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## A lagrangian relaxation approach to a multi-echelon consolidation of perishable products

Durga Ravi Kiran Jinagam

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## ABSTRACT

### A LAGRANGIAN RELAXATION APPROACH TO A MULTI-ECHELON CONSOLIDATION OF PERISHABLE PRODUCTS

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A consolidation center was established near a set of suppliers with perishable products that have a deterioration rate to consolidate their products and reduce transportation costs. A deterministic demand was considered to solve the model for optimal transportation and holding costs to the suppliers via consolidation center. The products from all suppliers were consolidated at the center to reduce freight transportation costs, which would otherwise have been individually shipped by each supplier. A mixed integer programming (MIP) model was developed to solve the optimal cost of shipping considering the holding cost, transportation cost and deterioration rate. The MIP model was then solved using IBM ILOG CPLEX software. The results of the CPLEX model were compared to new model, which involves decomposition of the original model and using a Lagrangian relaxation heuristic.

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DE KALB, ILLINOIS

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A LAGRANGIAN RELAXATION APPROACH TO A MULTI-ECHELON  
CONSOLIDATION OF PERISHABLE PRODUCTS

BY

DURGA RAVI KIRAN JINAGAM

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A THESIS SUBMITTED TO THE GRADUATE SCHOOL  
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Thesis Director:

Christine Nguyen

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## CHAPTER 1

### INTRODUCTION

The research under study is an extension of the work done in the paper “Evaluation of Transportation Practices in the California Cut Flower Industry” (Nguyen, Toriello, Dessouky, & Moore, 2013), which deals with the distribution of cut flowers. California cut flower growers transported their shipments individually in either full truck loads (FTL), less than truck loads (LTL), or courier services based on the demand and the size of the grower. High transportation costs were incurred by these growers, which is one of the major factors for shipping in larger shipments versus smaller shipments. The cost of shipping FTLs is much less expensive than LTLs, however the advantage only exists if volume to be shipped is large. Consolidation of the shipments can make a large difference in cost savings for the transportation network. Suppliers can also benefit from the lower transportation costs. This problem considers perishable products and the value of the product deteriorates with time.

In this problem, we considered a set of suppliers taking orders from customers in  $j$  different destinations. These orders were originally shipped by suppliers to the respective destinations individually using either FTL units, LTL units, or by courier. The disadvantage of the original method of shipping individually is that the suppliers had to bear higher transportation costs for shipments with smaller volumes. These higher transportation costs can occur in following ways:

1. Small volume shipments: All the small-scale suppliers mostly have smaller orders, and because of that, they are forced to transport their goods through LTL units, which is at a higher cost than the FTL units relatively.
2. Multiple Destinations: The orders come from multiple small suppliers with shipments to multiple different destinations, which requires separate LTL units for each destination.

This problem focuses on minimizing these costs by establishing a consolidation center. All of the orders of all suppliers go to respective destinations through this consolidation center. The advantages of having a consolidation center are:

- At suppliers: A supplier can send all the shipments to the consolidation center. This may help to reduce the number of shipments sent to each destination. It offers an opportunity for orders to be consolidated and shipped at the more economical FTL rate. Therefore, savings can be made for part of the transportation
- At consolidation center: All the orders can be consolidated at the consolidation center if they have the same destination. Shipments from different suppliers can be combined to minimize the costs associated with transportation.

In this model, we considered holding inventory at the suppliers as well as the consolidation center. Each supplier will have different inventory capacities, and the consolidation center will have relatively larger inventory capacity. The costs of LTL units and FTL units are different for each supplier to the consolidation center and from the consolidation center to the respective destinations. Each order originates from a destination and must be satisfied by an arbitrary supplier. The products are perishable, so time is an important factor. In

other words, we cannot store or consolidate orders for a long time because the product will deteriorate. For this reason, we have included a deterioration rate to increase the cost of storing the products, which causes the orders to be delivered quickly while simultaneously optimizing transportation costs. We have created a mixed integer programming model to find the number of FTL and LTL units to be shipped and when they should be shipped for various scenarios.

## CHAPTER 2

### LITERATURE REVIEW

This section summarizes the literature related to this work. We have considered the literature related to freight consolidation, perishable products, and other inventory models in multi-echelon systems.

#### Consolidation

In early 1980s, the focus on cost saving practices from freight consolidation accelerated, which led to the development of various models and heuristics to determine the optimal methods for transportation practices. The following are major articles in chronological order on the consolidation practices in freight transportation.

In 1984, Martha Cooper published an article on cost and delivery time implications of freight consolidation and warehousing strategies. She used a branch and bound algorithm and simulation to determine the lowest cost, shortest mean delivery time, and lowest delivery time variance among direct shipment from plants, consolidating at plants, consolidating at warehouses and combinations of all the scenarios. If the cost of shipments is high and the shipment size is small, it is not profitable to wait till the truck is full, but if the distance between origin and destination is high, the inventory costs will be comparatively low. Blumenfeld, Burns, Diltz, and Daganzo (1985) analyzed trade-offs among transportation, inventory, and production costs on

different freight networks like direct shipment between origin and destination pair, shipment through consolidation, and both combined by network decomposition method. Gupta and Bagchi (1987) developed a tool, which calculates the minimum economical quantity to be dispatched from the consolidation center, for logistics managers of companies who follow just-in-time procurement system and have small shipments that need to be shipped regularly. Closs and Cook (1987) explain which markets should be consolidated, where the consolidation center(s) should be located, and the optimum quantity or time for the freight consolidation by using dynamic simulation. Daganzo (1988) that enhancement in shipment composition at a consolidation terminal may result in a truck carrying more goods rather than sending them directly from the origin to destination even though vehicle-miles may be increased if the shipments are sent through consolidation center.

