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Valia Allori

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“The Mistake in Monty's Fall” by Christopher Pynes

Comments by Valia Allori

In his paper, Christopher argues against J. Rosenthal's interpretation of the classic solution of the Monty Hall Problem.

In the Monty Hall problem there is a contestant who has to choose among three doors, knowing that behind one there is a prize. Monty, who knows where the prize is, opens one of the remaining doors, showing that there is nothing behind it, and offers the contestant to switch. Does the contestant raise her probability of success by switching, or not?

The answer, according to the classical solution (against which Rosenthal is arguing and that Christopher is defending), is: yes. The reason for this (the “classical solution”) is that we should not forget how many choices we had at the beginning (namely 3), and given that the probability we got it right the first time is $1/3$, it is always convenient to switch. What is the role of Monty opening one (empty) door, then? The answer is: nothing!

Just compare what you would do in the following situation:

Consider a lottery with 1000 tickets that can be scratched off and only one contains a prize. You buy one, while I buy all the others. If I were to ask you if you would switch your ticket with all of mine, would you switch? Sure! If I were to ask you to switch your ticket with all of mine after I have scratched them all but one, would you switch? The answer should be: yes, since the relevant probability (the relevant cases we should count) is the one in the moment we buy the tickets, independently on whether I scratched my tickets or not.

The case of Monty is exactly like this, but with 3 choices, and not 1000.

Rosenthal argues that the classical solution is misguided because it does not really capture the essence of the solution of the problem and proposes his own solution, which (according to him) better handles a variant of the Monty Hall, that he calls “Monty Fall”: the contestant chooses one door but Monty, instead of choosing which door to open, just falls down and it just happens he hits the door without the prize. Rosenthal claims that in this case the situation is completely different, and that in this case there is no reason for the contestant to switch: the probability of winning is 50/50. Rosenthal's attempt to justify this is clumsy and obscure, referring to a principle he calls the “proportionality principle”.

Christopher tears apart the thought experiment, and concludes that the Monty Fall is logically equivalent to the Monty Hall, so Rosenthal is mistaken in his claim: the contestant should always switch. Since, as explained before, the fact that another door is opened does not play any role in deciding what to do (switch or not switch) then how the door is opened does not matter, so the Monty Hall is equivalent to the Monty Fall.

I entirely agree with Christopher: Monty Fall is presented in a misleading way so to induce to say, mistakenly, that the chances are 50/50. In fact it describes only one of the situations possible: the one in which Monty just happens to hit the door with no prize. But it could happen that Monty is hitting the door with the prize. In focusing on the occasions in which Monty just happens to fall in the one in which there is no prize, one makes the implicit assumption that the probability that Monty will reveal the prize is zero. But that is not so.

Christopher says that the game Rosenthal is talking about, that has indeed a 50/50 chance is not Monty Fall, but Monty Fall*. What happens here is that we actually consider all that can happen: it is possible for Monty to fall into the door with the prize, and this makes the game end, or it is possible for Monty to fall into the door without the prize, so that the game can proceed. This game is different from the previous ones because we are also considering the cases in which Monty opens the prize door. In this

case, using Bayes theorem, we indeed get that the probabilities are 50/50.

To cut the discussion short, I am showing you a computer simulation of the three games, to show that indeed they lead to the right probabilities Christopher (and not Rosenthal) predicts. The simulation does not make use of Bayes theorem, just counts the winning and the losing instances for both switching and not switching.

Monty Hall has been simulated as follows:

- 1-a random number is extracted to select where the prize is;
 - 2-a second random number is extracted to select the choice of the contestant;
 - 3a-if the contestant chose the prize, then a third random number is extracted to select (among the left two) the door to be opened by Monty;
 - 3b-if the contestant did not choose the prize, then it already decided what door Monty will open (the one that does not contain the prize)
 - 4-the contestant may or may not switch (there is a button for that)
 - 5-the prize is revealed and the game ends (with the contestant winning or loosing)
- The game is iterated, and the number of winning and loosing is counted, for both switching and not switching.

Monty Fall has been simulated as follows:

- 1-a random number is extracted to select where the prize is;
 - 2-a second random number is extracted to select the choice of the contestant;
 - 3a-whatever the contestant has chosen, a third random number is extracted to select (among the left two) the door to be opened by Monty;
 - 3b-if Monty hits the prize, then a new number is extracted, until Monty hits the empty door.
 - 4-the contestant may or may not switch (there is a button for that)
 - 5-the prize is revealed and the game ends (with the contestant winning or loosing)
- The game is iterated, and the number of winning and loosing is counted, for both switching and not switching.

(The difference between Monty Hall and Monty Fall is that in Monty Fall step 3b always forces Monty's fall to be on the empty door, so that the door just "happens to be" empty .

This is no difference at all, since we are also assuming that in Monty Hall, step 3b. That is, step 3a and 3b in both games will produce the same results.)

Monty Fall* has been simulated as follows:

- 1-a random number is extracted to select where the prize is;
 - 2-a second random number is extracted to select the choice of the contestant;
 - 3-a third random number is extracted to select (among the left two) the door that Monty opens;
 - 4a-if Monty hits the prize, the game ends (without anyone having won);
 - 4b-if Monty hits the empty door, the game continues;
 - 5-the contestant may or may not switch (there is a button for that)
 - 6-the prize is revealed and the game ends (with the contestant winning or loosing)
- The game is iterated, and the number of winning and loosing is counted, for both switching and not switching.

(The difference between Monty Fall and Monty Fall* is that in Monty Fall it can never happen that Monty hits the prize –just like in Monty Hall-, while in Monty Fall* these occurrences correspond to the end of the game without score).

So, there is no much of philosophical discussion here: Christopher is clearly right. What seems interesting to me is why it is that case that so many people, and so many smart people (like Rosenthal) are getting probability reasoning wrong. This is something that Christopher explicitly says he does not

want to discuss, but I think it is interesting anyway. So, does the fact that many people get Monty Hall wrong mean that humans are irrational? My response will be: No, the wrong responses just depend on how the question is formulated.

(For this discussion, I am relying on papers by Steven Stich and collaborators.)

Since Aristotle, people have believed that man is, essentially, a rational animal, and for 2300 years philosophers have (more or less) agreed. In the 1970s, Daniel Kahneman & Amos Tversky began a research program which many have interpreted as showing that Aristotle was wrong. This program claims that most people do not have the correct principles for reasoning, and we get by with much simpler principles which sometimes get the right answer, and sometimes do not. A cluster of studies have been made, in particular to study mistaken probabilistic judgments. The Monty Hall paradox just seems an instance of that.

For example, this question was proposed to a group of faculty, staff and advanced students at Harvard Medical School:

“Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

If Program A is adopted, 200 people will be saved.

If Program B is adopted, there is a 1/3 probability that 600 people will be saved, and a 2/3 probability that no people will be saved.”

A second group of subjects was given an identical problem, except that the programs were described as follows:

“If Program C is adopted, 400 people will die.

If Program D is adopted, there is a 1/3 probability that nobody will die and a 2/3 probability that 600 people will die.”

On the first version the vast majority chose Program A. But on the second version most chose Program D. The implication of this study is that the decisions we make are strongly influenced by the manner in which the options are described or framed.

Something similar seems to be going on in Monty Fall (or in the example of the three cards that Rosenthal gave us): the question was phrased in such a way that will make us respond in a wrong way. That is, the fact that Monty knows or does not know which door to open is irrelevant for the probabilistic evaluation. But putting the emphasis on the fact that “Monty knows where the prize is”, is misleading. This is what I said earlier, when I was trying to explain why it is convenient to switch in Monty Hall and Monty Fall, while it is indifferent in Monty Fall*.

To continue with the experimental evidence, the situation gets worse in more complicated situations, like the following:

“If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5%, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person’s symptoms or signs? _____%”

The correct Bayesian answer is 2%. But only 18% of the Harvard audience gave a correct answer. 45% said that the answer was 95%.

Why is that? Here is the response of Evolutionary Psychology. According to this theory, the mind is made of lots of interacting systems. Many of these are information processing devices that were designed by natural selection to deal with the problems posed in the Pleistocene and earlier. For problems involving probabilities, one can notice that probabilistic information was widely available in the environment in which we evolved. Organisms that could use it effectively would have had significant advantages in the competition to survive and reproduce. But useful information about

probabilities was available to our ancestors in the form of frequencies (e.g. 3 of the last 12 hunts near the river were successful), while information about the probabilities of single events was not available. So perhaps we evolved one or more mental organs that are good at dealing with probabilistic information, but only when that information is presented in a frequency format.

The idea is that ordinary people have access to principles of reasoning that will enable them to get the correct answer on many reasoning tasks. But these principles are not engaged by many reasoning tasks of great importance in the Contemporary world.

If that is true, then we should expect that the quizzes above, framed in frequencies terminology, would lead to better result. This is indeed what actually happens.

Here is the reformulation of the problem above in frequency terms: 1 out of every 1000 Americans has disease X. A test has been developed to detect when a person has disease X. Every time the test is given to a person who has the disease, the test comes out positive. But sometimes the test also comes out positive when it is given to a person who is completely healthy. Specifically, out of every 1000 people who are perfectly healthy, 50 of them test positive for the disease.

Imagine that we have assembled a random sample of 1000 Americans. They were selected by lottery.

Those who conducted the lottery had no information about the health status of any of these people.

Given the information above: on average, how many people who test positive for the disease will actually have the disease? _____ out of _____. “

In contrast to original experiment, now the correct answer was given by 76% of the subjects.

Let us come back to the Monty Hall/Fall problem. The original formulation is something like this: “You are presented with a situation in which there are 3 doors, only one with a prize behind it, and you choose one door. Monty knows where the prize is, and Monty opens one of the remaining doors, showing that it contains no prize. What are the probabilities for you to win if you switch and if you do not switch? ”. This formulation both contains probability talk AND misleading language... so it is the worst possible formulation for our brain to process. So no wonder a lot of people get it wrong!

Here is a formulation of the problem that does not mention probabilities, and that avoids misleading language:

“You are presented with a situation in which there are 3 doors, only one with a prize behind it, and you choose one door. Monty, the host of the show, opens one of the remaining doors, showing that it contains no prize. Should you switch and take two doors instead of one, or should you stick with the original choice?”

I think we will most likely have better results with the second formulation. Maybe we should post both these questions in our Facebook walls and see what different answers we get....

In any case, let me conclude with a joke: “Man is a rational animal who always loses his temper when he is called upon to act in accordance with the dictates of reason.” - Oscar Wilde