a = F_{acc} + M \cdot a \] 

(6.5b)

Where,

\[ M = \text{Mass of vehicle} \quad \text{Kg.} \]

\[ a = \text{Vehicle Acceleration} \quad m \cdot s^{-2} \]

\[ \sum J_r = \text{Total inertia of all the rotating parts} \quad Kg \cdot m^2 \]

\[ r_r = \text{Rolling radius of the vehicle' styre} \quad m \]

\[ F_{acc} = \frac{\sum J_r}{r_r^2} \cdot a \]

In a moving vehicle, there are both large and small rotating parts. The small rotating parts include bearings and pulleys while large rotating parts include motors, driveline, and tire/wheel assembles.

Comparing the weight of the small rotating parts as compared to the large rotating parts, only the inertia for the large shall be considered.

The summation of the inertia for the larger rotating parts is:

\[ \sum J_r = 4 \cdot J_w + (J_{Dl} + J_M) \cdot N_T^2 \cdot N_F^2 \] 

(6.6)

Where,

\[ J_w = \text{Inertia for the wheels or tyres} \quad Kg \cdot m^2 \]

\[ J_{Dl} = \text{Inertia for the driveline} \quad Kg \cdot m^2 \]

\[ J_M = \text{Inertia fot the motor} \quad Kg \cdot m^2 \]

\[ N_T = \text{Transmission gear ratio} \]
\[ N_F = \text{Final drive gear ratio} \]

So,

\[ M_{eff} = \left( M + \frac{4 \cdot J_w + (J_{DL} + J_M) \cdot N_t^2 \cdot N_F^2}{r_r^2} \right) \]

And

\[ F_a = \left( M + \frac{4 \cdot J_w + (J_{DL} + J_M) \cdot N_t^2 \cdot N_F^2}{r_r^2} \right) \cdot a \] \hspace{1cm} (6.7)

Power required (\( P_{REQ} \)) by the vehicle to overcome the total opposing forces is:

\[ P_{REQ} = F_{OPP} \cdot V \]

The general dynamic equation for a vehicle in motion is:

\[ F_T = M_{eff} \cdot a + F_r + F_g + F_w \] \hspace{1cm} (6.8)

That is,

\[ M \cdot a = F_T - (F_r + F_w + F_g + F_{acc}) \]

By combining figures (6.2, 6.3, 6.4, and 6.5) above,

\[ M \cdot a = F_T - \left\{ M \cdot g \cdot f_r \cdot \cos(\theta) + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 + M \cdot g \cdot \sin(\theta) + \left( M + \frac{4 \cdot J_w + (J_{DL} + J_M) \cdot N_t^2 \cdot N_F^2}{r_r^2} \right) \cdot a \right\} \] \hspace{1cm} (6.9)

Where,
\( F_T = \text{Traction force of a moving vehicle} \)

\( F_r = \text{Rolling force of a vehicle in motion} \)

\( F_w = \text{Aerodynamic force of a vehicle in motion} \)

\( F_g = \text{Grading resistant of a vehicle in motion} \)

\( F_a = \text{Accellerating/inertia force of a vehicle in motion} \)

\( M = \text{Mass of the vehicle} \)

\( a = \text{Accelleration of a vehicle in motion} \)

In calculations in appendix (2), the moment of inertia for both final drive and transmission are combined as one. Most data sheet also combine the moment inertia of the two.

**Drivetrain power calculation**

According to electromechanical literatures, power train is the mechanism that transmit the drive from the engine of a vehicle to its axle [45]. It generates power and delivers it to the drive wheel in order to overcome the opposing forces acting on a moving vehicle.

In hybrid electric vehicle (HEV), its main electric motor is the engine of the vehicle. In HEV industry, electric motor is the hearth of power train of a vehicle.

The specification for rating the power train of an electric drive vehicle is determined by:

- Drive cycle under which the vehicle operates
- The power dynamic equation of a vehicle in motion
**Final drive**

The last part of the transmission is the final drive, which incorporates the differential and is sometimes called the differential. [45]

During cornering, they allow one road wheel to turn faster than the other. The differential has three functions: to turn the direction of drive through 90 degrees to the rear wheels; to allow either rear wheel to turn faster than the other when cornering; and to affect a final gear reduction. [45]

**Drive cycle**

A typical drive cycle of a hybrid electric vehicle is as shown in Fig.6.2 [33]

![Drive cycle of Hybrid vehicle](image)
The standard used for Northern Illinois University previous work in hybrid electric vehicle is based on the following [13]:

1. Rated vehicle speed is 60 mph
2. Maximum speed of vehicle is 80 mph
3. Acceleration from 0 to 60 mph should be equal or less than 20 seconds.
4. The speed of vehicle on non-flat road should be 50 mph
5. Grade of 6.5% is assumed on non-flat road
6. 1st cruise speed is 60 mph
7. 2nd cruise speed is the maximum speed (80 mph)

As shown in Fig.6.2, a drive cycle consists of:

- Starting point.
- Initial acceleration from 0 to 60 mph
- 1st cruising at rated vehicle speed
- Further acceleration at maximum speed (80 mph)
- 2nd cruising at maximum speed
- Retardation from maximum speed (80 mph) to 0
- End.

In this thesis, cruising with rated and maximum, is assumed to be done with 0% grade (flat road). Under cruising, the vehicle is going at constant velocity, hence, acceleration is zero.
The grade ability of vehicle is the grade angle that the vehicle can negotiate at a certain constant speed. Also, acceleration is defined at rated speed of a vehicle with no grade.

In electromechanical engineering, power is defined as product of force (torque) and velocity. Hybrid electric vehicle (HEV), been an electromechanical equipment, its power train rating, when in motion, is the product of its opposing force and the velocity of the vehicle.

**Power train equations.**

Power required ($P_{REQ}$) by the vehicle to overcome the total opposing forces is:

From equation (6.1),

$$F_{OPP} = F_w + F_r + F_g + F_a$$

But,

$$P_{REQ} = F_{OPP} \cdot V \tag{6.10}$$

Equating (6.1), (6.2), (6.3), (6.4) and (6.7) into equation (6.10),

Then,

$$P_{REQ} = \left\{ M \cdot g \cdot f_r \cdot \cos(\theta) + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 + M \cdot g \cdot \sin(\theta) + \left( M + \frac{4 \cdot J_w + (J_D + J_M) \cdot N_f^2 + N_e^2}{r_f^2} \right) \cdot a \right\} \cdot V \tag{6.11}$$
This will be the propulsion power required to overcome opposing power caused by Rolling force (F_r), Aerodynamic drag/force (F_w), Inertia or accelerating force (F_a), and Grading resistant (F_g).

The power required are:

\[
P_{NAC} = \left\{ M \cdot g \cdot f_r \cdot \cos(\theta) + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 + M \cdot g \cdot \sin(\theta) \right\} \cdot V
\]

(6.12)

\[
P_{NAC+NOGRD} = \left\{ M \cdot g \cdot f_r + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 \right\} \cdot V
\]

(6.13)

\[
P_{AC+NOGRD} = \left\{ M \cdot g \cdot f_r + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 + \left( M + \frac{4\cdot J_w + (J_{DI} + J_M) + N_T^2 + N_F^2}{r_f^2} \right) a \right\} \cdot V
\]

(6.14)

Where,

\[
P_{NAC+NOGRD} = \text{Power required without acceleration and no grading resistant}
\]

\[
P_{NAC} = \text{Power required without acceleration.}
\]

\[
P_{AC+NOGRD} = \text{Power required for acceleration without grading resistant}
\]

Above calculation is based on ideal situation: where the efficiencies of all the parts in the vehicle are not considered. However, ideal situation does not exist in a moving vehicle.
Efficiencies for the power train (motor) and large rolling parts in a moving vehicle need to be considered. In this thesis, the effect of small rotating parts was not considered, hence, their efficiencies will be ignored.

Efficiency ($\eta$) is defined as:

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

So,

$$\text{Power input} = \frac{\text{Power output}}{1000 \cdot \eta} \quad (6.15)$$

This power at Kilowatts.

Here, the expected power required, is the same as the input power. Propulsion power is the power out from the power train.

The total efficiency is the product of all the efficiencies under consideration.

That is,

$$\eta = \eta_M \cdot \eta_T \cdot \eta_F$$

Where,

$$\eta_M = \text{Efficiency of motor}$$

$$\eta_T = \text{Efficiency of transmission}$$

$$\eta_F = \text{Efficiency of final drive}$$

Equations now become,
\[ P_{NAC} = \left\{ M \cdot g \cdot f_r \cdot \cos(\theta) + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 + M \cdot g \cdot \sin(\theta) \right\} \frac{V}{1000 \cdot \eta_M \eta_T \eta_F} \]  
(6.16)

\[ P_{NAC+NOGRD} = \left\{ M \cdot g \cdot f_r \cdot \cos(\theta) + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 \right\} \frac{V}{1000 \cdot \eta_M \eta_T \eta_F} \]  
(6.17)

\[ P_{AC+NOGRD} = \left\{ M \cdot g \cdot f_r \cdot \cos(\theta) + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 + \left( M \cdot \frac{4 \cdot J_w + (J_D + J_M) \cdot N_T^2 + N_T^2}{r_T^2} \right) \right\} a \frac{V}{1000 \cdot \eta_M \eta_T \eta_F} \]  
(6.18)

Power obtained are in kilowatts (kW).
Chapter 7

MODEL FORMULATION

Northern Illinois University has data for works previously done on electric vehicle. Data are as shown in Tab.7.1.