Bookbinder and Barkhouse (1993) developed an information system for simultaneous consolidation of the inbound as well as outbound shipments, which can effectively save costs incurred in truck routing and integrated logistics information systems (LIS) with just in time (JIT) manufacturing. Higginson and Bookbinder (1994) discussed quantity-policy, time-policy and hybrid quantity and time policy in their paper, and Higginson and Bookbinder (1995) used a Markovian decision policy to find the optimal time and quantity for dispatching after consolidation with respect to cost as well as customer satisfaction. Higginson (1995) also researched recurrent and non-recurrent decision approaches where in non-recurrent decisions a target time or quantity is set and the truck is dispatched when it reaches the target. This is efficient in the long run but does not guarantee optimality in every decision, since the decision of dispatch is uncertain and can only be decided after the customer order is received. He developed a heuristic that evaluates the truck dispatch decision within each accumulating cycle, which

avoids the delays caused if the last shipment quantity exceeds the size that can fit into the truck that is waiting.

Bookbinder and Higginson (2002) considered orders according to a Poisson distribution with Gamma distributed weight and used a stochastic clearing system to analyze the time and quantity policy. Tyan, Wang, and Du (2003) discussed reducing third party logistics aircraft utilization through a mixed integer programming formulation to find optimal shipment quantities of the various goods, mainly from Dell and Compaq, while maintaining the service requirements. Çetinkaya and Bookbinder (2003) considered three scenarios of transportations: the first considers shipments from many suppliers to a manufacturing unit or an assembly unit like in car production; second is manufacturers to distributors; and the third is bulk loads of multiple products from one manufacturer to destinations transported either by its own truck or by public carriage. Using analytical models, Çetinkaya and Bookbinder derived the optimal shipping quantity in quantity policy and optimal time a shipment can be withheld in time policy for consolidation.

Çetinkaya (2005) extensively reviewed freight consolidation. She explained the concept of pure consolidation where inventory decisions are not included while consolidating the shipments. She also explained an integrated inventory/shipment consolidation policy where inventory decisions are taken in coordination with shipment consolidation as well as other inventory costs. She also mentioned two fundamental questions that need to be answered while consolidating shipments: when to dispatch a vehicle so customer demand is satisfied and how large the dispatch quantity should be such that transportation costs are minimized (Çetinkaya, 2005). Çetinkaya discusses the challenges related to customer service – i.e., how long a first order in a consolidation cycle can be held in the inventory such that it has to reach the customer

within the deadline. In addition to that, the shipment holding and waiting costs are also included and should not exceed the savings incurred in consolidation of shipments. The complexity of the problem increases to maintain coordination of inventory replenishment decisions along with system wide costs that are integrated with shipment release decisions. She also identified the complications regarding structure of transportation costs such as mode of transportation, routing policies and carriage types like private carriage or common carriage. And complications in other areas like cargo capacity and multiple products and market areas are also indicated in the chapter.

Browne, Sweet, Woodburn, and Allen (2005) discussed the benefits of urban consolidation centers (UCC) for small scale retailers. Marcucci and Danielis (2008) compared two alternatives: 1) using a private vehicle with various traffic regulations and 2) using a freight consolidation center in Fiano, Italy, while considering various factors. They concluded that urban consolidation can help in reduction of pollution in cities. Çetinkaya, Tekin, and Lee (2008) considered vendor managed inventory with stochastic order arrivals as well as order sizes to find the outbound economical dispatch quantities while consolidating long-run average costs along with inbound inventory replenishment quantities and timing. Çetinkaya et al. also considered the trade offs between inventory decisions and shipment release decisions using analytical models and urgent shipments where immediate deliveries are preferred. Ülkü (2009) and Mutlu, Çetinkaya, and Bookbinder (2010) considered arrival of orders as poisson distribution and showed that the quantity policy has more savings than the time policy while transporting in private carriage irrespective of real life constraints for delivery deadlines. Çetinkaya and Mutlu (2010) developed an algorithm based on analytical models to obtain numerical solutions for integrated consolidation policies to vendors who use common carriages for freight transportation

for scheduling outbound dispatch and to maintain optimal inventory. Ülkü (2012) proved that consolidation of freight not only reduces cost of transportation but also helps to mitigate carbon and energy waste by calculating CO<sub>2</sub> emissions associated with dispatched vehicle. Nguyen, Dessouky, and Toriello (2014) compared a rolling horizon algorithm and a stochastic dynamic programming model for the consolidation of shipments from multiple small suppliers. They also proposed a cost allocation for the suppliers such that all the suppliers benefit from consolidating compared to individually transporting their shipments. They also developed a heuristic similar to a time policy that works better than the rolling horizon algorithm.

### Perishable Products

The products considered in this research are perishable, which means they deteriorate with time. In this paper, we are considering the value or cost of the product along with the quality of the product. The research on perishable product in freight consolidation practices is limited but below are a few studies that relate to perishability and deterioration in recent years.

Faizul, Asnani, Jones, and Cutright (2005) developed a heuristic to assess the issuing policy and inventory allocation to maximize revenue for a product with multiple expiration dates. In this algorithm the allocation is done based on the shelf life of remaining items to prevent the items from expiring. