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Performance analysis of adaptive algorithms and enhancement using Kalman filter

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ABSTRACT

PERFORMANCE ANALYSIS OF ADAPTIVE ALGORITHMS AND ENHANCEMENT USING KALMAN FILTER

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A new platform for designing robust adaptive filter is introduced. An adaptive filter is a filter that adjusts its transfer function according to an optimizing adaptive algorithm. The efficiency of the adaptive algorithm being used plays a key role in the working of the adaptive filter. The Least Mean Square (LMS) and the Normalized Least Mean Square (NLMS) adaptive algorithms are studied. The core part of this research is to use the theory of Kalman filter and use it in adaptive filtering process. The adaptive filtering problem can be updated to a new theory of state estimation problem. The main objective of the research is to evaluate and characterize the efficiency of the adaptive algorithms being used in the adaptive filtering process. The adaptive filtering process will be carried out using different adaptive algorithms and its efficiency is measured in terms of filter convergence speed and the variation in the power of the error signal with changes in the input signal power obtained during the adaptation process. A Kalman-based Normalized Least mean Square algorithm which is developed outperforms the existing Least Mean Square (LMS) and Normalized Least Mean Square (NLMS) Algorithms. The simulations are carried out by using MATLAB.

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NORTHERN ILLINOIS UNIVERSITY
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DECEMBER 2015

PERFORMANCE ANALYSIS OF ADAPTIVE ALGORITHMS AND
ENHANCEMENT USING KALMAN FILTER

BY

ANUSHA RAVVA
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A THESIS SUBMITTED TO THE GRADUATE SCHOOL
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE

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Thesis Director:

Dr. Mansour Tehernezhadi

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DEDICATION

To all my teachers who have moulded me into what I am and my friends and family.

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1 INTRODUCTION

1.1 Motivation

Digital Signal Processing (DSP) plays a major role in current technical advancements. When a particular signal changes with time, we are not only concerned about the magnitude but also how it changes. Oscilloscopes and other analog devices were used to make the record of the continuously changing signals in the time domain. Later on when digital computers were used for this purpose, the magnitude of the signal is sampled only at fixed intervals of time with a complete loss of continuity in between. The mathematics of the digital signal processing can be used to analyze the signal in both the time and frequency domains, which means we cannot only have the information about the magnitude of the signal which is varying by time but also about the amplitudes of the signals.

Digital signal processing (DSP) is used in the major fields of technology such as noise cancellation, echo cancellation and system identification. However, the standard DSP techniques are not quick solution for the problems and obtain acceptable results.

Adaptive filtering techniques must be implemented to have accurate, quick solutions and fast convergence to the solution [1]. An adaptive filter is defined as a self-adjusting system which functions according to a recursive algorithm, which makes it possible to have satisfactory results when the knowledge of the relevant statistics is not available [1].

Adaptive filters can be classified into two groups; one is the linear adaptive filters in which the estimate of the desired response is calculated using a linear combination of available

set of inputs to the filter and otherwise it is called as a nonlinear adaptive filter, which is the second group of adaptive filters

1.2 Literature Review

Haykin [1] made a clear and brief explanation about the adaptive filter. Adaptive filters can be divided into two main groups: linear adaptive filters and the other being nonlinear adaptive filters. They can further be divided into supervised and unsupervised adaptive filters.

Widrow and Hoff [2] developed the first adaptive algorithm which was used in designing a linear adaptive filter which is called the Least Mean Square algorithm (LMS). It is famous for the simplicity of implementation and it is a robust method. It is basically a stochastic gradient algorithm. The LMS algorithm has two disadvantages which is the slow convergence rate and sensitivity to the eigenvalue spread.

J. Homer [3] explains that the Least Mean Square adaptive algorithm has the slowest convergence rate with the increase in the length of the filter. A good expression is proposed regarding the relationship between the convergence speed and the length of the filter.

Cho et al. [4] proposed NLMS algorithm in order to increase the mean square error performance and also the convergence rate. One of the drawbacks of the NLMS algorithm is higher computational complexity and there is higher mismatch between the true and the estimated coefficients.

Mathe et al. [5] described about the Kalman filter used for the speech enhancement. The Kalman filter is based on state space formulation of a continuous or discrete system but it has high computational complexity involved in it.

Lopes et al. [6] described the Kalman-based NLMS algorithm by drawing similarities between the parameters used in Kalman algorithm. An idea of implementing Kalman filter in an adaptive way was proposed. It has an adaptive filter update and the step size update as listed in the NLMS algorithm. It has good convergence rate compared to the NLMS algorithm.

Shanmugan et al. [7] carried out a study about the common problem being occurred in the speech processing which is noise. Different adaptive algorithms were used to perform the noise cancellation in the noise-added input speech signal and the efficiencies of different algorithms were compared based on a few parameters such as mean square error, mis-adjustment, speed of convergence and output power of the speech signal being transmitted.

Shinde et al. [8] described the new framework for designing efficient adaptive filters. System identification which is one of the greatest applications of adaptive filters is explained using different algorithms and the superiority of the algorithms is compared by taking a few parameters into consideration. A theoretical model for knowing the steady-state behavior and a proof of filter convergence is provided.

Mendiratta and Jha [9] explain about the steady-state additive noise. The method explained has two inputs: one is the primary and the other is the reference signal. The primary signal has the input signal which is corrupted with the noise and the reference signal is a noise signal which is not correlated with the signal but related to the noise in some way. The reference signal input is adaptively filtered to become equal to the primary input noise signal to produce the clean uncorrupted signal which is the adaptive noise cancellation output. Least Mean Square algorithm was used to update the adaptive filter coefficients.

1.3 Objective

The research work is aimed at developing an adaptive filter which uses different algorithms to update its filter coefficients. The efficiency of each adaptive algorithm is analyzed taking mean square error convergence curves for each algorithm

The main objective of the work is to analyze the working of Least Mean Square (LMS) algorithm and Normalized Least Mean Square (NLMS) algorithm; the key point is to apply Kalman filter to the adaptive filtering algorithm. The Kalman filter can be used in adaptive filtering by making a few correspondences. The Normalized Least Mean Square (NLMS) algorithm can be combined with the Kalman and this new Kalman-based NLMS algorithm optimizes the key parameters such as the filter convergence speed, mean square error and the power of output signal.

Our efforts will be focused on analyzing the efficiency of the adaptive algorithms by comparing the variation in the power of error signals with the change in the power of the input signal and the stability of the algorithm.

2 ADAPTIVE FILTER

An adaptive filter is a self-modifying digital filter that adjusts its coefficients so as to minimize the error function, also called the cost function, which is the distance measurement between the reference signal and the output of the adaptive filter.

2.1 Adaptive Filtering

This is a process in which the parameters of a signal changes according to some criterion. Estimated mean square error is said to be one criterion to change the parameters. These are considered to be time varying since its parameters change continuously with time in order to satisfy the performance requirements. The performance can be judged only when we have a reference signal which is hidden in the approximation step of a fixed-filter design. The general setup of adaptive filter is shown in the Figure 2-1.

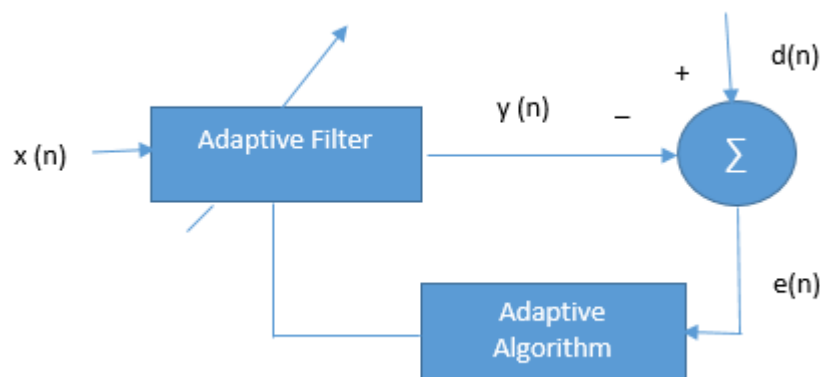


Figure 2-1 Block diagram of adaptive filter.

