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Seven Steps Toward the Classical World: The Problem of the Classical Limit in Quantum Mechanics

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Quantum mechanics is the most powerful of the theories developed by the human beings. In fact it is able to predict successfully the experimental results with an impressive degree of accuracy. For this reason, the majority of physicists tends to believe that quantum mechanics has no problem and that we are close to find a real "theory of everything". As it has been underlined many times, starting from the famous Bohr–Einstein debate and finally arriving to the writing of John Bell, it is far from being true.

The basic problem of quantum mechanics is that it is not clear what it is about. It might seem that quantum mechanics is basically about the behaviour of the wave function but Schrödinger, with his famous cat paradox, has shown that it could not be possible. According to the rules of quantum mechanics, the cat turns out to be in the impossible superposition of states with macroscopic disjoint supports, as the dead-cat state and the alive-cat state. To somehow solve this problem, it is necessary to introduce a different time evolution, parallel to Schrödinger's equation, according to which an *observer* selects (collapses) one of the terms composing the superposition wave function.

The introduction of this process, the collapse of the wave function, makes a lot of question arise: what is the role of the observer? Who has the power to reduce the wave function? Where is the border between the microscopic world, in which superposition exists, and the macroscopic one, in which they cannot? There is no possible answer to this question in the framework of quantum mechanics and we must conclude, with Bell, that there are only two possibility to solve the problem: add something to Schrödinger's equation, or modify Schrödinger's evolution. Bohmian mechanics is a theory that follows the first of this options and I believe it is the simpler and the most powerful quantum theory that we have developed so far.

In my Ph. D. thesis in physics I studied the problem of the classical limit of quantum mechanics. It is not difficult to realise that this is exactly the problem of the border between microscopic and macroscopic world. Classical world is about real objects whose motion is governed by Newton's law. In standard quantum mechanics only the wave function exists and to answer the question of how the classical world emerges from the quantum world seems rather impossible: how can we deduce classical trajectories from a theory like quantum mechanics in which they do not even exist? This is not the case of Bohmian mechanics, which is a theory of particles following trajectories and satisfying some law of motion. The question then becomes simple: when do Bwe have classical limit? When ohmian trajectory become classical trajectories.

It is always more frequent to find people claiming that decoherence is the right tool to solve every problem of standard quantum mechanics, specially the problem of the classical limit. Note that

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decoherence is simply taking into account the effects of the external environment in the behaviour of the system of interest. This means that we are still moving within quantum mechanics and thus all the problem are still present: we need to add the collapse rule to solve them.

The crucial problem of the classical limit of quantum mechanics is spreading: in ordinary quantum mechanics the classical limit is somehow defined for wave packets but, as soon as there is some interaction, they start spreading. Due to the fact that there are no trajectories, a spread out wave function can in no way show a classical limit in standard quantum mechanics. Moreover, in my Ph. D. thesis it is argued that the hallmark of the classical limit is indeed the formation of a typical wave function: what we called a local plane wave. This is a wave function that locally looks like a plane wave and that mathematically can be thought as a sum of spatially disjoint non interacting wave packets. We claim that whenever there is a local plane wave there is classical limit. Note that a local plane wave a very spread out wave functionand it is important to underline that only in the framework of Bohmian mechanics, a theory in which we do have trajectories, we are able to *explain* the classical behaviour: in standard quantum mechanics it is completely impossible. On the contrary, in Bohmian mechanics to each configuration is attached a guiding wave packet with a definite wave vector that locally determines the particle dynamics according to the classical laws (in some suitable limit): only this guiding wave packet is important for the dynamics and we can forget all the rest of the local plane wave. This means that the classical limit can be symbolically be expressed as $(\psi, X) \to (P, X)$, that is, the complete description of the quantum state tends to the complete description of the classical state in terms of position and momentum.