5-1-2010

Deriving the Wage-Wage and Price-Price Phillips Curves from a Model with Efficiency Wages and Imperfect Information

Carl M. Campbell III

Original Citation
Campbell, Carl M., III "Deriving the Wage-Wage and Price-Price Phillips Curves from a Model with Efficiency Wages and Imperfect Information," Economics Letters, Vol. 107, May 2010, pp. 242-245.

Economics Letters 107 (2010) 242–245

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Deriving the wage–wage and price–price Phillips curves from a model with efficiency wages and imperfect information

Carl M. Campbell III *

Department of Economics, Northern Illinois University DeKalb, IL 60115, USA

A R T I C L E I N F O

Article history:
Received 22 January 2009
Received in revised form 1 December 2009
Accepted 22 January 2010
Available online 1 February 2010

JEL classfication:
E24
E31

Keywords:
Phillips curve
Efficiency wages

A B S T R A C T

This study derives reduced-form equations for the wage–wage Phillips curve and the price–price Phillips curve from firms' optimizing behavior, under the assumptions that firms pay efficiency wages and that workers' expectations of average wages or prices are partly adaptive.

1. Introduction

While the Phillips curve has been an important part of empirical macroeconomic modeling since the publication of Phillips' (1958) seminal paper, it has been a challenge to provide theoretical justification for it. This study demonstrates that a Phillips curve can be derived from the profit-maximizing behavior of firms if it is assumed that firms pay efficiency wages and that, in making decisions that affect their efficiency, workers' expectations of wages at other firms or of the price level are partly adaptive. A wage–wage Phillips curve is derived if it is assumed that efficiency depends on relative wages, and a price–price Phillips curve is derived if it is assumed that efficiency depends on real wages. The framework developed in this study differs from the New Keynesian Phillips curve in that all firms can change wages and prices each period, and it differs from Mankiw and Reis' (2002) sticky information model in that the informational imperfection lies in workers' expectations of average wages or prices, rather than in firms' expectations of optimal prices.

* Tel.: +1 815 753 6974; fax: +1 815 752 1019.
E-mail address: carlcamp@niu.edu.

0165-1765/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.econlet.2010.01.036

2. A model of the wage–wage Phillips curve

In deriving the wage–wage Phillips curve, the following assumptions are made:

1. Workers' efficiency depends on the ratio of their current wage to their expectations of wages at other firms and on the unemployment rate. Thus, efficiency can be expressed as

$$e = e[W_t / \overline{W}^e_t, u_t] \text{ with } e_W > 0,\ e_u > 0,\ e_{WW} < 0,\ e_{Wu} < 0, \quad (1)$$

where W_t is the wage at a worker's current firm, $\overline{W}^e_t$ denotes workers' expectations of the average wage rate (to be defined more rigorously below), and u_t represents the unemployment rate.[1]

2. As in Campbell (2008a), it is assumed that workers have imperfect information about average wages and that their expectations of average wages are a mixture of rational and adaptive expectations. In particular, expectations are assumed to be a geometric weighted average

[1] Reasons given for a positive relationship between wages and productivity include the shirking model of Shapiro and Stiglitz (1984), the morale models of Akerlof (1982) and Akerlof and Yellen (1990), the labor turnover models of Stiglitz (1974), Schlicht (1978), and Salop (1979), and the adverse selection model of Weiss (1980). Campbell (2006, 2008a) discusses justification for the assumption that $e_{Wu}<0$.

of the actual current average wage and last period's average wage adjusted by a geometric weighted average of past wage inflation, so that

$$\overline{W}_t^e = \overline{W}_t^{\omega}\left[\overline{W}_{t-1}\left(\frac{\overline{W}_{t-1}}{\overline{W}_{t-2}}\right)^{\lambda_1}\left(\frac{\overline{W}_{t-2}}{\overline{W}_{t-3}}\right)^{\lambda_2}\cdots\left(\frac{\overline{W}_{t-T}}{\overline{W}_{t-T-1}}\right)^{\lambda_T}\right]^{1-\omega} \quad (2)$$

with $\lambda_1 + \lambda_2 + \cdots + \lambda_T = 1$,

where ω represents the degree to which expectations are rational, and the λ's represent the weights given to the various lags of wage inflation in forming adaptive expectations.[2]