- n is the iteration number
- $x(n)$ denoted the input signal
- $d(n)$ denotes the desired signal
- $y(n)$ is the adaptive filter output
- $e(n)$ is the error signal which is defined as the difference of the adaptive filter output $y(n)$ and the desired signal $d(n)$. It is used to form an objective function which is required by the adaptive algorithm.

The calculated error signal $e(n)$ at the end of each iteration is used by an adaptive algorithm to find the appropriate updating coefficients for the adaptive filter. The minimization of this error signal $e(n)$ indicates that the adaptive filter output is becoming equal to the desired signal .

An adaptive filter is a filter which changes its transfer function taking an adaptive algorithm into consideration which generates the filter updates. To avoid the complexity of these optimization algorithm, most of the adaptive filters are digital filters.

A non-adaptive filter has static transfer function. Adaptive filters are used in some applications where the parameters of the desired processing operation is not known ahead of time. The adaptive filter uses the error signal as the feedback to update the transfer function according to the requirement. With the increase in the power of the digital signal processing, adaptive filters have become very common and are being used in mobile phones and other communication devices, digital cameras and medical monitoring equipment. The figure 2-2 demonstrates the working of the adaptive filters. Coefficients of the adaptive filter weight is given by w [10]. $x(n)$ represents the input speech vector samples, $y(n)$ represents the output of the adaptive filter, $d(n)$ is the desired signal, z^{-1} is the delay introduced between the samples

and $e(n)$ is the error estimated at time n [10]. The objective of the adaptive filter is to calculate the difference between the desired signal and the adaptive filter output [10]. The error signal is fed back to the adaptive filter and the coefficients of the adaptive filter are updated according to an adaptive algorithm which is used to minimize the difference, also called cost function.

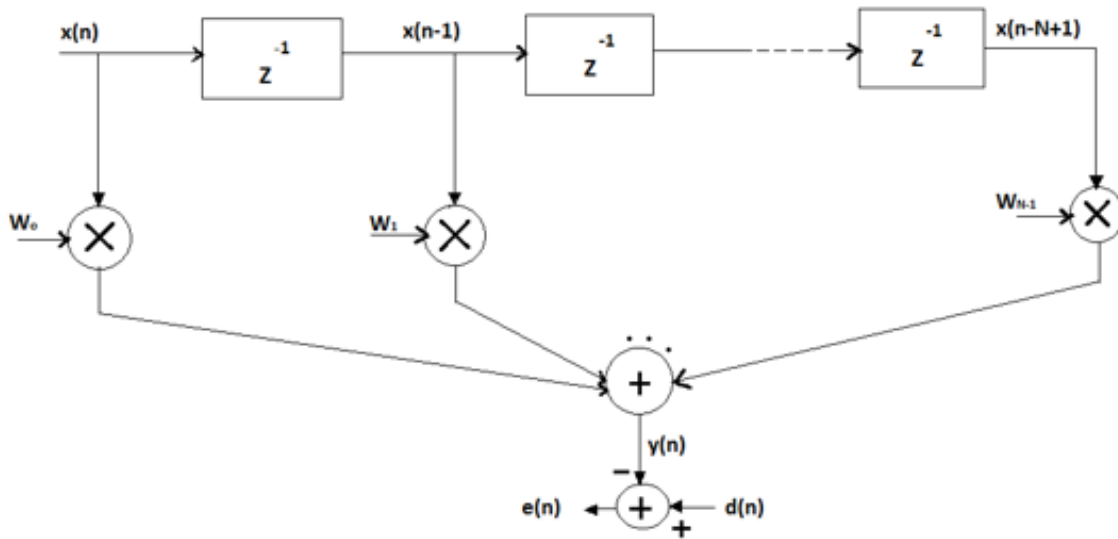


Figure 2-2 Detailed block diagram of adaptive filter.

The Figure 2-2 shows the detailed view of the adaptive filter coefficients. Here w represents the tap weights, z^{-1} represents the delay of one sample period, $x(n)$ is the input signal given, $d(n)$ is the desired signal and $e(n)$ is the error signal.

2.2 Applications of Adaptive Filters

There are many applications of adaptive filters, which include Noise Cancellation, Echo Cancellation, Adaptive feedback Cancellation, System Identification, etc.

2.2.1 System Identification

Among all the applications available for the adaptive filters, system identification is one of the most interesting applications. The error signal measured at the end of each iteration is used to update the adaptive filter coefficients. This process is done till the output of the adaptive filter becomes equal to the desired signal. This application in speech processing helps in making remarkable developments in research of automation and prediction.

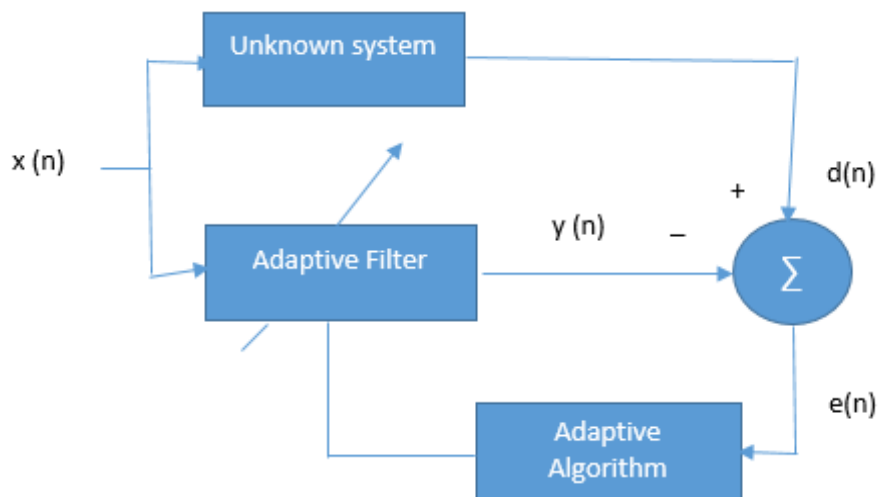


Figure 2-3 Block diagram for system identification.

System identification is one of the prominent applications of the adaptive filters. Adaptive filters are used to identify an unknown system which includes finding the response of an unknown communication channel or the frequency response of any system. The Figure 2-3 shown below is one of the possible structures of the system identification.

An unknown system is placed in parallel to the adaptive filter; when $e(n)$ is equal to zero then the adaptive filter response is very close to that of the unknown system by which we can determine the unknown system's characteristics. In this process of system identification, the same input feeds to the adaptive filter and the unknown system.

2.2.2 Noise Cancellation

One of the most common problems that occur in speech processing is the interference of unwanted noise in the speech signal being transmitted. This interference may come from the sources such as traffic crowds, ventilation equipment, or reverberation.

Noise cancellation is a process in which the background noise is filtered from the actual speech signal and helps the speech signal from degradation. The degradation of the speech signal causes a lot of problems in some applications such as speech recognition. The quality of the speech signal is judged by its clarity and pleasantness.

Noise cancellation is also one of the applications of the adaptive filters. In this process an adaptive filter will be helpful to remove the noise from the signal in real time. The desired signal $d(n)$ is the signal which has noise-added signal, which means it contains both noise $n(n)$ and the desired information. To eliminate this unnecessary noise from the signal another noise signal $n'(n)$ is fed to the adaptive filter which is not related to the original speech signal but correlated to the noise signal $n(n)$ in some way as shown in Figure 2-4.

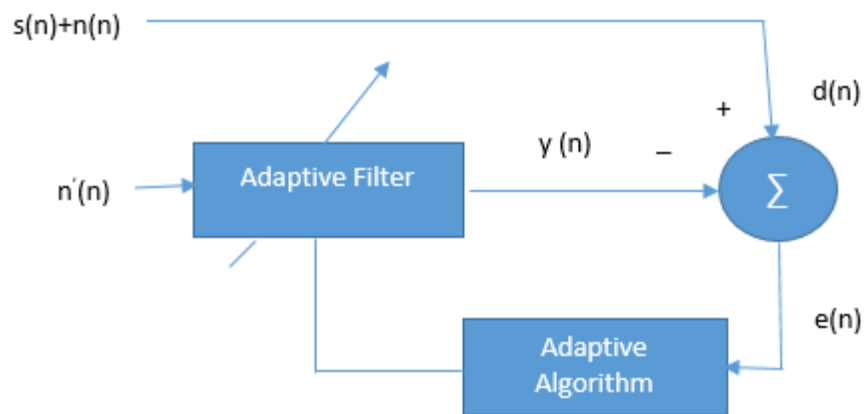


Figure 2-4 Block Diagram for Noise Cancellation.