Tab.7.1 Mechanical characteristics of a typical Hybrid electric vehicle. [38]

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>M</td>
<td>Kg</td>
</tr>
<tr>
<td>Frontal area</td>
<td>A</td>
<td>m²</td>
</tr>
<tr>
<td>Air density</td>
<td>ρₐ</td>
<td>-</td>
</tr>
<tr>
<td>Motor efficiency</td>
<td>ηₘ</td>
<td>%</td>
</tr>
<tr>
<td>Transmission efficiency</td>
<td>ηₜ</td>
<td>%</td>
</tr>
<tr>
<td>Final drive gear efficiency</td>
<td>ηₖ</td>
<td>%</td>
</tr>
<tr>
<td>Wheel mass</td>
<td>Mₜ</td>
<td>Kg</td>
</tr>
<tr>
<td>Tire mass</td>
<td>Mₜ</td>
<td>Kg</td>
</tr>
<tr>
<td>Rolling radius</td>
<td>rₜ</td>
<td>m</td>
</tr>
<tr>
<td>Wheel radius</td>
<td>rₚ</td>
<td>m</td>
</tr>
</tbody>
</table>

Continued on following page
Tab. 7.1 continued

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tire radius</td>
<td>( r_t )</td>
<td>m</td>
</tr>
<tr>
<td>Moment of inertia of motor and driveline</td>
<td>( J_M N_f^2 N_t^2 )</td>
<td>Kgm(^2)</td>
</tr>
<tr>
<td>Moment of inertia of a wheel/tire</td>
<td>( J_W )</td>
<td>Kgm(^2)</td>
</tr>
<tr>
<td>Rolling resistance coefficient</td>
<td>( f_r )</td>
<td>-</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>( C_d )</td>
<td>-</td>
</tr>
<tr>
<td>Transmission gear ratio</td>
<td>( N_T )</td>
<td>-</td>
</tr>
<tr>
<td>Final drive gear ratio</td>
<td>( N_F )</td>
<td>-</td>
</tr>
</tbody>
</table>

Tab. 7.2 Electrical characteristics of Lithium-ion Battery [39]

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>LITHIUM-ION BATTERY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specific energy</td>
<td>( S_E )</td>
<td>KWh/Kg</td>
</tr>
<tr>
<td>Specific power</td>
<td>( S_P )</td>
<td>KW/Kg</td>
</tr>
<tr>
<td>Specific density</td>
<td>( S_D )</td>
<td>KWh/L</td>
</tr>
<tr>
<td>Cost (specific)</td>
<td>( C_s )</td>
<td>$/Kg</td>
</tr>
<tr>
<td>Specific volume</td>
<td>( V_s )</td>
<td>L/Kg</td>
</tr>
</tbody>
</table>
Tab.7.3. Electrical characteristics of a supercapacitor BMOD0165 [30] [31]

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUPERCAPACITOR (BMOD0165)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specific energy</td>
<td>$S_E$</td>
<td>KWh/Kg</td>
</tr>
<tr>
<td>Specific power</td>
<td>$S_P$</td>
<td>KW/Kg</td>
</tr>
<tr>
<td>Specific density</td>
<td>$S_D$</td>
<td>KWh/L</td>
</tr>
<tr>
<td>Cost (specific)</td>
<td>$C_S$</td>
<td>$$/Kg</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$C$</td>
<td>F</td>
</tr>
<tr>
<td>Voltage (rated)</td>
<td></td>
<td>Volts</td>
</tr>
<tr>
<td>ESR</td>
<td></td>
<td>mΩ</td>
</tr>
<tr>
<td>Mass</td>
<td>$M$</td>
<td>Kg</td>
</tr>
<tr>
<td>Specific volume</td>
<td>$V_S$</td>
<td>L/Kg</td>
</tr>
</tbody>
</table>
Tab. 7.4  Electrical characteristics and cost estimate of a supercapacitor BCAP3400

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUPERCAPACITOR (BCAP3400)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specific energy</td>
<td>SE</td>
<td>KWh/Kg</td>
</tr>
<tr>
<td>Specific power</td>
<td>SP</td>
<td>KW/Kg</td>
</tr>
<tr>
<td>Specific density</td>
<td>SD</td>
<td>KWh/L</td>
</tr>
<tr>
<td>Cost (specific)</td>
<td>CS</td>
<td>$/Kg</td>
</tr>
<tr>
<td>Capacitance</td>
<td>C</td>
<td>F</td>
</tr>
<tr>
<td>Voltage (rated)</td>
<td></td>
<td>Volts</td>
</tr>
<tr>
<td>ESR</td>
<td></td>
<td>mΩ</td>
</tr>
<tr>
<td>Mass</td>
<td>M</td>
<td>Kg</td>
</tr>
<tr>
<td>Specific volume</td>
<td>Vs</td>
<td>L/Kg</td>
</tr>
</tbody>
</table>
Tab. 7.5 Electrical characteristics AC55 Electric motor. [40]

<table>
<thead>
<tr>
<th>Specifications</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Torque</td>
<td>240Nm</td>
</tr>
<tr>
<td>Maximum Current</td>
<td>250A rms</td>
</tr>
<tr>
<td>Continuous Torque</td>
<td>55Nm</td>
</tr>
<tr>
<td>Continuous Power</td>
<td>34kW</td>
</tr>
<tr>
<td>Peak Efficiency</td>
<td>93%</td>
</tr>
<tr>
<td>Motor Controller</td>
<td>UMOC440TF</td>
</tr>
<tr>
<td>Peak Electrical Power</td>
<td>78kW</td>
</tr>
<tr>
<td>At Voltage of</td>
<td>312 VDC</td>
</tr>
<tr>
<td>Nominal Speed</td>
<td>2.5k rpm</td>
</tr>
<tr>
<td>Maximum Speed</td>
<td>8k rpm</td>
</tr>
<tr>
<td>Weight</td>
<td>106.3kg</td>
</tr>
<tr>
<td>Diameter</td>
<td>343mm</td>
</tr>
<tr>
<td>Length*</td>
<td>432 to 447mm</td>
</tr>
</tbody>
</table>

*Depending on end bells and options

Fig. 7.1 Efficiency vs Torque of AC55 [40]
Fig. 7.2  Torque vs Speed of AC55 [40]

Tab. 7.6 Electrical characteristics and cost estimate of fuel cell. [39]

<table>
<thead>
<tr>
<th>Assumption Description</th>
<th>Units</th>
<th>2005</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Type</td>
<td>--</td>
<td>hydrogen</td>
<td>hydrogen</td>
</tr>
<tr>
<td>Fuel Cell Peak Efficiency</td>
<td>%</td>
<td>62.9</td>
<td>62.9</td>
</tr>
<tr>
<td>Fuel Cell Efficiency at 25% Power</td>
<td>%</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Fuel Cell Efficiency at Rated Power</td>
<td>%</td>
<td>53.6</td>
<td>53.6</td>
</tr>
<tr>
<td>Fuel Cell System Specific Power</td>
<td>W/kg</td>
<td>500</td>
<td>650</td>
</tr>
<tr>
<td>Fuel Cell System Power Density</td>
<td>W/L</td>
<td>500</td>
<td>650</td>
</tr>
<tr>
<td>Fuel Cell System Cost</td>
<td>$/kW</td>
<td>96</td>
<td>27</td>
</tr>
<tr>
<td>Fuel Cell System 10-90% Power</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transient Response Capability</td>
<td>s</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Tab. 7.7  Electrical characteristics and cost estimate: Cylinder of Hydrogen. [39]

```
<table>
<thead>
<tr>
<th>Assumption Description</th>
<th>Units</th>
<th>2005</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>H2 Storage Energy Density</td>
<td>kWh/L</td>
<td>1.2</td>
<td>1.5</td>
</tr>
<tr>
<td>H2 Storage Specific Energy</td>
<td>kWh/kg</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>H2 Storage Cost</td>
<td>$/kWh</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>
```

Tab. 7.8  Electrical characteristics and cost estimate of two sources of energy. [39]

```
<table>
<thead>
<tr>
<th>Assumption Description</th>
<th>Units</th>
<th>PbA</th>
<th>NiMH</th>
<th>Li-Ion</th>
<th>Ultra-capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Storage Energy Density</td>
<td>Wh/L</td>
<td>75</td>
<td>100</td>
<td>190</td>
<td>5</td>
</tr>
<tr>
<td>Energy Storage Specific Energy</td>
<td>Wh/kg</td>
<td>35</td>
<td>55</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>Energy Storage Power Density</td>
<td>W/L</td>
<td>1600</td>
<td>2000</td>
<td>2800</td>
<td>4500</td>
</tr>
<tr>
<td>Energy Storage Specific Power</td>
<td>W/kg</td>
<td>550</td>
<td>1000</td>
<td>1300</td>
<td>3500</td>
</tr>
<tr>
<td>Energy Storage Cost (power)</td>
<td>$/kW</td>
<td>$10.00</td>
<td>$40.00</td>
<td>$60.00</td>
<td>$15.00</td>
</tr>
</tbody>
</table>
```
Fig. 7.3 Volume (liter) vs Power (KW) of fuel cell [39]

Fig. 7.4 Cost ($) vs Power (KW) of fuel cell [39]
Tab. 7.9  Electrical characteristics and cost estimate: Cylinder of hydrogen. [39]

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>HYDROGEN TANK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specific volume</td>
<td>( V_S )</td>
<td>L/Kg</td>
</tr>
<tr>
<td>Specific cost</td>
<td>( C_S )</td>
<td>$/Kg</td>
</tr>
<tr>
<td>Storage energy density</td>
<td></td>
<td>KWh/L</td>
</tr>
<tr>
<td>Storage specific energy</td>
<td></td>
<td>KWh/Kg</td>
</tr>
</tbody>
</table>

**Power calculation.**

Using the data obtained from Tab.7.1, the maximum power obtainable from equations (6.16), (6.17), and (6.18) formulated in chapter 6 of this write-up are as calculated in Appendix (2).