3. Each firm faces a downward-sloping demand curve in the product market of the following form:

$$Q_t^D = Y_t\left(\frac{P_t}{\overline{P}_t}\right)^{-\gamma}, \quad (3)$$

where P is the firm's price, $\overline{P}$ is the aggregate price level, γ is the price elasticity of demand, and Y is real aggregate demand per firm, which is assumed to be determined from the constant velocity specification, $Y_t = M_t/\overline{P}_t$, where M is nominal demand. Solving Eq. (3) for P_t and multiplying by Q_t yields the following equation for total revenue:

$$P_tQ_t = Y_t^{\frac{1}{\gamma}}Q_t^{\frac{\gamma-1}{\gamma}}\overline{P}_t.$$

4. Firms produce output (Q) with the Cobb–Douglas production function,

$$Q_t = A_t^{\phi}L_t^{\phi}K_0^{1-\phi}e\left[W_t/\overline{W}_t^e, u_t\right]^{\phi}, \quad (4)$$

where A represents technology (assumed to be exogenous and labor augmenting), L represents labor, and K represents capital (assumed to be fixed).

5. Labor supply per firm is $N(W_t/\overline{P}_t^e)$, where $\overline{P}_t^e$ is the expected price level. It is assumed that parameters are such that firms pay efficiency wages, yielding excess supply of labor.[3]

Given the model's assumptions, profits in period t can be expressed as

$$\Pi = Y_t^{\frac{1}{\gamma}}\left[A_t^{\phi}L_t^{\phi}K_0^{1-\phi}e\left[W_t/\overline{W}_t^e, u_t\right]^{\phi}\right]^{\frac{\gamma-1}{\gamma}}\overline{P}_t - W_tL_t - rK_0. \quad (5)$$

Differentiating Eq. (5) with respect to L_t, setting the derivative equal to 0, and solving for L_t yields the following expression for labor demand:

$$L_t = W_t^{\frac{\gamma}{\phi(\gamma-1)-\gamma}}\left(\frac{\phi(\gamma-1)}{\gamma}\right)^{-\frac{\gamma}{\phi(\gamma-1)-\gamma}}Y_t^{-\frac{1}{\phi(\gamma-1)-\gamma}}A_t^{-\frac{\phi(\gamma-1)}{\phi(\gamma-1)-\gamma}}K_0^{-\frac{(1-\phi)(\gamma-1)}{\phi(\gamma-1)-\gamma}} \times e\left[W_t/\overline{W}_t^e, u_t\right]^{-\frac{\phi(\gamma-1)}{\phi(\gamma-1)-\gamma}}\overline{P}_t^{-\frac{\gamma}{\phi(\gamma-1)-\gamma}}. \quad (6)$$

The other first-order condition is

$$\frac{d\Pi}{dW_t} = 0 = \frac{\phi(\gamma-1)}{\gamma}Y_t^{\frac{1}{\gamma}}A_t^{\frac{\phi(\gamma-1)}{\gamma}}L_t^{\frac{\phi(\gamma-1)}{\gamma}}K_0^{\frac{(1-\phi)(\gamma-1)}{\gamma}}e[\bullet]^{\frac{\phi(\gamma-1)}{\gamma}-1}e_W[\bullet]\frac{1}{\overline{W}_t^e}\overline{P}_t - L_t. \quad (7)$$

If Eq. (6) is substituted into Eq. (7), the following condition, which is analogous to the Solow (1979) condition, is obtained:

$$W_te\left[W_t/\overline{W}_t^e, u_t\right]^{-1}e_W\left[W_t/\overline{W}_t^e, u_t\right]\frac{1}{\overline{W}_t^e} = 1. \quad (8)$$