The adaptive filter is a filter which adjusts the filter parameters according to some adaptive algorithm, thereby reducing the error function which is calculated as the difference between the desired signal and the output of the adaptive filter, resulting in a clean signal obtained as $e(n)$.

2.2.3 Echo Cancellation

In today's telecommunication systems echo cancellation is the most crucial point to be taken into care. This occurs when an audio source operates in the full duplex mode. Hands-free speaker phone is a good example of this. The interference caused due to the acoustic echo in the actual speech signal distracts the users and also reduces the quality of the speech signal being transmitted. Adaptive filters are the filters which change the filter characteristics in order

to minimize the distance between the desired signal and the output of the adaptive filter. The Figure 2-5 shows the block diagram for adaptive echo cancellation implementation.

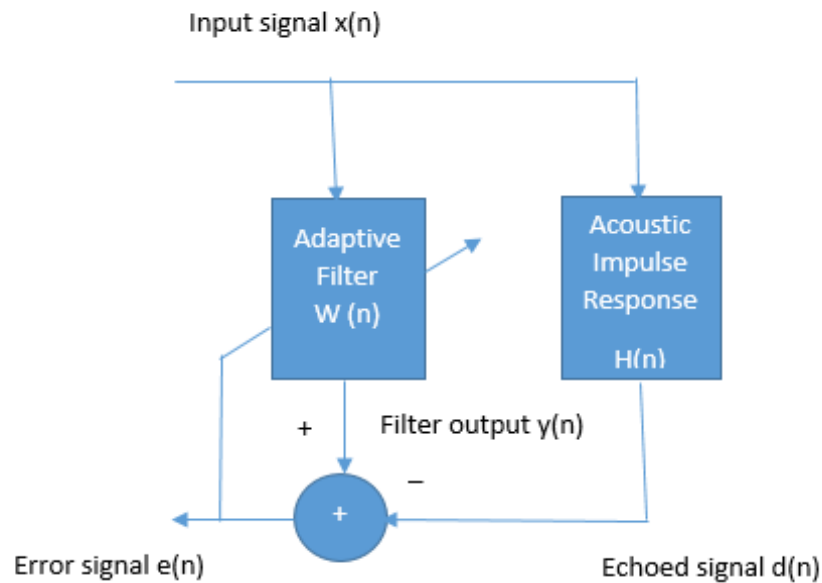


Figure 2-5 Block diagram for echo cancellation.

Acoustic echo occurs when an input speech signal is reverberated which results in the original speech signal plus the time-delayed emulated versions of the speech signal. If a system which has an active loud speaker and a microphone is considered, then the system is said to be working in the full-duplex mode. The speech signal which is received by the system contains the output of the loud speaker which is nothing but the echoed signal and then it is returned to the system which is fed as the input to the microphone. These echoed signals contain the images of the original signal. This causes the degradation of the quality of the speech signal.

Adaptive filters are dynamic filters which alter or change their parameters to meet with an optimized desired signal. The adaptive filter uses an algorithm to change its filter parameters to make the adaptive filter output $y(n)$ equal to the desired signal $d(n)$. The error signal is calculated, which can also be referred to as the cost function.

The impulse response of the filter considered in the environment is given by $H(n)$. The adaptive filter is represented by $W(n)$, which is the key responsible factor in removing the echo signal [10].

The adaptive filter tries to make the output of the adaptive filter $y(n)$ and the desired signal $d(n)$ which is the echoed signal to be equal. At the end of each iteration the error signal $e(n)$ is fed back to the adaptive filter to update its filter coefficients. At some point the error signal becomes equal to the echo-cancelled speech signal [10].

In this case of the echo cancellation, the output of the adaptive filter and the unwanted echo signal have the same amount of data. The error signal becomes zero at a point of time. This is the case where the echoed signal would be cancelled completely and the far user will not hear any of the original speech signals returned to them.

2.3 Adaptive Algorithm

There are a wide variety of adaptive filter algorithms available. These algorithms when applied through the filter result in adaptive filters. These filters are used to remove the unwanted noise present in a speech signal to enhance the speech signal being transmitted. The following aspects are taken into account for comparing the wide variety of algorithms.

2.3.1 Filter Structure

The transfer function of the adaptive filters gives relationship between the input and the output. Due to the efficiency and simplicity of the transversal filter associated with the standard duration impulse response (FIR) filter, it is mostly used in the adaptive filters. This is the factor which greatly influences the speed of the adaptation process and computational complexity of the adaptive algorithm.

2.2.2 Rate of Convergence, Mis-Adjustment and Tracking

The coefficients of the adaptive filter can be made to converge slow or fast to the optimum solution in a noiseless situation, but in general they will not reach any optimum values but they stay close to the optimum values. Mis-adjustment is nothing but the measurement of the estimated and the optimum in steady state.

2.3.3 Computational Aspects

The adaptive filter must consider the computational complexity and limited-precision representation of the associated signals and coefficients due to the desired real-time characteristic. The main reason behind making fast versions of more complex algorithms is to reduce the computational requirements and also to reduce the amount of memory needed to run these algorithms in the practical world.

3 ADAPTIVE ALGORITHMS

3.1 LMS Algorithm

Least Mean Square (LMS) algorithm is a good approximation of steepest descent procedure. It does not require any matrix inversions and to solve this difficulty we can use an efficient manner where the coefficient vector w is iteratively adjusted with a time instant of n . The initial weight vector of the filter is set to be zero. The LMS algorithm is then applied using the following equations:

$$y(n) = w^T(n) * x(n)$$

$$e(n) = d(n) - y(n)$$

$$w(n + 1) = w(n) + \mu * x(n) * e(n)$$

where

- $x(n)$ is the input speech signal
- $y(n)$ is the output of the adaptive filter
- $d(n)$ is the desired signal
- $w(n)$ is the tap weight vector
- μ is the step size, $\mu_{max} = \frac{2}{\lambda_{max}}$
- $e(n)$ is the error signal which is given as a feed back to the adaptive filter

Step size is represented by μ , which is parameter, and is used to know the convergence rate of the algorithm. The higher value of the step size shows that the algorithm converges very fast but we need to make sure that the value is within the bounds to avoid the divergence of the algorithm. The bound on μ is not known exactly because it depends on different statistics

which use a scalar value. Higher values of μ result in the use of higher tap weights which cause distortion in the combiner outputs which increases the value of mean square error. The coefficients of the filter are updated using a cost function which is represented by a factor $[\mu * x(n) * e(n)]$.

Disadvantages:

1. It has a difficulty of choosing efficient step size μ which is the stabilizing factor of the algorithm.
2. It is very sensitive to the scaling of the input signals.

3.2 Normalized Least Mean Square Algorithm (NLMS)

In the traditional LMS algorithm when the step size which is the convergence factor is large the algorithm experiences instability and a gradient noise amplification. To overcome the disadvantages that are found in LMS algorithm we use the Normalized Least Mean Square (NLMS) algorithm. The NLMS algorithm has an advantage over LMS algorithm by normalizing the power of the input signal, which means the variations of the signal at the input of the filter are taken into account and a normalized step size parameter is decided.

$$y(n) = w^T(n) * x(n)$$

$$e(n) = d(n) - y(n)$$

$$w(n + 1) = w(n) + \frac{\alpha}{c + \|x\|^2} * e(n) * x(n)$$

where

- $x(n)$ is the input speech signal
- $y(n)$ is the output of the adaptive filter
- $d(n)$ is the desired signal
- $w(n)$ is the tap weight vector
- $e(n)$ is the error signal which is given as a feed back to the adaptive filter
- α is the NLMS adaptation constant, which optimizes the convergence rate of the algorithm
- c is the constant term for normalization and is always less than 1

NLMS algorithm has two advantages over the LMS algorithm, which are the rate of convergence and the stable behavior for a known range of parameter values.

Disadvantages:

1. The NLMS algorithm has high computational complexity, It requires a minimum of one additional multiply, divide and addition over the LMS algorithm to shift the input data.
2. Tradeoff between convergence speed and steady-state error.