From Appendix (2), power for all categories are:

- Under no acceleration with no grade using rated velocity = 17.931 KW
- Under no acceleration with no grade using maximum velocity = 17.931 KW
- Under no acceleration with grade using rated velocity = 37.774 KW
- Under acceleration with no grade using rated velocity = 114.242 KW
- Under acceleration with no grade using maximum velocity = 165.918 KW
Energy calculation

Thesis assumes that the hybrid electric vehicle under consideration should be able to cruise for five (5) hours at its rated speed of 60 mph.

Minimum energy needed for this is:

Energy required = Power needed during cruising * Time

= \(17.931 \text{ KW} \times 5 \text{ Hours}\)

= 89.655 \text{ KWh}.

Chapter 8

MATHEMATICAL MODEL FORMULATION

Power and energy profile.

In this thesis, we are proposing plug-in parallel configuration for our hybrid electric vehicle. Our three sources of energy are fuel cell, battery, and supercapacitor.

The power train configuration shown in Fig.8.1 is for the plug-in parallel hybrid vehicle been considered. The three sources of energy will be used, under certain condition, to contribute both power and energy to the power train.

Fig.8.1 Power train configuration of proposed hybrid electric vehicle
Aggregate of how these sources of energy are expected to interact with each other during a driving cycle is:

- **Start-up**: Both fuel cell and battery shall provide both the energy and power needed. Fuel-cell power does not get to the maximum at start-up (needs time to warm up) so battery shall be needed to augment both energy and power needed for start-up.

- **Cruising**: Both fuel cell and battery shall provide both the energy and power needed.

- **Acceleration**: During acceleration, more power is needed to overcome all opposing forces on a vehicle in motion, supercapacitor can provide more specific power than any of the two sources of energy. However, it has less specific energy than the two. The combination of all sources of energy and power shall be needed during acceleration.

Each of this aggregate requires a minimum amount of power and energy.

The amount of power needed for each aggregate are:

- **Start-up**: This is the minimum power needed under no acceleration with no grade using the rated velocity \( = 17.931 \text{ kW} \)

- **Cruising**: This is the minimum power needed under no acceleration with no grade using the rated velocity \( = 17.931 \text{ kW} \)

- **Acceleration**: This is the minimum power needed under acceleration with no grade using rated velocity \( = 114.242 \text{ kW} \)
All the electrical components been used in this thesis, has its unique electrical characteristics (as in Fig.7.1 to Fig.7.2 and Tab.7.2. to Tab.7.9.), that are been used, to model both the power and energy requirements for the plug-in hybrid electric vehicle under consideration.

Electrical and chemical characteristics for fuel cell, battery, and supercapacitor were discussed in the previous chapters of this write-up.

Some these characteristics are:

- Specific energy \( (S_E) \) = Energy/mass \( \text{KWh/Kg} \)
- Specific power \( (S_P) \) = Power/mass \( \text{KW/Kg} \)
- Specific density \( (S_D) \) = Energy/volume \( \text{KW/Liter (L)} \)
- Specific volume \( (V_S) \) = Volume/mass
  \[ \text{Liter/Kg} \]
- Specific cost \( (C_S) \) = Cost/mass \( \text{$/Kg} \)

So,

- Energy (E) = Specific energy \( (S_E) \) * Mass (M) \( \text{KWh} \)
- Power (P) = Specific power \( (S_P) \) * Mass (M) \( \text{KW} \)
- Cost (C) = Specific cost \( (C_S) \) * Mass (M) \( \text{$} \)
- Volume (V) = Specific volume \( (V_S) \) * Mass (M) \( \text{Liter} \)

During a normal vehicle operation, hydrogen tank attached to the fuel cell and battery should provide energy needed. At start-up and cruising, fuel cell and battery should provide
power needed. In addition, during acceleration, all the three of energies should provide power needed.

Based on these requirements, the energy and power requirements of the model are:

\[
E_M = S_{EB} \cdot M_B + S_{EH} \cdot M_H \tag{8.1}
\]

\[
P_A = S_{PB} \cdot M_B + S_{PC} \cdot M_C + S_{PFC} \cdot M_{FC} \tag{8.2}
\]

\[
P_O = S_{PB} \cdot M_B + S_{PFC} \cdot M_{FC} \tag{8.3}
\]

Where,

\(E_M\) = Minimum energy require by the system

\(E_H\) = Energy by hydrogen

\(E_B\) = Energy by battery

\(P_A\) = Minimum Power need during acceleration

\(P_{FC}\) = Power from Fuel-cell

\(P_B\) = Power from battery

\(P_{C}\) = Power from supercapacitor

\(P_O\) = Minimum Power needed for normal operation

\(P_{FC}\) = Power from Fuel-cell

\(P_B\) = Power from battery
With,

\[ E_H = S_{EH} \times M_H \]  \hspace{1cm} (8.4)  
\[ E_B = S_{EB} \times M_B \]  \hspace{1cm} (8.5) 
\[ P_B = S_{PB} \times M_B \]  \hspace{1cm} (8.6) 
\[ P_C = S_{PC} \times M_C \]  \hspace{1cm} (8.7) 
\[ P_{FC} = S_{PFC} \times M_{FC} \]  \hspace{1cm} (8.8) 

Where,

\[ S_{EH} = \text{Specific energy of hydrogen} \]  
\[ M_H = \text{Hydrogen mass} \]  
\[ S_{EB} = \text{Specific energy of battery} \]  
\[ M_B = \text{Battery mass} \]  
\[ S_{PC} = \text{Specific power of supercapacitor} \]  
\[ M_C = \text{Supercapacitor mass} \]  
\[ S_{PFC} = \text{Specific power of fuel – cell} \]  
\[ M_{FC} = \text{Fuel – cell mass} \]  

Therefore,

Base on design criterial,
\[ E_M \geq 90 \text{ kWh} \quad (8.9) \]

\[ P_A \geq 80 \text{ kW} \quad (8.10) \]

\[ P_O \geq 40 \text{ kW} \quad (8.11) \]

That is,

\[ S_{EB} \times M_B + S_{EH} \times M_H \geq 90,000 \quad (8.12) \]

\[ S_{PB} \times M_B + S_{PC} \times M_C + S_{PFC} \times M_{FC} \geq 80,000 \quad (8.13) \]

\[ S_{PB} \times M_B + S_{PFC} \times M_{FC} \geq 40,000 \quad (8.14) \]

Both the energy and power models for this hybrid electric vehicle must meet above criteria.

The mathematical model governing the total cost, total volume, and total mass of this vehicle is as follow:

Total mass \((M_T) = \text{Battery mass (} M_B \text{) + Supercapacitor mass (} M_C \text{)}\)
+ Fuel-cell mass ($M_{FC}$) + Hydrogen mass ($M_H$) \hspace{1cm} (8.16)

Total volume ($V_T$) = Battery volume ($V_B$) + Supercapacitor volume ($V_C$) + Fuel-cell volume ($V_{FC}$) + Hydrogen volume ($V_H$) \hspace{1cm} (8.17)

Total cost ($C_T$) = Battery cost ($C_B$) + Supercapacitor cost ($C_C$) + Fuel-cell cost ($C_{FC}$) + Hydrogen cost ($C_H$) \hspace{1cm} (8.18)

But,

$V_H = V_{SH} \times M_H$ \hspace{1cm} (8.19)

$V_B = V_{SB} \times M_B$ \hspace{1cm} (8.20)

$V_C = V_{SC} \times M_C$ \hspace{1cm} (8.21)

$V_{FC} = V_{SFC} \times M_{FC}$ \hspace{1cm} (8.22)

Where,

$V_{SH} = Specific \text{ volume of hydrogen}$

$V_{SB} = Specific \text{ volume of battery}$

$V_{SC} = Specific \text{ volume of supercapacitor}$

$V_{SFC} = Specific \text{ volume of fuel – cell}$

And,
\[ C_H = C_{SH} \times M_H \]  
\[ C_B = C_{SB} \times M_B \]  
\[ C_C = C_{SC} \times M_C \]  
\[ C_{FC} = C_{SFC} \times M_{FC} \]

Where,

- \( C_{SH} = \text{Hydrogen cost} \)
- \( C_{SB} = \text{Battery cost} \)
- \( C_{SC} = \text{Supercapacitor cost} \)
- \( C_{SFC} = \text{Fuel cell cost} \)

Therefore,

\[ M_T = M_B + M_C + M_{FC} + M_H \]  
\[ V_T = V_{SB} \times M_B + V_{SC} \times M_C + V_{SFC} \times M_{FC} + V_{SH} \times M_H \]  
\[ C_T = C_{SB} \times M_B + C_{SC} \times M_C + C_{SFC} \times M_{FC} + C_{SH} \times M_H \]

The specific power, specific energy, specific cost, and energy density are unique and are known for each of the components been used in this model.
To minimize the total cost, total volume, and total mass of the vehicle, mass of individual components is been used as determining factor to optimize the performance of our vehicle. In other to get the best variable values for the mathematical model, optimization theorem of linear programming shall be used to optimize the values for each sources of energy.

Using linear programming, the objective functions been minimized are total cost, total mass and total volume of the optimization model. Power and energy requirements in our mathematical model become the constraints to be satisfied in our optimization model.

The mathematical model then becomes:

\[
\text{Mathematical model}
\]

\[
M_T = M_B + M_C + M_{FC} + M_H
\]

\[
V_T = V_{SB} \times M_B + V_{SC} \times M_C + V_{SFC} \times M_{FC} + V_{SH} \times M_H
\]

\[
C_T = C_{SB} \times M_B + C_{SC} \times M_C + C_{SFC} \times M_{FC} + C_{SH} \times M_H
\]

\[
S_{EB} \times M_B + S_{EH} \times M_H \geq 90,000
\]

\[
S_{PB} \times M_B + S_{PC} \times M_C + S_{PFC} \times M_{FC} \geq 80,000
\]

\[
S_{PB} \times M_B + S_{PFC} \times M_{FC} \geq 40,000
\]

The optimization model can then be formulated from the mathematical model.
The optimization model for our system is:

**Optimization model**

Minimize:

\[ M_T = M_B + M_C + M_{FC} + M_H \]
\[ V_T = V_{SB} \ast M_B + V_{SC} \ast M_C + V_{SFC} \ast M_{FC} + V_{SH} \ast M_H \]
\[ C_T = C_{SB} \ast M_B + C_{SC} \ast M_C + C_{SFC} \ast M_{FC} + C_{SH} \ast M_H \]

Subject to

Constraints:

\[ S_{EB} \ast M_B + S_{EH} \ast M_H \geq 90,000 \]
\[ S_{PB} \ast M_B + S_{PC} \ast M_C + S_{PFC} \ast M_{FC} \geq 80,000 \]
\[ S_{PB} \ast M_B + S_{PFC} \ast M_{FC} \geq 40,000 \]

\[ M_B \geq 0 \]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]
To obtain values for both hydrogen cylinder and relationship between fuel cell and its electrical parameters, consider Figures (7.1, 7.2) and Tables (7.2 to 7.9) in chapter 7.

\[
V_{SB} = \text{Specific volume of battery} = 0.5263
\]

\[
V_{SC} = \text{Specific volume of supercapacitor} = 0.9176
\]

\[
C_{SB} = \text{Battery cost} = 78
\]

\[
C_{SC} = \text{Supercapacitor cost} = 118.22 \ [6]
\]

\[
S_{EB} = \text{Specific energy of battery} = 100
\]

\[
S_{Ec} = \text{Specific energy of supercapacitor} = 3.9
\]

\[
S_{PC} = \text{Specific power of battery} = 1300
\]

\[
S_{PC} = \text{Specific power of supercapacitor} = 3300
\]

To estimate the relationship between power, mass, and volume of fuel cell, previous work done by the department of electrical engineering (NIU) and reference [14], shall be considered.

\[
V_{SFC} = \text{Specific volume of fuel – cell}
\]

\[
= \frac{\text{Specific power of fuel–cell}}{\text{Specific power density of fuel–cell}}
\]  \hspace{1cm} (8.30)

Also,
\[ V_{SH} = \text{Specific volume of Hydrogen} \]
\[ = \frac{\text{Specific energy of hydrogen}}{\text{Specific energy density of hydrogen}} \]  
\[ (8.31) \]

But, from Tab.7.6 and Tab.7.7 for electrical characteristics of fuel cell and cylinder of hydrogen respectively.

\[ V_{SFC} = \frac{500}{500} = 1 \text{ Liter/Kg} \]  
\[ (8.32a) \]

And,

\[ V_{SH} = \frac{1500}{1200} = 1.25 \text{ Liter/Kg} \]  
\[ (8.32b) \]

From Fig.7.3, the relationship between volume of fuel cell and its power is:

\[ V_{FC} = 3 \times P_{FC} + 73 \]  
[14]  
\[ (8.33) \]

However, from literature, mass is directly proportional to volume.

That is,

\[ M_{FC} = K \times V_{FC} \]  
\[ (8.34) \]

Where,

K is in Kg/Liter

So,

For this Fuel cell,

\[ K = \frac{1}{V_{SFC}} \]
But from (8.33a), \( V_{SFC} = 1 \)

Therefore,

\( K = 1 \)

Then,

\[ M_{FC} = 1 \times V_{FC} \]

Substituting this into equation (8.34)

Then,

\[ M_{FC} = 3 \times P_{FC} + 73 \]

Where,

\[ P_{FC} = \frac{M_{FC} - 73}{3} \]

\( P_{FC} = 0.333 \times M_{FC} - 24.33 \) \hspace{1cm} (8.35a)

\( P_{FC} = 333 \times M_{FC} - 24330 \) (With a power factor of 1000) \hspace{1cm} (8.35b)

From Fig.7.4

\[ C_{FC} = 53 \times P_{FC} + 1290 \] \hspace{1cm} [14] \hspace{1cm} (8.36)

Equating (8.35B) into (8.36)

Then,

\[ C_{FC} = 52.7 \times (0.333 \times M_{FC} - 24.33) + 1282 \]

\[ = 17.55 \times M_{FC} \] \hspace{1cm} (8.37)

Tab.8.1. New relationship between cost, specific volume and power of fuel cell
From equations (8.35b), (8.37) and Tab.8.1, and values for battery and supercapacitor above, the optimization model then becomes:

**Optimization model**

*(For Supercapacitor BMOD0165)*

Minimize:

\[
M_B + M_C + M_{FC} + M_H \\
0.5263 M_B + 0.9176 M_C + M_{FC} + 1.25 M_H \\
78M_B + C_{SC} \times 118.22C_{SFC} + 17.55M_{FC} + 9 M_H
\]

Subject to

Constraints:

\[
100M_B + 1500M_H \geq 90,000 \\
1300M_B + 3300S_{PC} \times M_C + 333M_{FC} - 24333 \geq 80,000 \\
1300 M_B + 333M_{FC} - 24333 \geq 40,000
\]
\[ M_B \geq 0 \]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]

By rearranging above,

**Optimization model**

*(For Supercapacitor BMOD0165)*

Minimize:

\[ M_B + M_C + M_{FC} + M_H \]
\[ 0.5263 M_B + 0.9176 M_C + M_{FC} + 1.25 M_H \]
\[ 78 M_B + 118.22 C_{SFC} + 17.55 M_{FC} + 9 M_H \]

Subject to

Constraints:

\[ 100 M_B + 1500 M_H \geq 90,000 \]
\[ 1300 M_B + 3300 M_C + 333 M_{FC} \geq 104,333 \]
\[ 1300 M_B + 333 M_{FC} \geq 64,333 \]

\[ M_B \geq 0 \]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]

(For Supercapacitor BCAP3400)

Minimize:
\[ M_B + M_C + M_{FC} + M_H \]
\[ 0.5263 M_B + 0.7604 M_C + 1.25 M_H \]
\[ 78M_B + 278.4C_{SFC} + 17.55M_{FC} + 9 M_H \]

Subject to

Constraints:
\[ 100M_B + 1500M_H \geq 90,000 \]
\[ 1300M_B + 8500S_{PC} \times M_C + 333M_{FC} \geq 104,333 \]
\[ 1300 M_B + 333M_{FC} \geq 64,333 \]

\[ M_B \geq 0 \]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]
Optimization is a mathematical technique for finding a maximum or minimum value of a function of several variables subject to a set of constraints. [41]

It is the process of finding the greatest or least value of a function for some constraint, which must be true regardless of the solution. [42]

In other words, optimization finds the most suitable value for a function within a given domain.

For a function $f(x)$, called the objective function, that has a domain of real numbers of set $A$, the maximum optimal solution occurs where $f(x_o) \geq f(x)$ over set $A$ and the minimum optimal solution occurs where $f(x_o) \leq f(x)$ over set $A$. [42]

In engineering, a mathematical model of a process or system’s operation is formulated. To, optimize the performance of the system’s operation, the mathematical model is then converted to optimization model.

Optimization model represents problem choices as decision variables and seek values that either maximize or minimize objective function of decision variables subject to constraints on variables values expressing the limits on possible decision choices.
With the help of analytical tool such as Linear programming, optimal solution is then found to
the objective function and system variables of the model.

A standard optimization model has the form of:

Minimize or Maximize (Objective function)

Subject to: Main constraints

Variable-type constraints.

Linear programs are problems that can be expressed in standard form as

Minimize or Maximize \[ c^T x \] \[ \text{[3]} \]

Subject to: \[ Ax \leq (\geq) b \]

And \[ x \geq 0 \]

Where, \( x \) represents the vector of variables (decision variables) to be
determined, \( c \) and \( b \) are vectors (single row or single column matrix) of coefficients of both
the objective function (s) and RHS (Right hand side) of the main constraints respectively. \( c^T x \)
= Transpose of the coefficients of the objective function. \[ \text{[28]} \]

While, \( A \) is the matrix of the coefficients of the main constraints.

The expression to be maximized or minimized is called the objective function (i.e.

\( c^T x \)).

The inequalities \( Ax \leq b \) and \( x \geq 0 \) are the main constraints and variable-type constrains
respectively.
In this thesis, from the optimization model.

The objective functions are:

\[ M_B + M_C + M_{FC} + M_H \]
\[ 0.5263 M_B + 0.7604 M_C + M_{FC} + 1.25 M_H \]
\[ 78M_B + 278.4C_{SFC} + 17.55M_{FC} + 9 M_H \]

The decision is to minimize above objective functions subject to:

Main constraints:

\[ 100M_B + 1500M_H \geq 90,000 \]
\[ 1300M_B + 8500S_{PC} \times M_C + 333M_{FC} \geq 104,333 \]
\[ 1300M_B + 333M_{FC} \geq 64,333 \]

Variable-type constraints:

\[ M_B \geq 0 \]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]

Where the decision variables are:

\[ M_B = \text{Lithium – ion mass} \]
\[ M_C = \text{Supercapacitor mass} \]
\[ M_{FC} = \text{Fuel – cell mass} \]
\[ M_c = \text{Hydrogen cylinder mass} \]

Optimization model and solution must make sure:

- There is feasible set of choices for the decision variables that satisfy all constrains and meet the objective function.
- Optimization model must be bounded.
- To find optima solution to both the objective function and decision variables.

Optimal solution is a feasible choice for decision variables with objective function value at least equal to that of any other solution that satisfy all constraints.

A model is said to be feasible if all decision variables satisfy all constraints.

A model is not feasible if all decision variables do not satisfy all constraints.