Totally differentiating Eq. (8) and dividing by the original equation yields

$$0 = \left[1 - e^{-1}e_W\frac{W_t}{\overline{W}_t^e} + \frac{e_{WW}}{e_W}\frac{W_t}{\overline{W}_t^e}\right]\hat{W}_t + \left[-1 + e^{-1}e_W\frac{W_t}{\overline{W}_t^e} - \frac{e_{WW}}{e_W}\frac{W_t}{\overline{W}_t^e}\right]\hat{\overline{W}}_t^e + \left[\frac{e_{Wu}}{e_W} - e^{-1}e_u\right]du_t, \quad (9)$$

where $\hat{W}_t = dW_t/W_t$ and $\hat{\overline{W}}_t^e = d\overline{W}_t^e/\overline{W}_t^e$. The above equation can be viewed as the relationship between percentage deviations in W_t, percentage deviations in $\overline{W}_t^e$, and absolute deviations in u_t from their initial equilibrium values. If we consider small deviations of W, $\overline{W}^e$, and u from their initial equilibrium values, we can treat the coefficients on these variables as constants, with these constants determined by the steady-state values of W_t, $\overline{W}_t^e$, e, e_W, e_u, e_{WW}, and e_{Wu}. In a steady-state equilibrium, $W_t = \overline{W}_t^e$ and $\bar{e}\bar{e}_W{}^1 = (W_t/\overline{W}_t^e) = 1$ (from Eq. (8)). These substitutions allow Eq. (9) to be expressed as

$$\hat{W}_t = \hat{\overline{W}}_t^e + \frac{e_u - e_{Wu}}{e_{WW}}du_t. \quad (10)$$

If Eq. (2) is totally differentiated and divided by the original equation, we obtain the following expression for percentage deviations from steady-state values:

$$\hat{\overline{W}}_t^e = \omega\hat{\overline{W}}_t + (1-\omega)\left[\hat{\overline{W}}_{t-1} + \sum_{i=1}^{T}\lambda_i\left(\hat{\overline{W}}_{t-i} - \hat{\overline{W}}_{t-i-1}\right)\right]. \quad (11)$$

Substituting Eq. (11) into Eq. (10) and aggregating across firms yields

$$\left(\hat{\overline{W}}_t - \hat{\overline{W}}_{t-1}\right) = \frac{e_u - e_{Wu}}{(1-\omega)e_{WW}}du_t + \sum_{i=1}^{T}\lambda_i\left(\hat{\overline{W}}_{t-i} - \hat{\overline{W}}_{t-i-1}\right) \quad (12)$$

with $\lambda_1 + \lambda_2 + \cdots + \lambda_T = 1$.

Thus, the model produces a reduced-form relationship between current wage inflation, unemployment, and a weighted average of lagged wage inflation with the weights summing to 1. Since $e_u > 0$, $e_{Wu} < 0$, and $e_{WW} < 0$, the coefficient on the unemployment rate is unambiguously negative.[4] Also, since Eq. (12) is a reduced-form relationship, the coefficients on unemployment and lagged wage inflation will be the same for both demand shocks and technology shocks and will be the same for any process governing the shocks (e.g., stochastic or deterministic).

Eq. (12) is an expression for the wage–wage Phillips curve. However, Phillips curves are generally estimated by regressing either wage inflation or price inflation on unemployment and lagged price inflation.[5] Campbell (2008b) demonstrates that the model developed

[2] Campbell (2008a) discusses justification for the assumption that expectations are a mixture of rational and adaptive expectations, based on previous studies that examine the nature of agents' expectations. It should be noted that Campbell (2008a) uses a simpler specification in which expectations are a weighted average of the actual current average wage and last period's average wage. However, the specification used in the present study is more appropriate for an economy in which wages tend to rise over time.

[3] Assuming a positive relationship between wages and efficiency does not guarantee that there will be excess supply of labor. Whether a firm operates on its labor supply curve or to the left of its labor supply curve (i.e., pays an efficiency wage) depends on the elasticity of output with respect to the wage, calculated at the market-clearing wage. It is assumed that parameters are chosen so that firms maximize profits by operating to the left of their labor supply curves.

[4] In the model of Campbell (2008a), wage inflation depends negatively on unemployment at the national level. In this previous study, however, wage inflation does not depend on lagged wage inflation (nor on lagged price inflation).

[5] It is not clear why there is not more empirical work that involves estimating wage–wage Phillips curves.

in this section yields asymptotic wage–price and price–price Phillips curves in response to stochastic aggregate demand shocks. In these versions of the Phillips curve, the coefficient on lagged price inflation asymptotically approaches 1 as the sample size increases, and it is close to 1 even when the sample size is small.

3. A model of the price–price Phillips curve

In deriving the wage–wage Phillips curve, it is assumed that workers' efficiency depends on the ratio between their wages and their expectations of average wages. It could also be assumed that their efficiency depends on the ratio between their wages and their expectations of the price level. While efficiency should depend on relative wages in the long run, there are two reasons why workers' efficiency may depend on their expectations of the price level in the short run. First, in the fair wage model of Akerlof and Yellen (1990), workers may view the fair wage as a function of the real wage. Second, even if workers are concerned about their relative wages, they may use information about price inflation to predict how much wages are rising at other firms, since wage inflation and price inflation are correlated and since price inflation data are more highly publicized than wage inflation data.[6]

If efficiency depends on price expectations, Eqs. (4), (6), (8), and (10) become

$$Q_t = A_t^{\phi} L_t^{\phi} K_0^{1-\phi} e\left[W_t / \overline{P}_t^e, u_t\right]^{\phi}, \qquad (13)$$

$$L_t = W_t^{\frac{\gamma}{\phi(\gamma-1)-\gamma}} \left(\frac{\phi(\gamma-1)}{\gamma}\right)^{-\frac{\gamma}{\phi(\gamma-1)-\gamma}} Y_t^{-\frac{1}{\phi(\gamma-1)-\gamma}} A_t^{-\frac{\phi(\gamma-1)}{\phi(\gamma-1)-\gamma}} K_0^{-\frac{(1-\phi)(\gamma-1)}{\phi(\gamma-1)-\gamma}} \times e\left[W_t / \overline{P}_t^e, u_t\right]^{-\frac{\phi(\gamma-1)}{\phi(\gamma-1)-\gamma}} \overline{P}_t^{-\frac{\gamma}{\phi(\gamma-1)-\gamma}}, \qquad (14)$$

$$W_t e\left[W_t / \overline{P}_t^e, u_t\right]^{-1} e_W\left[W_t / \overline{P}_t^e, u_t\right] \frac{1}{\overline{P}_t^e} = 1, \text{ and} \qquad (15)$$

$$\hat{W}_t = \hat{\overline{P}}_t^e + \frac{e_u - e_{Wu}}{e_{WW}} du_t. \qquad (16)$$

The unemployment rate can be expressed as $u_t = [N(W_t/\overline{P}_t^e) - L_t]/N(W_t/\overline{P}_t^e)$. Letting s_L equal the steady-state value of $L_t/N(W_t/\overline{P}_t^e)$ and ψ represent the steady-state value of the short-run labor supply elasticity (with $\psi \geq 0$), du_t can be approximated by

$$du_t = \frac{-dL_t}{N} + L_t N^{-2} N' \left[\frac{1}{\overline{P}_t^e} dW_t - \frac{W_t}{(\overline{P}_t^e)^2} d\overline{P}_t^e\right] \approx -s_L \hat{L}_t + s_L \psi \hat{W}_t - s_L \psi \hat{\overline{P}}_t^e. \qquad (17)$$

From Eqs. (13)–(17), it can be demonstrated that

$$\hat{\overline{P}}_t = \hat{\overline{P}}_t^e - \frac{(1-\phi)[e_{WW} - s_L(e_u - e_{Wu})(1+\psi)] + \phi e^{-1} e_u s_L e_{WW}}{s_L e_{WW}} du_t - \phi \hat{A}_t.^{7} \qquad (18)$$

Similar to the assumption about wage expectations, workers' expectations about the price level are assumed to be a mixture of rational and adaptive expectations, so that

$$\overline{P}_t^e = \overline{P}_t^{\omega} \left[\overline{P}_{t-1} \left(\frac{\overline{P}_{t-1}}{\overline{P}_{t-2}}\right)^{\lambda_1} \left(\frac{\overline{P}_{t-2}}{\overline{P}_{t-3}}\right)^{\lambda_2} \cdots \left(\frac{\overline{P}_{t-T}}{\overline{P}_{t-T-1}}\right)^{\lambda_T}\right]^{1-\omega}, \qquad (19)$$

and $\hat{\overline{P}}_t^e = \omega \hat{\overline{P}}_t + (1-\omega)\left[\hat{\overline{P}}_{t-1} + \sum_{i=1}^{T} \lambda_i \left(\hat{\overline{P}}_{t-i} - \hat{\overline{P}}_{t-i-1}\right)\right]$

with $\lambda_1 + \lambda_2 + \cdots + \lambda_T = 1$.