3.3 Kalman Filter

In general the Kalman filter is based on state space formulation. It can be considered to be a continuous or discrete time system. The Kalman filter gives the estimate of the state of the system given a set of inputs [6]. Kalman filters can also be considered as the generalization of Weiner filter. The Kalman filter accommodates the vector signals and noises which are non-stationary [11].

It can also be approached as a Bayesian sequential MMSE estimator of a signal which is added with noise and the system is defined as a state model [11]. The Bayesian linear model form can be assumed to fit the data we are transmitting as shown in Figure 3-1.

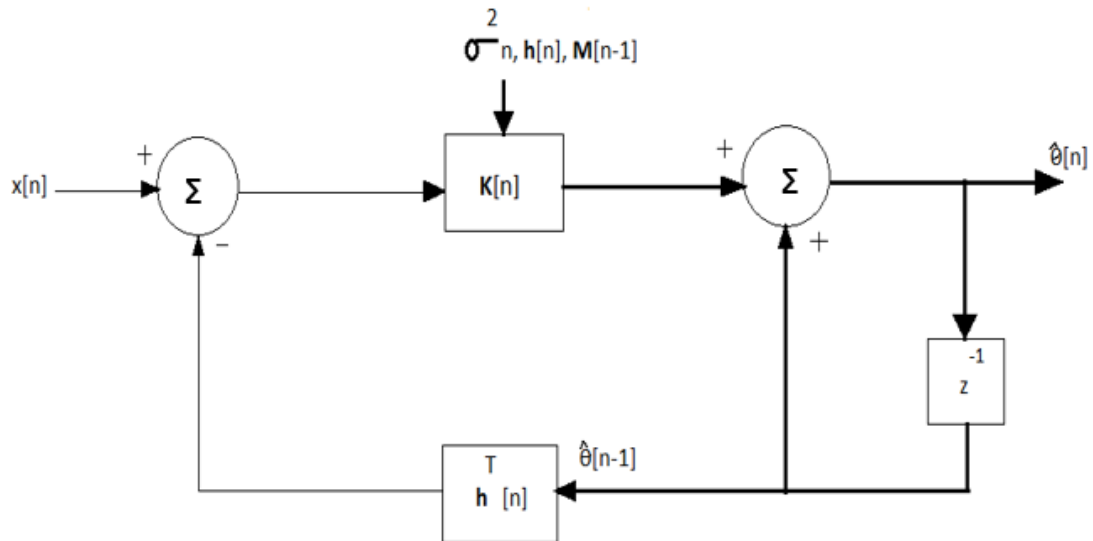


Figure 3-1 Linear minimum mean square estimator.

The data can be described in the form of the Bayesian linear model form:

$$x = H\theta + W$$

where x is an $N \times 1$ data vector, H is a $N \times P$ observation matrix and θ is a $P \times 1$ random vector.

W is also a random vector which has a variance σ_n^2 .

The error covariance matrix which is also called as the minimum mean square error matrix denoted by M and the order of this matrix is $P \times P$.

The Bayesian estimation has two steps: one is the estimator update and the other is the Mean square error update.

Estimator Update:

$$K[n] = \frac{M[n-1]h[n]}{\sigma_n^2 + h^T[n]M[n-1]h[n]}$$

$$\hat{\theta}[n] = \hat{\theta}[n-1] + K[n](x[n] - h^T[n]\hat{\theta}[n-1])$$

Mean square error matrix update:

$$M[n] = (I - K[n]h^T[n])M[n-1]$$

where K is considered to be the Kalman gain in our scenario of designing the Kalman filter and the order of K is P×1.

To use Kalman filter in adaptive filtering process we make a few correspondences with the Bayesian way of approach to the traditional way of adaptive filtering. The input vector x(n) taken in general adaptive filtering is considered to be h(n) and the filter weight vector w(n) used in the traditional adaptive filtering is considered to be the random vector $\theta(n)$ in the present Kalman filter scenario. It can be summarized as $\theta(n) \rightarrow w(n)$ and $h(n) \rightarrow x(n)$.

In implementing the traditional way of adaptive filtering we require an error update equation and also filter coefficients update equations. So the above defined equations can be reformulated as:

Error update:

$$e(n) = d(n) - \text{dot}(X, W)$$

Adaptive filter update:

$$w[n] = w[n-1] + K[n]e(n)$$

where the Kalman gain $K[n]$ is given by

$$K[n] = \frac{M[n-1]x[n]}{\sigma_n^2 + x^T[n]M[n-1]x[n]}$$

$$M[n] = (I - K[n]x^T[n])M[n-1]$$

where $M[n]$ is a mean square error matrix

$x[n]$ is the input signal vector

$K[n]$ is the Kalman gain

$w[n]$ is the adaptive filter update

3.4 Kalman NLMS Algorithm

We can combine the algorithm based on Kalman filter with the Normalized Least Mean Square (NLMS) algorithm, a new type of algorithm called Kalman NLMS algorithm can be defined. The equations governing this algorithm are given by:

Error update:

$$e(n) = x(n) - d(n)$$

Adaptive filter update:

$$w(n+1) = w(n) + K(n)e(n)x(n)$$

where $K(n) = \frac{M(n)}{x^T(n)M(n)x(n) + \sigma^2}$

$$M(n+1) = (I - M_d(n)) * M(n)$$

where $M_d(n) = \text{diag}[K(n) * x(n)]$

$e(n)$ is the error signal

$x(n)$ is the input signal vector

$d(n)$ is the desired signal

$w(n)$ is the adaptive filter tap weight vector

$M(n)$ is the variance matrix which should be made diagonal carefully by extracting its diagonal elements. The factor $diag[K(n) * x(n)]$ is calculated to reduce the computational complexity involved in the Kalman filter.

Stability factor:

In Normalized Least Mean Square (NLMS) algorithm the stability factor step size μ is given by using the squared norm of the input vector as a factor.

$$\mu = \frac{1}{\|x\|^2}$$

where μ is the step size.

The selection of the maximum step size μ in the Least Mean Square (LMS) Algorithm is given by the following equation:

$$\mu_{max} = \frac{2}{\lambda_{max}}$$

where λ_{max} is the maximum eigenvalue.

The main factor to be considered while having comparison between the Least Mean Square (LMS) algorithm and the Normalized Least Mean Square (NLMS) Algorithm is the step size. The eigenvalue spread plays a main role in the stability of these two algorithms. As the eigenvalue spread increases there will be a decrease in the convergence rate of the algorithm. For non-stationary signals, the step size of Normalized Least Mean Square (NLMS) algorithm should always be greater than the step size of the Least Mean Square (LMS) algorithm.

4 RESULTS AND DISCUSSION

4.1 Simulation Model

The simulation model for this research is shown in the figure 4-1. An input speech signal is given to the system using the predefined MATLAB function. This is denoted by $x(n)$. A FIR filter is designed using a few known parameters and it is denoted by $h(n)$. The input speech signal is convolved with the FIR filter and produces a signal $d(n)$ which is the desired signal. The input speech signal is also fed simultaneously to the adaptive filter which produces an output $y(n)$. The error signal is calculated at the end of each iteration and it is fed back to the adaptive filter to update its filter coefficients and eventually $y(n)$ becomes equal to the desired signal $d(n)$, which implies that the filter coefficients of both the filters are equal when the error signal becomes zero.

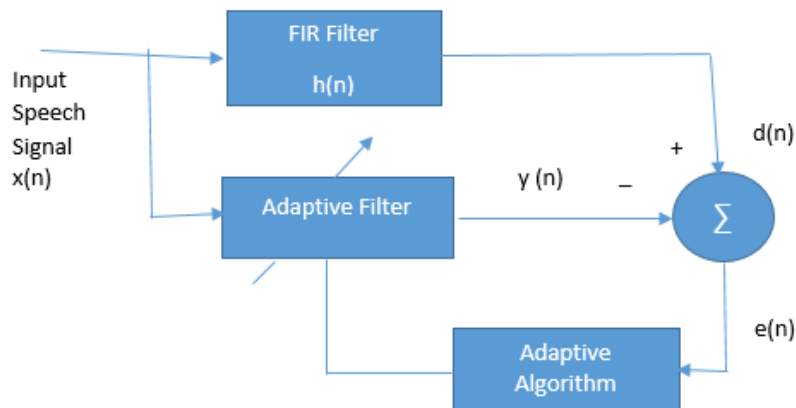


Figure 4-1 Simulation model.