Optimization model is said to be unbounded if optimal objective value does continue to improve without a limit. In our case, keep on decreasing without a limit. It is bounded, if the optimization is feasible and has definite optimal objective value.
Multiple objective functions

The optimization model of this thesis contains multiple objective functions. Optimization model was defined so far with single objective function. In linear programming, multiobjective optimization model can be solved in different ways. This thesis shall concentrate on two approaches. They are:

1. Weighted sums of objective functions
2. Preemptive optimization

Weighted sums of objective functions

In this approach, multiple objective functions are combined to form a single composite objective function. The new single composite objective function is then the combination of objective functions with each assign weight. [27] The new single composite then becomes the weighted sum of the previous objective functions. If the composite single function is to be minimized, weights on previous maximized objective function should be negative, and weights on previous minimized objective function should be positive.

Minimize or Maximize \[ \sum_{i=1}^{n} (y_i \cdot c_i^T x) \quad i = 1 \text{ to } n \] [27]
Subject to: \[ Ax \leq (\geq) b \]

And \[ x \geq 0 \]

Where,

\[ \sum_{i=1}^{n} (\gamma_i \cdot c_i^T x) = Weighted \ sum \ of \ multi \ objective \ functions \]

\[ \gamma_i = Weight \ value \ with \ each \ objective \ function \]

\[ c_i x = Individual \ objective \ function \]

\[ \sum_{i=1}^{n} \gamma_i c_i^T x = weight1 \cdot objective1 + weight2 \cdot objective2 \quad \text{when } n = 2 \]

In a weighted sum approach, the weight attached to any objective function in the weighted sum will reflect the importance we attach to that objective function.

If value for weight1 is higher than value for weight 2 in above example, then, objective function 1 is more important to us than objective function 2 and vice versa.

In this thesis, cost objective function is assigned a higher weight than both volume objective function and mass objective function. Cost objective function is treated as more important than that of mass and volume.

**Preemptive optimization approach.**

Preemptive optimization performs multiobjective optimization by considering objectives one at a time.
The most important is first optimized, then the second most important optimized subject to a requirement that the first achieve its optimal value and continue until the last least important is optimized.\[27\]

Decision must be made to determine the hierarchy of importance.

Preemptive approach provides efficient primal solution. No improvement in objective value when the last and least important objective function has been optimized.

In this thesis the order of hierarchy for the objective functions are: cost, volume, and mass.
Chapter 10

SENSITIVITY ANALYSIS OF PRIMAL OPTIMIZATION MODEL

**Primal optimization model**

This the optimization model obtained from the mathematical model. It formulates the application of primary interest.

**Dual optimization model**

It is an alternative/subsidiary optimization model, defined over the same input parameters as the primal optimization model but charactering the sensitivity of the primal results to changes in inputs. Dual allows the sensitivity information of the primal to be obtained. [27]

For an optimization model to be feasible and bounded, both the primal solution and dual solution must be feasible and obtainable.

The mathematical model of the system is as shown in previous chapter.

The optimization model was obtained from the mathematical model.

The model of primary interest is the optimization model which represents the primal optimization model.
The dual optimization model of the primal is the following standard form:

Minimize or Maximize \( b^T v \) [3]

Subject to: \( Dv \leq (\geq)c \)

And \( v \geq 0 \)

Where, \( v \) represents the vector of dual variables to be determined, \( b \) and \( c \) are vectors (single row or single column matrix) of coefficients of both the dual objective function (s) and RHS (Right hand side) of the dual main constraints respectively. \( b^T v = \) Transpose of the coefficients of the dual objective function. [28]

While, \( D \) is the matrix of the coefficients of the dual main constraints.

The dual expression to be maximized or minimized is called the dual objective function (i.e. \( b^T v \)).

The inequalities \( Dv \leq (\geq)c \) and \( v \geq 0 \) are the dual main constraints and dual variable-type constrains respectively.

Primal optimization model shall be subjected to sensitivity analysis by using its equivalent dual optimization model.

With its equivalent dual, the following information about primal can be obtained:

- The rate of change of the optima objective value with one unit increase of the RHS(right hand side) of the main constraints
- The feasibility or not of the primal can be confirm
- Strong duality and complementary slackness properties of linear programming can be ascertained.
• How change(s) in the coefficient of decision variables can affect optima objective value.

• How removal or addition of decision variables to the optimization model can affect the optimal objective value.

**Relaxing and tightening of constraints**

Relaxing and tightening of constraints could be caused by changes in number of decision variable (adding or dropping), changes in the coefficients of the decision variables, changes (increase or decrease) in RHS of the main constraints, and changes in the coefficients of the objective functions. Relaxing means making optimal value(s) better or the same. Tightening means making optimal value(s) worst or the same.

In this thesis, minimization of objective functions is been carried out. Also, the LHS (left hand side) is greater ($\geq$) than RHS (right hand side) of both the main constraints and variable-type constraints.

With minimization of objective function and $\text{LHS} \geq \text{RHS}$, increasing the RHS will tightening feasible space/region and conversely, decreasing the RHS will relax the feasible space/region. Also, increasing the LHS will relax feasible space/region and conversely, decreasing the RHS will tightening the feasible space/region. [27]

Adding constraints to an optimization model tightens its feasible space/region/set, and conversely, dropping constraints relaxes feasible space/region/set.
Effect of changes in the RHS of constraints, changes in number of decision variable (adding or dropping), changes in the coefficients of the decision variables, changes (increase or decrease) in LHS of the main constraints, and changes in the coefficients of the objective functions to optimal values shall be explored. [27]

**Strong duality**

If primal and its dual have solutions, and their objective functions are the same, then, the primal optimization model is said to have established a strong duality. This helps us to establish both the feasibility and boundedness of our optimization model. It means we can find optimal value for the decision variables and objective function(s)
Chapter 11

LINEAR PROGRAMMING.

Linear programming is a high-level computer algorithm used as a tool to obtain primal solution(s) to optimization model. It is used to obtain optimal objective value and optimal values for decision variable(s).

The use of linear programming as optimization model, requires both the objective function and constraints of the optimization model to be linear. A function is linear if it is a constant weighed sum of decision variables. [27]

The approach to finding the optimal values for both the objective function and decision variables in this thesis are as following:

- Formulate the mathematical model for power train equations
- Convert the mathematical model to optimization model
- Convert multiple objective functions to single one using weighted sum approach in optimization method.
- Test and confirm that optimization model is feasible and bounded
- Use Linear programming on the primal optimization model to find the optimal values for objective function and decision variables
• Use linear programming on the dual optimization model to confirm both the feasibility and boundedness of the primal optimization model.

• To use dual optimization model to do sensitivity analysis of the optimization model

Two types of supercapacitor are been considered to show how the choice of decision variable can affect the optimal values of our system.

**Optimization model**

As formulated in previous chapter, the optimization model for our system is:

**Supercapacitor BMOD0165**

Minimize:

\[
M_B + M_C + M_{FC} + M_H \\
0.5263 M_B + 0.9176 M_C + M_{FC} + 1.25 M_H \\
78M_B + 118.22C_{SFC} + 17.55M_{FC} + 9 M_H
\]

Subject to

Constraints:

\[
100M_B + 1500M_H \geq 90,000 \\
1300M_B + 3300 M_C + 333M_{FC} \geq 104,333 \\
1300 M_B + 333M_{FC} \geq 64,333
\]
\[ M_B \geq 0 \]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]
Supercapacitor BCAP3400

Minimize:

\[ M_B + M_C + M_{FC} + M_H \]
\[ 0.5263 M_B + 0.7604 M_C + M_{FC} + 1.25 M_H \]
\[ 78M_B + 278.4C_{SFC} + 17.55M_{FC} + 9 M_H \]

Subject to

Constraints:

\[ 100M_B + 1500M_H \geq 90,000 \]
\[ 1300M_B + 8500S_{PC} \cdot M_C + 333M_{FC} \geq 104,333 \]
\[ 1300 M_B + 333M_{FC} \geq 64,333 \]

\[ M_B \geq 0 \]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]
Two approaches to solving this multiobjective optimization model were discussed in the previous chapter.

These two approaches (weighted sums of objective functions and Preemptive optimization solution) will be used to find optimal values for both the objective functions and decision variables.

**Weighted sums of objective functions**

In this thesis, total cost objective function is assigned a higher weight than both total volume and total mass objective functions. Equal weight is assigned to both volume and total mass objective functions.

Total cost assigned weight = 1000

Total cost assigned weight = 10

Total cost assigned weight = 10

The weighted sum of the objective functions is:

**Supercapacitor BMOD0165**

Weighted sum objective function

$$= 10*(M_B + M_C + M_{FC} + M_H) + 10 * (0.5263 M_B + 0.9176 M_C + M_{FC} + 1.25 M_H) + 1000 * (78 M_B + 118.22 C_{SFC} + 17.55 M_{FC} + 9 M_H)$$
Supercapacitor BCAP3400

Weighted objective function

\[ = 10(M_B + M_C + M_{FC} + M_H) + 10 \times (0.5263M_B + 0.7604M_C + M_{FC} + 1.25M_H) + 1000 \times (78M_B + 278.4C_{SFC} + 17.55M_{FC} + 9M_H) \]

\[ = 78015.263 \times M_B + 278417.604 \times M_C + 17570 \times M_{FC} + 9022.5M_H \]

So, for the two supercapacitors, the modified optimization model with one objective function is:

**Primal LP**

**Supercapacitor BMOD0165**

Minimize:

\[ 78015.263 \times M_B + 118239.176 \times M_C + 17570 \times M_{FC} + 9022.5M_H \]

Subject to

Constraints:

\[ 100M_B + 1500M_H \geq 90,000 \]

\[ 1300M_B + 3300M_C + 333M_{FC} \geq 104,333 \]

\[ 1300M_B + 333M_{FC} \geq 64,333 \]

\[ M_B \geq 0 \]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]
**Supercapacitor BCAP3400**

Minimize:

\[ 78015.263 \times M_B + 278417.604 \times M_C + 17570 \times M_{FC} + 9022.5 M_H \]