Substituting Eq. (19) into Eq. (18) results in the Phillips curve relationship:

$$\left(\hat{\overline{P}}_t - \hat{\overline{P}}_{t-1}\right) = -\frac{(1-\phi)[e_{WW} - s_L(e_u - e_{Wu})(1+\psi)] + \phi e^{-1} e_u s_L e_{WW}}{(1-\omega) s_L e_{WW}} du_t + \sum_{i=1}^{T} \lambda_i \left(\hat{\overline{P}}_{t-i} - \hat{\overline{P}}_{t-i-1}\right) - \frac{\phi}{1-\omega} \hat{A}_t. \qquad (20)$$

Thus, if workers' efficiency depends on their wages relative to their expectations of the price level, a reduced-form equation for the price–price Phillips curve can be derived. In this equation, the sum of coefficients on lagged inflation equals 1, the coefficient on unemployment is negative (since $1-\phi>0$, $e_u>0$, $e_{WW}<0$, $e_{Wu}<0$, $\psi \geq 0$, and $1-\omega>0$), and the rate of price inflation depends both on the unemployment rate and on technology shocks.

4. Conclusion

This study assumes that firms pay efficiency wages and that workers' expectations of average wages or prices are a mixture of rational and adaptive expectations. It is demonstrated that the profit-maximizing behavior of firms results in a wage–wage Phillips curve when workers' efficiency depends on their wages relative to their expectations of average wages and results in a price–price Phillips curve when their efficiency depends on their wages relative to their expectations of the price level.

This study considers the profit-maximization problem of firms and does not explicitly model the behavior of workers. Two decisions made by workers that are relevant to a firm's wage and employment choices are their decisions concerning labor supply and efficiency. In a simple leisure-consumption framework, workers' utility maximization yields a relationship between real wages and labor supply, as assumed in this study. In addition, Campbell (2006) demonstrates that an effort function with $e_W>0$, $e_u>0$, $e_{WW}<0$, and $e_{Wu}<0$ can be derived from workers' utility-maximizing behavior. Thus, the Phillips curves derived in this study can be viewed as being determined from profit maximization and utility maximization.

References

Akerlof, G.A., 1982. Labor contracts as partial gift exchange. Quarterly Journal of Economics 97, 543–569.
Akerlof, G.A., Yellen, J.L., 1990. The fair wage — effort hypothesis and unemployment. Quarterly Journal of Economics 105, 255–283.
Campbell, C.M., 2006. A model of the determinants of effort. Economic Modelling 23, 215–237.
Campbell, C.M., 2008a. An efficiency wage approach to reconciling the wage curve and the Phillips curve. Labour Economics 15, 1388–1415.
Campbell, C.M., 2008b. An efficiency wage — imperfect information model of the Phillips curve. Working Paper 2008-1. Northern Illinois University.
Mankiw, N.G., Reis, R., 2002. Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve. Quarterly Journal of Economics 117, 1295–1328.
Phillips, A.W., 1958. The relationship between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957. Economica 25, 283–299.
Salop, S.C., 1979. A model of the natural rate of unemployment. American Economic Review 69, 117–125.

[6] Workers' efficiency probably depends on both average wages and average prices in the short run, although there has been little examination of this issue.
[7] An appendix deriving this equation is available at http://www.niu.edu/econ/Directory/Campbell/PhillipsPaperELAppendix.pdf.

Schlicht, E., 1978. Labor turnover, wage structure and natural unemployment. Zeitschrift für die Gesamte Staatswissewnschaft 134, 337–346.

Shapiro, C., Stiglitz, J.E., 1984. Equilibrium unemployment as a worker discipline device. American Economic Review 74, 433–444.

Solow, R.M., 1979. Another possible source of wage stickiness. Journal of Macroeconomics 1, 79–82.

Stiglitz, J.E., 1974. Alternative theories of wage determination and unemployment in L.D.C'.s: the labor turnover model. Quarterly Journal of Economics 88, 194–227.

Weiss, A., 1980. Job queues and layoffs in labor markets with flexible wages. Journal of Political Economy 88, 526–538.