The key point to be noticed and analyzed is the speed of convergence of the adaptive filter coefficients. The convergence speed depends on the adaptive algorithm being used in the system, which is responsible for updating the adaptive filter coefficients and producing an output signal $y(n)$.

The power of the error signal and the power of the desired signal are taken into consideration and analyzed. When the input signal power changes there will be a lot of changes observed in the power of the error signal. The adaptive algorithm which maintains the stability in the power of the error without any fluctuations with the changes in the input signal power is said to be efficient. In this thesis work Least Mean Square (LMS) algorithm, Normalized Least Mean Square (NLMS) algorithm, Kalman filter and a new Normalized Least Mean Square algorithm based on Kalman filter are used. All these algorithms are used in the adaptive algorithm block shown in the Figure 4-1.

The performance analysis of the mentioned algorithms is carried out in a systematic way using MATLAB.

Input Speech Signal:

Results presented in the Figure 4-2 are for a speech signal which is given as an input using the MATLAB command 'wavrecord' and it is sampled at 8 KHz.

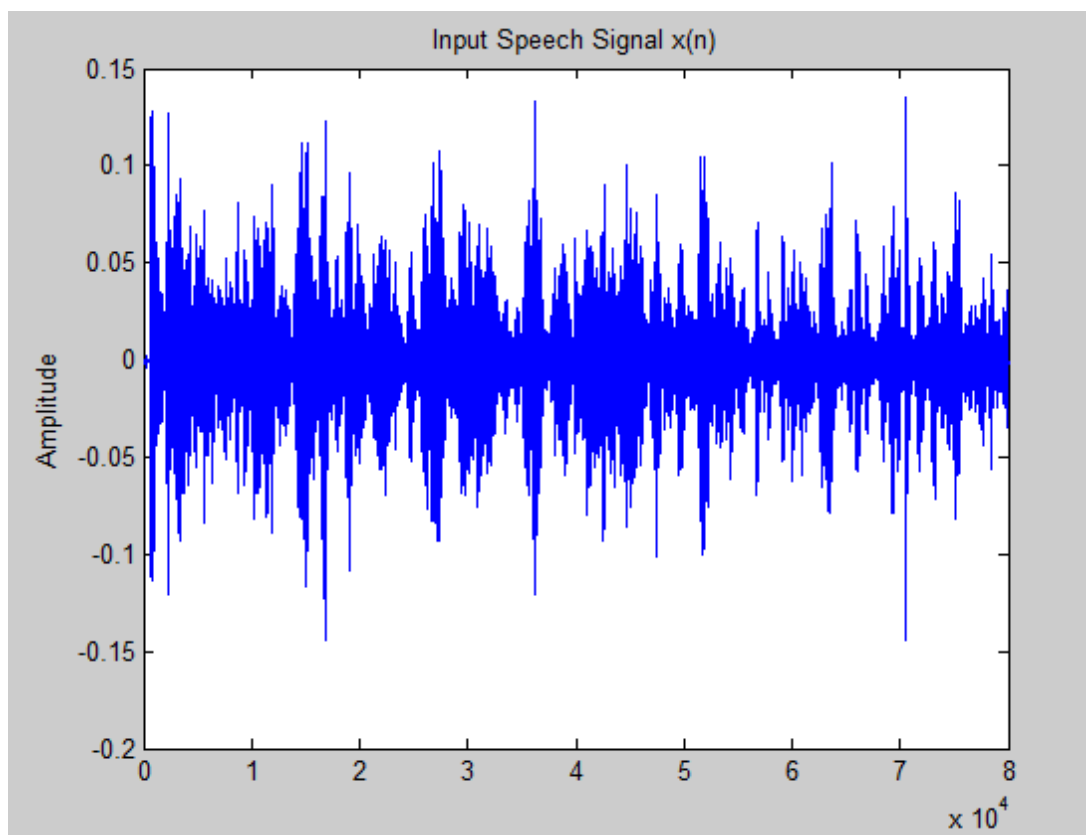


Figure 4-2 Input speech signal.

FIR Filter:

A filter $h(n)$ is designed using the filter coefficients $h = [-.1 \ -.15 \ 0.05 \ 0.5 \ 0.9 \ 0.4 \ -0.3 \ -0.2 \ -.05]$

and adding additional zeros on both the sides and finally h is defined in MATLAB as

$h = [\text{zeros}(1,90) \ h \ \text{zeros}(1,100)]$. The designed filter $h(n)$ is as represented in the Figure 4-3.

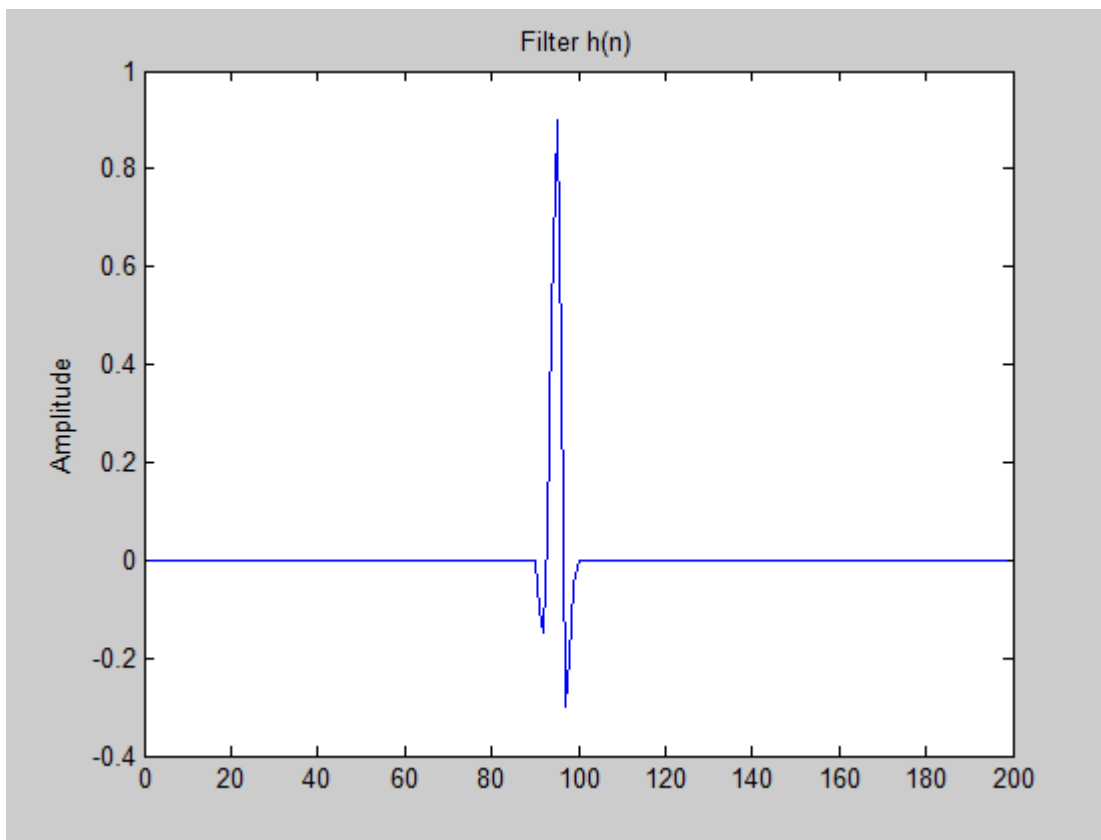


Figure 4-3 FIR filter response.

Desired Signal:

The desired signal $d(n)$ is generated by convolution of the input speech signal $x(n)$ and the impulse response $h(n)$. The desired signal generated is as shown in the Figure 4-4.