Subject to

Constraints:

\[ 100 M_B + 1500 M_H \geq 90,000 \]

\[ 1300 M_B + 8500 M_C + 333 M_{FC} \geq 104,333 \]

\[ 1300 M_B + 333 M_{FC} \geq 64,333 \]

\[ M_B \geq 0 \]

\[ M_C \geq 0 \]

\[ M_{FC} \geq 0 \]

\[ M_H \geq 0 \]
DUAL LP

The Dual Linear programming for above primal is shown below:

**Supercapacitor BMOD0165**

Maximize:

$$90000 \times v_1 + 104333 \times v_2 + 64333 \times v_3$$

Subject to

Constraints:

$$100v_1 + 1300v_2 + 1300v_3 \leq 78015.263$$

$$3300v_2 \leq 118239.176$$

$$333v_2 + 333v_3 \leq 17570$$

$$1500v_1 \leq 9022.5$$

$$v_1 \geq 0$$

$$v_2 \geq 0$$

$$v_3 \geq 0$$
**Supercapacitor BCAP3400**

Maximize:

\[ 90000 \times v_1 + 104333 \times v_2 + 64333 \times v_3 \]

Subject to

Constraints:

\[ 100v_1 + 1300v_2 + 1300v_3 \leq 78015.263 \]
\[ 8500v_2 \leq 278417.604 \]
\[ 333 v_2 + 333v_3 \leq 17570 \]
\[ 1500v_1 \leq 9022 \]

\[ v_1 \geq 0 \]
\[ v_2 \geq 0 \]
\[ v_3 \geq 0 \]
Preemptive optimization

In this approach, hierarchy of the importance of the multiple objective functions must be determined. In this thesis the order of hierarchy for the objective functions are: cost, volume, and mass.

Steps taken in using preemptive optimization approach are:

1. Optimization model was 1st optimized with cost objective as the only objective function to be minimized, using the constraints in the optimization model.

2. New constraint was obtained by turning the cost objective function into equation with primal objective value from step 1 as its RHS. LHS must be less than RHS.

3. This newly formulated constraint is added to the previous 3-main constraints making 4-main constraints.

4. Optimization model with these 4-main constraints was optimized with volume objective function being the only objective function to be minimized.

5. Another constraint was formulated by turning the volume objective function into equation with primal objective value from step 4 as its RHS. LHS must be less than RHS.

6. Another linear programming’ equation obtained from step 5 is added to the 4-main constraints from step 3.

7. With now 5-main constraints, a final optimization procedure was carried out, using mass object function as the only objective function to be minimized.

8. Our optimal values are ones obtained from step 7.
Preemptive Optimal solution to LP with BMOD0165 Supercapacitor

Step 1.

Minimize:

\[ 78M_B + 118.22C_{SFC} + 17.55M_{FC} + 9M_H \]

Subject to

Constraints:

\[ 100M_B + 1500M_H \geq 90,000 \]

\[ 1300M_B + 3300M_C + 333M_{FC} \geq 104,333 \]

\[ 1300M_B + 333M_{FC} \geq 64,333 \]

\[ M_B \geq 0 \]

\[ M_C \geq 0 \]

\[ M_{FC} \geq 0 \]

\[ M_H \geq 0 \]

Step 2.

Minimize:

\[ 0.5263M_B + 0.9176M_C + M_{FC} + 1.25M_H \]

Subject to

Constraints:
\[78M_B + 278.4C_{SFC} + 17.55M_{FC} + 9M_H \leq 5363.5\]

\[100M_B + 1500M_H \geq 90,000\]

\[1300M_B + 33000M_C + 333M_{FC} \geq 104,333\]

\[1300M_B + 333M_{FC} \geq 64,333\]

\[M_B \geq 0\]

\[M_C \geq 0\]

\[M_{FC} \geq 0\]

\[M_H \geq 0\]

**Step 3**

Minimize:

\[M_B + M_C + M_{FC} + M_H\]

Subject to

Constraints:

\[0.5263M_B + 0.9176M_C + M_{FC} + 1.25M_H \leq 279.3118\]

\[78M_B + 118.22C_{SFC} + 17.55M_{FC} + 9M_H \leq 5363.5\]

\[100M_B + 1500M_H \geq 90,000\]

\[1300M_B + 3300M_C + 333M_{FC} \geq 104,333\]

\[1300M_B + 333M_{FC} \geq 64,333\]

\[M_B \geq 0\]
\[ M_C \geq 0 \]
\[ M_{FC} \geq 0 \]
\[ M_H \geq 0 \]

**Analysis of result**

Primal and Dual results for both BMOD0165 and BCAP3400 supercapacitors are contained in Appendix (1). The optimal objective value for both the dual and primal are the same. This establishes the optimization model is feasible and bounded. It also shows the strong duality of the primal optimization model. Dual optimal variable values indicate the rate of change of the optimal objective value to a single increase of the RHS of the main constraints.

The Dual values confirmed the rate of change of the optimal objective value to a unit change in the RHS of main constraints of the primal optimization model.

The successful implementation of both the dual and primal with observations from previous discussions under sensitivity analysis show that this hybrid vehicle’s performance could be optimized with our design.

By comparing the optimal values for the decision variables and objective functions of both optimization models (models with BMOD0165 Supercapacitor and BCAP3400 Supercapacitor), the following can be observed:

- Both the specific power and specific energy of BCAP3400 Supercapacitor were higher than that of BMOD0165 Supercapacitor.
• From the optimum solutions in appendix (1) (Solution 11.1 and Solution 11.2)

<table>
<thead>
<tr>
<th>BMOD0165</th>
<th>BCAP3400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_B = 0$</td>
<td>$M_B = 0$</td>
</tr>
<tr>
<td>$M_C = 12.12$</td>
<td>$M_C = 4.7059$</td>
</tr>
<tr>
<td>$M_{FC} = 193.1922$</td>
<td>$M_{FC} = 193.1922$</td>
</tr>
<tr>
<td>$M_H = 60$</td>
<td>$M_H = 60$</td>
</tr>
</tbody>
</table>

• The optimal value of $M_C$ for optimization model with BCAP3400 Supercapacitor is lower than that with BMOD0165 Supercapacitor.

• The specific power for both capacitors is higher than that of Lithium-ion battery.

• The optimal value of $M_B = 0$ for both optimization models.

In this thesis, the objective is to minimize the total cost, the total mass, and the total volume. By using a Supercacitor with better electrical characteristics, both the total cost and total mass are reduced drastically. Also, this thesis shows that with advancement in supercapacitor technology, we can design an optimum performance hybrid fuel cell electric vehicle without using a battery.
Advanced vehicle simulator (ADVISOR) was developed in 1994 by National Renewable Energy Laboratory to support the U.S. Department of Energy hybrid propulsion system program. [34] It is designed to be used as a modelling/analyzing tool. It could be used accurately, to model and analyses hybrid vehicle, its design, and compare different drivetrain configurations. ADVISOR predicts acceleration time to within 0.7% and energy use on the demanding US06 to within 0.6% for an underpowered series hybrid vehicle (0-100 km/h in 20 seconds) [36] An ADVISOR model can be used to stimulate vehicle performance on a standard driving cycles much faster than its contemporary forward-facing vehicle modelling tools. ADVISOR runs on MATLAB platform. [32] [34] Its graphic user interface (GUI) is simple and easy to use. It is freely available to designers and engineers.

The block diagram capability of Simulink of MATLAB is heavily utilized by ADVISOR.

The block diagram of drivetrain of a fuel cell vehicle with energy storage system is shown in Fig. 12.1 [34]

ADVISOR has three (3) GUI screens:

- Vehicle input Screen: It is used to input vehicle’s specifications.
- Simulation setup Screen
- Result Screen: This shows summary results of the system performance.

Fig. 12.1 Drivetrain of Fuel Cell vehicle with energy storage system [34]
ADVISOR was used to analyze the feasibility and the performance of our proposed plug-in hybrid electric vehicle.

Due to limitation of combining both the supercapacitor and Lithium-ion battery in ADVISOR model, 2-models of our design were carried out:

- 1st with Supercapacitor only as shown in Fig.12.2
- 2nd with Lithium-ion battery only as shown in Fig.12.4

ADVISOR’S input screen for our design are shown in Fig.12.2 and Fig.12.4

Some of the data contained in Fig were used.

The simulation setup Screen of our design are as shown in Fig.12.3 and Fig.12.5

Finally, Fig.12.6 and Fig.12.7 show multiple (2) simulation run carried out respectively for:

1. CYC_UDDS drive cycle as shown in Fig. 12.3 and Fig.12.5
2. CYC_NYCC drive cycle as shown in Fig. 12.6.
RESULTS

Fig. 12.2  Input Screen of ADVISOR with Supercapacitor as Storage device
Fig. 12.3. Setup Screen of ADVISOR with Supercapacitor as Storage device

Fig. 12.4. Input Screen of ADVISOR with Lithium-ion as Storage device
Fig. 12.5. Setup Screen of ADVISOR with Lithium-ion as Storage device

Fig. 12.6. CYC NYCC obtained from result Screen of our ADVISOR
Two (2) driving cycles are being used to analyze the results of our design.

Driving cycle is a series of data points representing the speed of a vehicle versus time.

Driving cycles are produced by different countries and organizations to assess the performance of vehicles in various ways, as for example fuel consumption and polluting emissions.

Driving cycles is used in vehicle simulations to predict performance of internal combustion engines, transmissions, electric drive.

ADVISOR model’s results are usually compared with standard drive cycles to ascertain any improvement.

In this thesis, two of the drive cycles are used to ascertain improvement of our design over the requirements for these drive cycles.

A standard CYC_UDDS drive cycle is as shown in Fig. 12.7 [43]

A standard CYC_NYCC drive cycle is as shown in Fig.12.8 [44]

UDDS= Urban Dynamometer Driving Schedule

NYCC= New York City Cycle

**Thesis basic parameters are:**

- Maximum speed: 80 mph
- Average speed: 60 mph
- Maximum power: 114.242 KW
- Maximum acceleration: 2.4 ms$^{-2}$
The basic parameters for CYC UDDS cycle are:

Urban route of 7.5 mi (12.07 km).

The average speed is 19.6 mph (31.5 km/h).

The maximum speed is 56.7 mph (91.25)

Maximum power 15 KW.

Maximum braking power 15 KW. Maximum acceleration 0.57 ms⁻²
The following are basic parameters of the cycle:

- Duration: 598 seconds
- Distance: 1.18 miles = 1.89 km
- Average speed: 7.1 mi/h = 11.4 km/h
- Maximum speed: 27.7 mi/h = 44.6 km/h
- Maximum power 85 KW
- Maximum braking power 80KW
- Maximum acceleration 2.7 ms\(^2\)
Analysis of result

Comparing our UDDS drive cycle (Fig. 11.5) to that of Standard UDDS (Fig. 11.7), the speed vs time graphs are closely identical. This indicates that the optimized results were able to maintain this standard driving cycle.

Comparing our NYCC drive cycle (Fig. 11.6) to that of Standard NYCC (Fig. 11.8), the speed vs time graphs are closely identical. This again indicates the optimized design was able to maintain this alternative average driving cycle.

With some limitations (No original ADVISOR in the department and limited Student’s version MATLAB platform), the results of our design greatly improve on the performance of hybrid electric vehicle.
Chapter 13

ANALYSIS OF RESULT

Based on the solutions obtained by applying optimization theorems on both the primal and dual model of our system, we were able show the feasibility and boundedness of our optimization model by proving the strong duality (optimal objective values for both the dual and primal are the same). This shows that our model and its dynamic equations can be used to determine the static optimization of our system.

Also, sensitivity analysis was carried out on our optimization model. The convergent of the primal objective function of both dual and primal to the same value established design feasibility. Also, sensitivity analysis enables us to understand and prove changes in our objective functions by varying the values of our energy sources’ components. It was proved, that our primal objective values reduced (minimized) by relaxing the coefficient values in our constrain equations. Increase in optimal value of our objective functions (as against decrease i.e. due to minimization) means reduction in performance of our system (hybrid electric vehicle). Conversely, reduction in optimal value of the objective functions (by relaxing our constraint equation) will indicate a better performance of our system. With this, we were able established that our design will lead to increase performance of the hybrid electric vehicle.
By carrying out various static optimization on our model by changing different values for decision variables (particularly specific power for both supercapacitor and lithium-ion), we were able to show that supercapacitor with its better specific power over both lithium-ion and fuel cell can greatly improve the performance of hybrid electric vehicles. It was shown, supercapacitor can adequately carry out the energy storage function of battery in hybrid electric vehicle.

By comparing the optimal values for the decision variables and objective functions of both optimization models (models with BMOD0165 Supercapacitor and BCAP3400 Supercapacitor), the following can be observed:

- Both the specific power and specific energy of BCAP3400 Supercapacitor were higher than that of BMOD0165 Supercapacitor.
- From the optimum solutions in appendix (1) (Solution 11.1 and Solution 11.2)

<table>
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- The optimal value of $M_C$ for optimization model with BCAP3400 Supercapacitor is lower than that with BMOD0165 Supercapacitor.
- The specific power for both capacitors is higher than that of Lithium-ion battery.
- The optimal value of $M_B = 0$ for both optimization models.
Primal and Dual results for both BMOD0165 and BCAP3400 supercapacitors are contained in Appendix (1).

Mass of both the supercapacitor and lithium-ion battery were calculated based on optimal values obtained. An ADVISOR was used to simulate our design using the calculated mass, the calculated specifications’ parameters, and design requirements of our design. The simulation was successful. Our ADVISOR values and drive cycles’ speed vs time graphs compared favorably with that of standard drive cycle parameters and speed vs time graphs. For example, by comparing our UDDS drive cycle (Fig.12.5) to that of Standard UDDS (Fig.12.7), our maximum speed, average speed, and maximum power are better than that of standard UDDS. The speed vs time graphs are closely identical.

Also, by comparing our NYCC drive cycle (Fig.12.6) to that of Standard NYCC (Fig.12.8), our maximum speed, average speed, maximum power was equally better than that of standard NYCC. The speed vs time graphs are closely identical.

This establishes that even by reducing the capacity values of both fuel cell and battery components, we were able to match and exceed the accepted standards by augmenting the reduced capacity value with that of supercapacitor (which uses no fuel, less weight, and volume). This will lead to more efficient vehicle and improve fuel consumption.

Combining that our operational model was feasible and bounded (resulting in optimal values for both our objective functions and decision variables) with the positive results from our simulation (with ADVISOR), we have shown that our design greatly improved the performance of hybrid electric vehicle.
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Solution 11.1

To find optimal values for:

PRIMAL LP with Supercapacitor BMOD0165

USING

Weighted sum optimization method

\[
f = [78015.263 \ 118239.176 \ 17570 \ 9022.5] \quad \% \text{Weighted Objective}
\]

\[
A = [-100 \ 0 \ 0 \ -1500; -1300 -3300 -333 \ 0; -1300 \ 0 -333 \ 0]; \quad \% \text{Main constraints coefficients}
\]

\[
b = [-90000 \ -104333 \ -64333] \quad \% \text{RHS of the main constraints}
\]

\[
lb = [0 \ 0 \ 0 \ 0];
\]

\[
up = [];
\]

\[
Aeq = [];
\]

\[
beq = [];
\]

\[
[x,fval] = \text{linprog}(f,b,Aeq,beq,lb,up)
\]

Objective function coefficients
1.0e+05 *

0.7802  1.1824  0.1757  0.0902

Main constraints coefficient

\[ A = \begin{bmatrix}
-100 & 0 & 0 & -1500 \\
-1300 & -3300 & -333 & 0 \\
-1300 & 0 & -333 & 0 \\
\end{bmatrix} \]

RHS COEFFICIENT

\[ b = \begin{bmatrix}
-90000 \\
-104333 \\
-64333 \\
\end{bmatrix} \]

Optimal solution found.

Optimal variable values:

\[ M_B = 0 \]
\[ M_C = 12.1212 \]
\[ M_{FC} = 193.1922 \]
\[ M_H = 60.0000 \]

Optimal objective value = 5.3689e+06
Solution 11.2

To find optimal values for:

PRIMAL LP with Supercapacitor BCAP3400

USING

Weighted sum optimization method

\[ f = [78015.263 \ 278417.604 \ 17570 \ 9022.5] \]  \% Weighted %objective function coefficients

\[ A = [-100 \ 0 \ 0 \ -1500; \ -1300 \ -8500 \ -333 \ 0; \ -1300 \ 0 \ -333 \ 0] \] \% Main % constraints coefficients

\[ b = [-90000 \ -104333 \ -64333] \]  \% RHS of the main constraints

\[ lb = [0 \ 0 \ 0 \ 0]; \]

\[ up = []; \]

\[ Aeq = []; \]

\[ beq = []; \]

\[ [x,fval] = \text{linprog}(f,A,b,Aeq,beq,lb,up) \]

Objective function coefficients
1.0e+05 *(0.7802  2.78417  0.1757  0.0902)

Main constraints coefficient

\[
\begin{align*}
A &= -100 \quad 0 \quad 0 \quad -1500 \\
   &= -1300 \quad -8500 \quad -333 \quad 0 \\
   &= -1300 \quad 0 \quad -333 \quad 0
\end{align*}
\]

RHS COEFFICIENT

\[
\begin{align*}
b &= -90000 \quad -104333 \quad -64333
\end{align*}
\]

Optimal solution found.

Optimal variable values:

\[
\begin{align*}
M_B &= 0 \\
M_C &= 4.7059 \\
M_{FC} &= 193.1922 \\
M_H &= 60.0000
\end{align*}
\]

Optimal objective value = 5.2459e+06
Solution 11.3

To find optimal values for:

DUAL LP with Supercapacitor BMOD0165

USING

Weighted sum optimization method

\[
f = [-90000 -104333 -64333] \quad \text{% Weighted objective}\]

\[
f\text{unction coefficients}\]

\[
A = [100 \quad 1300 \quad 1300; \quad 0 \quad 3300 \quad 0; \quad 0 \quad 333 \quad 333; \quad 1500 \quad 0 \quad 0] \quad \text{% Main}\]

\[
\text{Main constraints coefficients}\]

\[
b = [78015.263 \quad 118239.176 \quad 17570 \quad 9022.5] \quad \text{%RHS of the main}\]

\[
\text{Constraints}\]

\[
lb = [0 \quad 0 \quad 0];\]

\[
up = [];
\]

\[
Aeq = [];
\]

\[
beq = [];
\]

\[
[y,fval] = \text{linprog}(f,A,b,Aeq,beq,lb,up)
\]

Objective function coefficients
-90000  -104333  -64333

Main constraints coefficient

\[
A = \begin{bmatrix}
100 & 1300 & 1300 \\
0 & 3300 & 0 \\
0 & 333 & 333 \\
1500 & 0 & 0
\end{bmatrix}
\]

RHS COEFFICIENT

\[
b = 1.0 \cdot 10^5 \times \begin{bmatrix}
0.7802 & 1.1824 & 0.1757 & 0.0902
\end{bmatrix}
\]

Optimal solution found.

\[
v = 6.0150
\]

35.8301

16.9327

Optimal DUAL objective value = \(-5.3689 \cdot 10^6\)
Solution 11.4

To find optimal values for:

DUAL LP with Supercapacitor BCAP3400

USING

Weighted sum optimization method

\[
f = [-90000\, -104333\, -64333] \quad \% \text{Weighted objective function coefficients}
\]

\[
A = [100\, 1300\, 1300;\; 0\, 8500\, 0;\; 0\, 333\, 333;\; 1500\, 0\, 0] \quad \% \text{Main constraints coefficients}
\]

\[
b = [78015.263\, 278417.604\, 17570\, 9022.5] \quad \% \text{RHS of the Main constraints}
\]

\[
\text{lb} = [0\, 0\, 0];
\]

\[
\text{up} = [];
\]

\[
\text{Aeq} = [];
\]

\[
\text{beq} = [];
\]

\[
[v, fva] = \text{linprog}(f, A, b, \text{Aeq}, \text{beq}, \text{lb}, \text{up})
\]

Objective function coefficients

-90000  -104333  -64333
Main constraints coefficient

\[
A = \begin{bmatrix}
100 & 1300 & 1300 \\
0 & 8500 & 0 \\
0 & 333 & 333 \\
1500 & 0 & 0 \\
\end{bmatrix}
\]

RHS COEFFICIENT

\[
b = 1.0 \times 10^5 \times (0.7802 \ 2.7842 \ 0.1757 \ 0.0902)
\]

Optimal solution found.

\[
v = 6.0150 \\
32.7550 \\
20.0078
\]

Optimal DUAL objective value = 5.2459e+06
Solution 11.5

To find optimal values for:

PRIMAL LP with Supercapacitor BOD

USING

PREEMPTIVE optimization method

STEP 1:

\[ f = \begin{bmatrix} 78 & 118.22 & 17.55 & 9 \end{bmatrix} \] \quad \text{%Objective function coefficients}

\[ A = \begin{bmatrix} -100 & 0 & 0 & -1500 \\ -1300 & -3300 & -333 & 0 \\ -1300 & 0 & -333 & 0 \end{bmatrix} \] \quad \text{% Main constraints coefficients}

\[ b = \begin{bmatrix} -90000 \\ -104333 \\ -64333 \end{bmatrix} \] \quad \text{% RHS of the main constraints}

\[ lb = [0 \ 0 \ 0 \ 0]; \]
\[ up = []; \]
\[ Aeq = []; \]
\[ beq = []; \]
\[ [x,fval] = \text{linprog}(f,A,b,Aeq,beq,lb,up) \]

STEP 2:
f = [0.5263 0.9176 1 1.25] \hspace{1cm} \% Objective function coefficients

A = [78 118.22 17.55 9; -100 0 0 -1500; -1300 -3300 -333 0; -1300 0 -333 0] \hspace{1cm} \%
Main constraints Coefficients

b = [5363.5 -90000 -104333 -64333] \hspace{1cm} \% RHS of the main
\hspace{1cm} \% constraints

[x,fval] = linprog(f,A,b,Aeq,beq,lb,up)

\textbf{STEP 3:}

f = [1 1 1 1] \hspace{1cm} \% Objective function coefficients

A = [0.5263 0.9176 1 1.25; 78 118.22 17.55 9; -100 0 0 -1500; -1300 -3300 -333 0; -1300 0 -333 0] \hspace{1cm} \% Main constraints
\hspace{1cm} \% coefficients

b = [279.3118 5363.5 -90000 -104333 -64333] \hspace{1cm} \% RHS of the
\hspace{1cm} \% main constraints

[x,fval] = linprog(f,A,b,Aeq,beq,lb,up)
Results to preemptive optimization option

1st STEP

Cost objective function coefficients

\[ [78.0000 \quad 118.2200 \quad 17.5500 \quad 9.0000] \]

Initial Main constraints coefficient

\[ \begin{align*}
A &= -100 \quad 0 \quad 0 \quad -1500 \\
   &\quad -1300 \quad -3300 \quad -333 \quad 0 \\
   &\quad -1300 \quad 0 \quad -333 \quad 0
\end{align*} \]

Initial RHS COEFFICIENT

\[ b = [-90000 \quad -104333 \quad -64333] \]

1st Iteration Optimal solution found.

\[ M_B = 0 \]

\[ M_C = 12.1212 \]

\[ M_{FC} = 193.1922 \]

\[ M_H = 60.0000 \]

1st Iteration Optimal objective value = \(5.3635e+03\)

2nd STEP

2nd Iteration Objective function coefficients

\[ 0.5263 \quad 0.9176 \quad 1.0000 \quad 1.2500 \]
2nd Iteration Main constraints coefficient

A = 1.0e+03 * [0.0780  0.1182  0.0175  0.0090
                 -0.1000  0  0 -1.5000
                 -1.3000 -3.3000 -0.3330  0
                 -1.3000  0 -0.3330  0]

2nd Iteration RHS COEFFICIENT

b =1.0e+05 *[0.0536  -0.9000  -1.0433  -0.6433]

2nd Iteration Optimal solution found.

MB = 0.0008
MC =12.1212
MFC =193.1890
MH =59.9999

2nd Iteration Optimal objective value = 279.3118

3rd & Final Step

Final Iteration Objective function coefficients

[1 1 1 1]

Final Iteration Main constraints coefficient

A = 1.0e+03 *[0.0005  0.0009  0.0010  0.0013
                 0.0780  0.1182  0.0175  0.0090]
\begin{align*}
\begin{pmatrix}
-0.1000 & 0 & 0 & -1.5000 \\
-1.3000 & -3.3000 & -0.3330 & 0 \\
\end{pmatrix}
\end{align*}

**Final Iteration RHS COEFFICIENT**

\[ b = [279.3118 \ 5363.5 \ -90000 \ -104333 \ -64333] \]

**Final Iteration Optimal solution found.**

\[ M_B = 0.0008 \]

\[ M_C = 12.1212 \]

\[ M_{FC} = 193.1890 \]

\[ M_H = 59.9999 \]

**Final Iteration Optimal objective value = 265.310**
APPENDIX  B

Power required calculations

\[ M := 1400 \quad \text{Mass of the vehicle in Kg} \]

\[ g := 9.8 \quad \text{Gravity acceleration constant in } m \cdot s^{-2} \]

\[ f_r := 0.01 \quad \text{Rolling resistance coefficient} \]

\[ \rho_a := 1.25 \quad \text{Air density} \]

\[ C_d := 0.335 \quad \text{Drag coefficient} \]

\[ V := 96.558 \quad \text{Rated velocity of the vehicle} \]

\[ 3.6 \quad m \cdot s^{-1} \]

\[ J_W := 1.224 \quad \text{Moment of inertia of a wheel/tire} \]

\[ Kg \cdot m^2 \]
\[ r_r := 0.3 \quad \text{Rolling radius} \quad \text{m} \]

\[ J_{Di} := 6.78 \quad \text{Moment of inertia of motor and driveline} \]

\[ a := 96.558 = 1.788 \quad \text{Acceleration of the vehicle} \]

\[ 3.6 \cdot 15 \]

\[ \eta_M := 0.9 \quad \text{Motor efficiency} \]

\[ \eta_t := 0.92 \quad \text{Transmission efficiency} \]

\[ \eta_f := 0.92 \quad \text{Final drive gear efficiency} \]

\[ Kg \cdot m^2 \]

\[ m \cdot s \]

\[ A_f := 2.47 \quad \text{Frontal area} \quad \text{m} \]

\[ G_r := 0.065 \quad \text{Grade of the road} \]

**Power required at rated velocity with no grade and no Acceleration** (\( P_{NAC+NOGRD\text{rated}} \))

\[
P_{NAC+NOGRD\text{rated}} = \left( M \cdot g \cdot f_r + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 \right) \cdot \frac{V}{1000 \cdot \eta_M \cdot \eta_T \cdot \eta_F} \]

\[ P_{NAC+NOGRD\text{rated}} = 17.931 \text{ KW} \]

**Power required at maximum velocity with no grade and no Acceleration** (\( P_{NAC+NOGRD\text{max}} \))
\[ V = \frac{128.75}{3.6} = 35.764 \]

\[
P_{NAC+Nogradmax} = \left\{ M \cdot g \cdot f_r + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 \right\} \frac{V}{1000 \cdot \eta_M \eta_T \eta_F}
\]

\[ P_{NAC+Nogradmax} = 37.497 \text{ KW} \]

Power required at rated velocity with grade and no Acceleration (\(P_{NAC}\))

\[ V = \frac{128.75}{3.6} = 35.764 \]

\[
P_{NAC} = \left\{ M \cdot g \cdot f_r \cdot \cos(\theta) + \frac{1}{2} \cdot \rho \cdot A_f \cdot C_D \cdot V^2 + M \cdot g \cdot G_r \right\} \frac{V}{1000 \cdot \eta_M \eta_T \eta_F}
\]

\[ P_{NAC} = 37.774 \text{ KW} \]

Power required at rated velocity with no grade and with Acceleration (\(P_{AC+Nogradrated}\))

\[ V = \frac{128.75}{3.6} = 35.764 \]
\[ P_{AC+NOGRD\text{rated}} = \left\{ Mg f_r + \frac{1}{2} \rho A_f C_D V^2 + \left( M + \frac{4 * J_w + (J_{DL})}{r_f^2} \right) a \right\} \frac{V}{1000 \cdot \eta_M \eta_T \eta_F} \]

\[ P_{AC+NOGRADE\text{rated}} = 114.242 \text{ KW} \]

Power required at maximum velocity with no grade and with Acceleration (\( P_{AC+NOGRADE\text{max}} \))

\[ V = \frac{128.75}{3.6} = 35.764 \]

\[ P_{AC+NOGRD\text{max}} = \left\{ Mg f_r + \frac{1}{2} \rho A_f C_D V^2 + \left( M + \frac{4 * J_w + (J_{DL})}{r_f^2} \right) a \right\} \frac{V}{1000 \cdot \eta_M \eta_T \eta_F} \]

\[ P_{AC+NOGRADE\text{max}} = 165.918 \text{ KW} \]