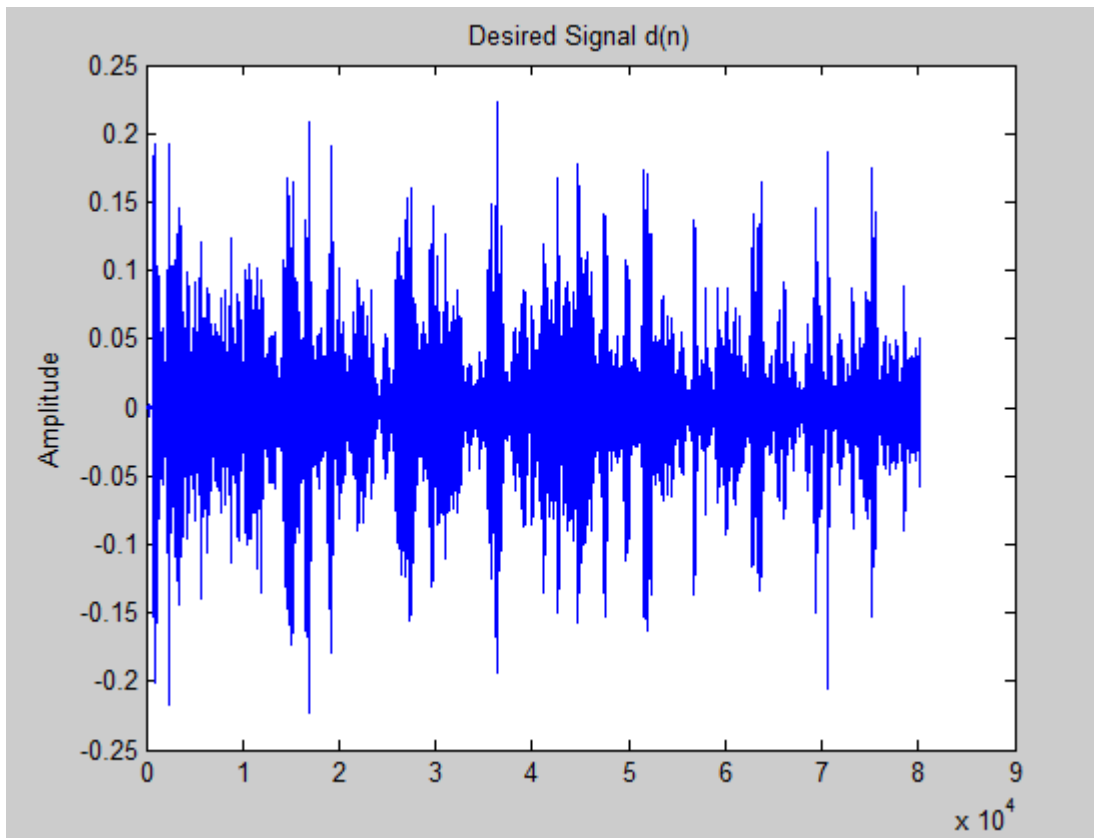


Figure 4-4 Desired signal.

Simulation Results Using LMS Algorithm:

Simulations of the LMS algorithm are carried out using the equations listed in the Section 3.1. The step size is taken as 0.008. The power of the desired signal $d(n)$ and the power of the error signal are plotted as shown in Figure 4-5 to analyze the error convergence. Only 300 samples are considered for the purpose of analyzing.

It can be observed that the power of the error signal becomes very unstable with the change in the power of the input signal.

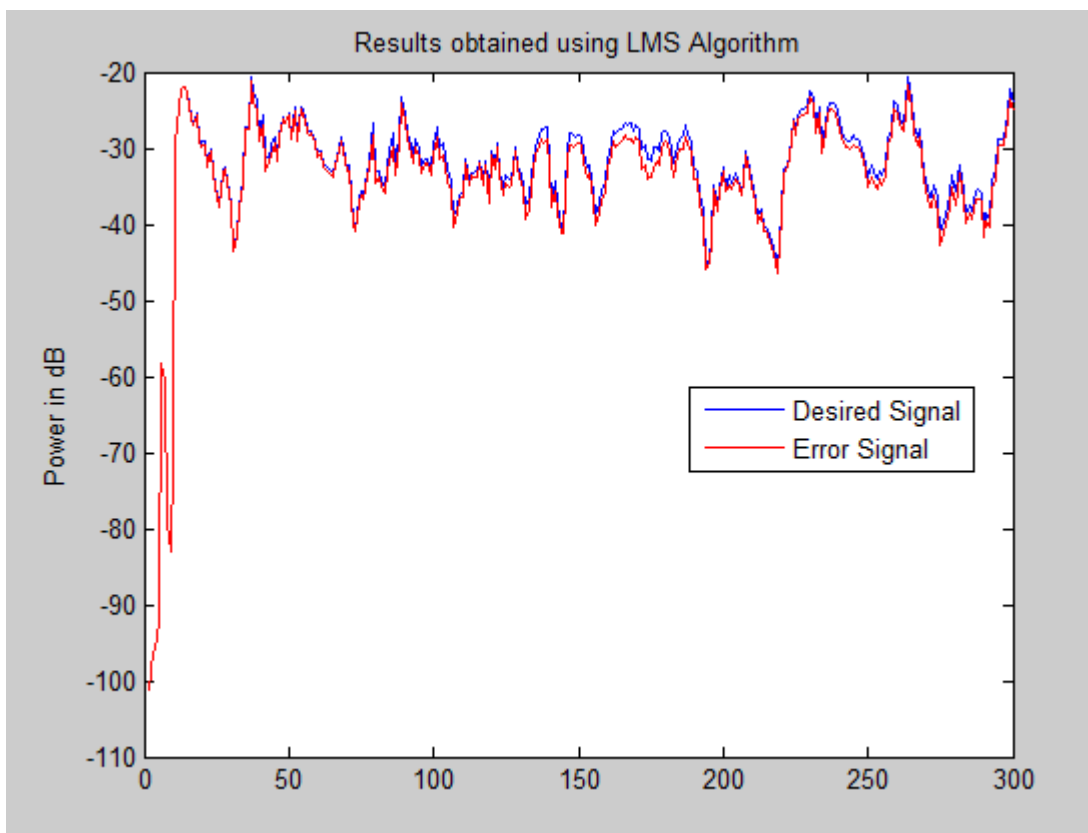


Figure 4-5 Mean square error convergence curve obtained for LMS algorithm.

Simulation Results Using NLMS Algorithm:

Simulations of the NLMS algorithm are carried out using the equations listed in the Section 3.2. The parameters taken are $c=0.00001$. The error signal considerably reduces when compared to the Least Mean Square (LMS) algorithm, but the power of the error signal experiences instability with the changes in the input power as shown in Figure 4-6.

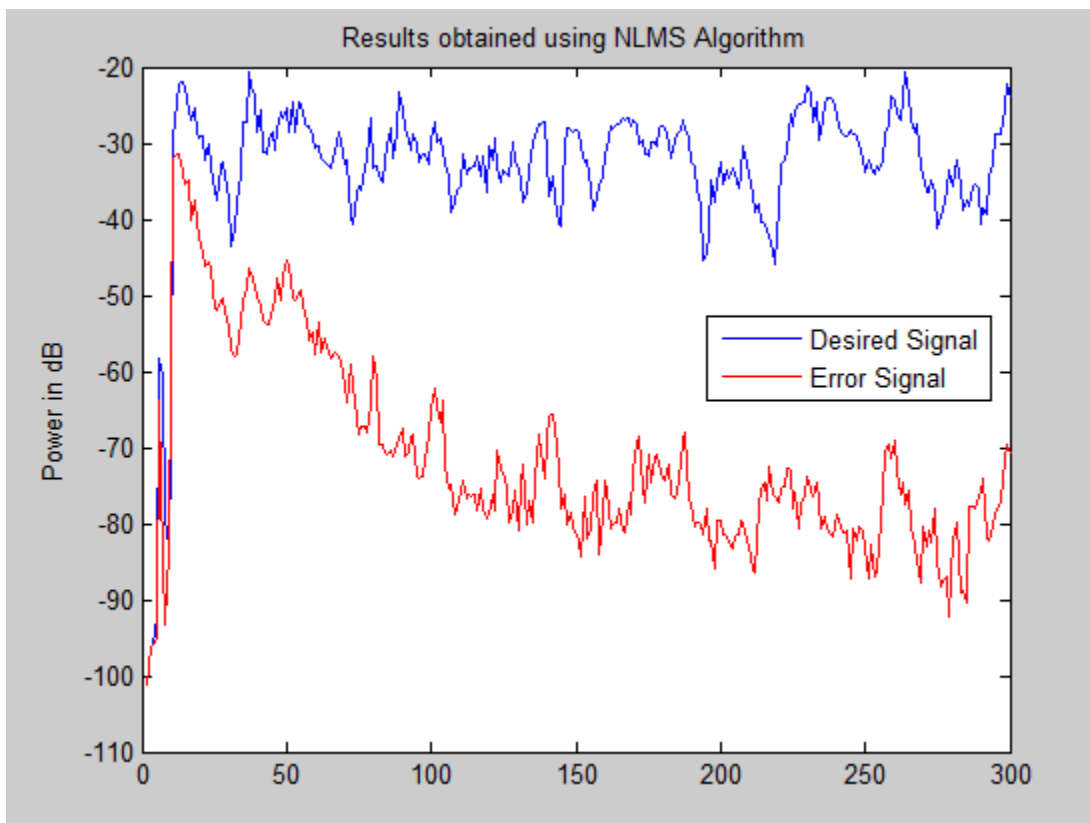


Figure 4-6 Mean square error convergence curve for NLMS algorithm.

Results Obtained Using Kalman Filter:

The simulations were carried out using Kalman filter based on Bayesian linear model form. These simulations were carried out by using the equations listed in the Section 3.3. The values are taken as follows. The dimensions for the mean square error M matrix are taken as

$$M = 10^5 * I_{199 \times 199}$$

The adaptive filter W is taken by making correspondence with the LMMSE estimator $\hat{\theta}[n]$ described in the concept and its value is taken as

$$W = \hat{\theta}[0] = [0, 0, \dots \dots]_{199 \times 1}$$

The results are observed only by taking the first 300 samples as shown in the Figure 4-7. The value for the variance σ_n^2 is taken to be 0.00001.

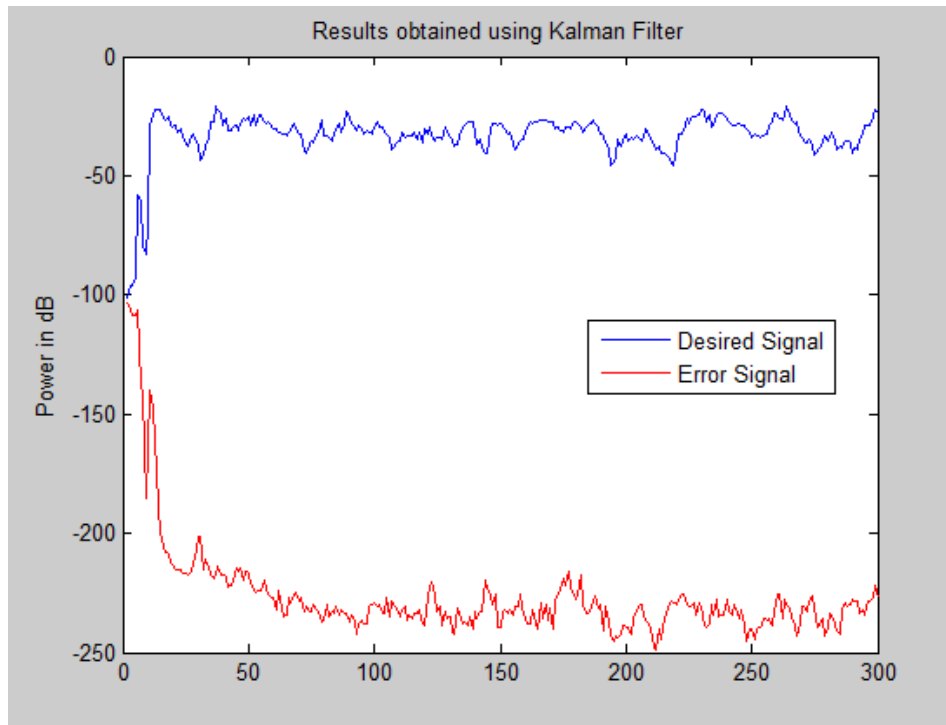


Figure 4-7 Mean square error convergence curve for Kalman algorithm.

The Kalman filter exhibits wonderful results as the power of the error signal reduces within a few number of iterations and it remains stable with the changes in the input signal power compared to the Normalized Least Mean Square (NLMS) algorithm.

Kalman NLMS Algorithm:

The Kalman filter and the Normalized Least Mean Square (NLMS) algorithm can be combined and used to create a new algorithm which is known as the Kalman NLMS algorithm which performs well when compared with the LMS and NLMS algorithms as shown in the Figure 4-8.

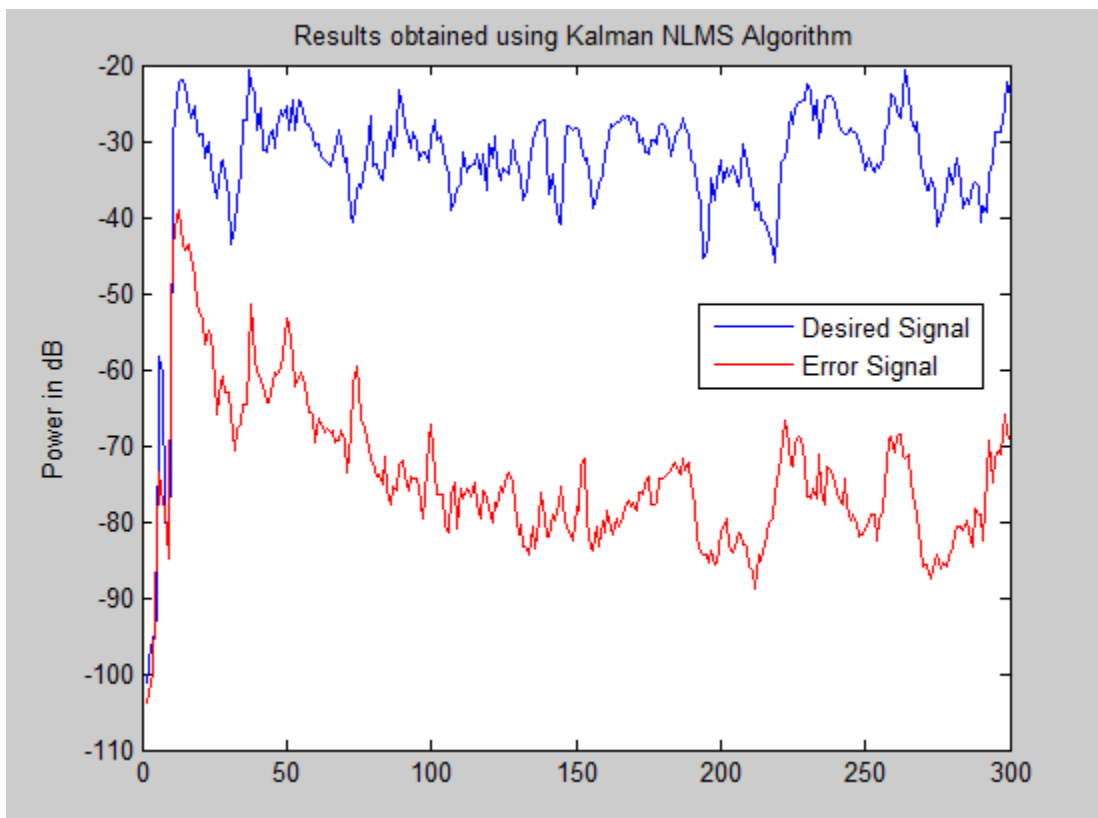


Figure 4-8 Mean square error convergence curve for Kalman NLMS algorithm

4.2 Summary:

The parameters of all the algorithms implemented are optimized to achieve best performance. Figure 4-9 represents the mean square error convergence curves. This mean square error is calculated as the difference between the output of the adaptive filter and the desired signal. The desired signal power is also represented in the figure just as a reference point and to analyze the error signal power. The step size used in the Least Mean Square and Normalized Least Mean Square algorithms is optimized to achieve desired results. The curves are the result of considering 300 iterations.

It is observed that the use of Kalman filtering algorithm has the best performance and outperforms all the other algorithms in the process of adaptive filtering. The mean square error power is very low, which helps in the fast convergence to the true filter coefficients.

From the Figure 4-9 we can observe that when the power of the reference signal increases, the LMS algorithm becomes very unstable and there is no reduction in the power of the error signal. The NLMS algorithm has fewer problems compared with the LMS but it suffers instability with the variation in the reference signal amplifications.

The Kalman NLMS algorithm proves to be efficient and to have accurate performance compared to LMS and NLMS algorithm since the error power seems to be stable even if there are fluctuations in the power of the reference signal.

The Kalman filter outperforms all the other algorithms; the error power drastically reduces within a few number of iterations and proves to be stable even with the changes in the power of the input reference signal. The Kalman algorithm has higher convergence compared to the LMS, NLMS, and Kalman NLMS adaptive algorithms.

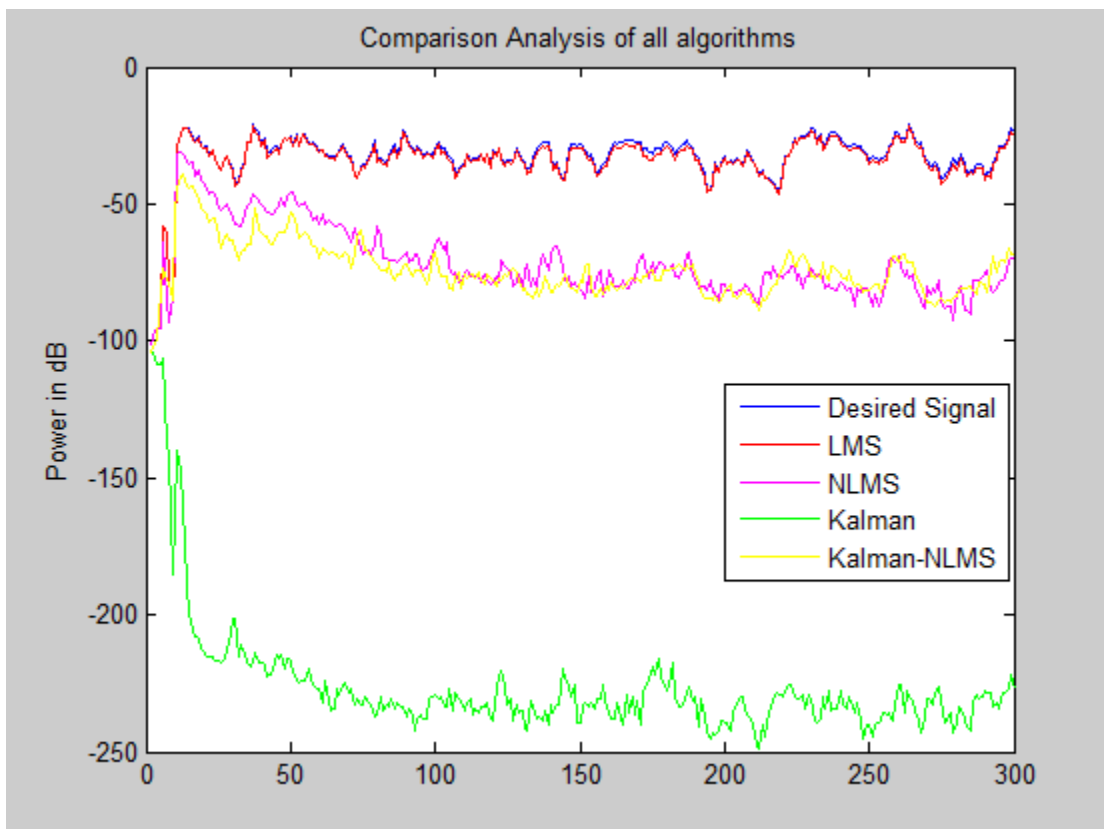


Figure 4-9 Mean square error convergence curves for LMS, NLMS, Kalman and Kalman NLMS algorithms.

Figure 4-10 provides the close comparison between the LMS, NLMS and Kalman NLMS algorithms. The results are magnified to make a clear comparison between the NLMS algorithm and the Kalman-based NLMS algorithm. The error power difference between the two algorithms is almost 8dB, which is highly considerable.

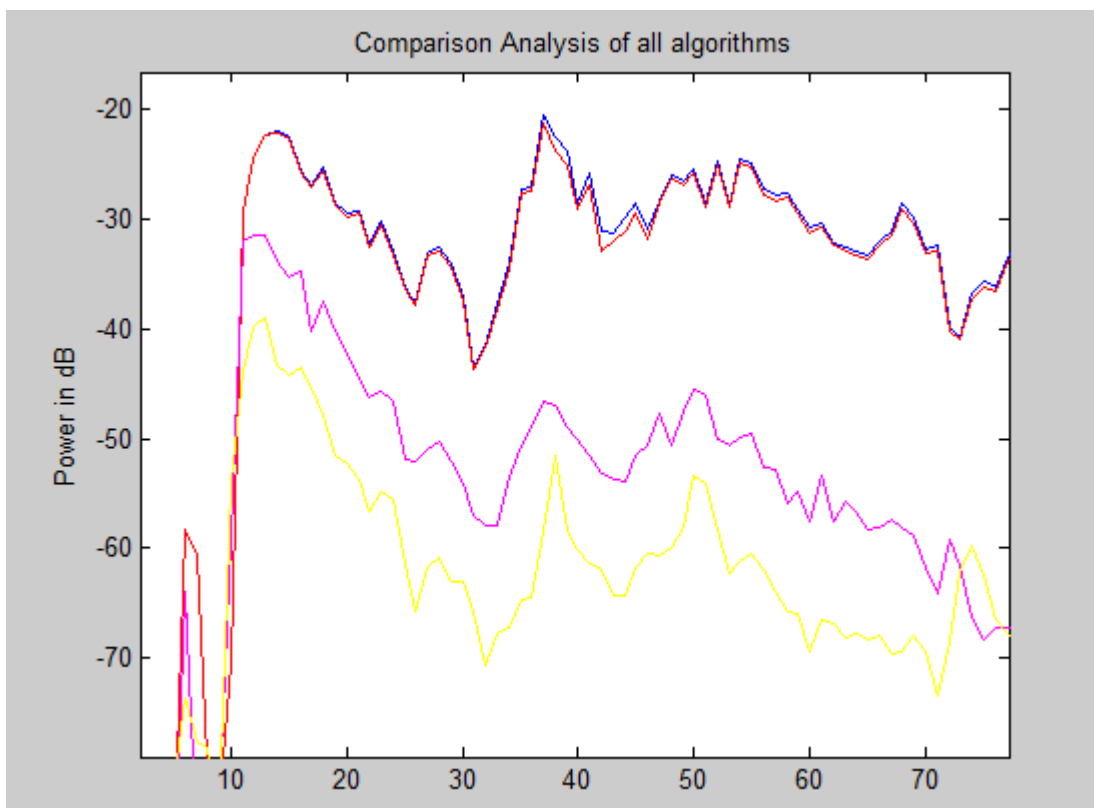


Figure 4-10 Magnified version of mean square error convergence curves for LMS, NLMS, Kalman and Kalman NLMS algorithms.

5 CONCLUSION AND FUTURE WORK

The adaptive filter is briefly studied through this research. The theory of the Kalman filter is clearly studied which uses the Bayesian approach of implementation. The Kalman filter theory can be combined with Normalized Least Mean Square (NLMS) algorithm, and a new type of NLMS algorithm based on Kalman filter is implemented. The adaptive algorithms are implemented in the adaptive filtering process. The Kalman filter outperforms all the algorithms implemented in the research work. The comparisons between the Normalized Least Mean Square (NLMS) algorithm and the Kalman-based NLMS algorithm are carried out in particular. The new Kalman-based Normalized Least Mean Square (NLMS) algorithm is stable since it is derived from the Kalman filter. It allows faster convergence of adaptive filter coefficients. The power of the error signal is compared between the adaptive algorithms. The simulation results prove that the NLMS algorithm based on Kalman gives better performance compared to the LMS and NLMS algorithms.

All the adaptive algorithms described and implemented in the thesis work can be used in other adaptive filtering applications such as noise cancellation; Kalman algorithm based on Bayesian approach if used in noise cancellation process can outperform all the other existing methods.

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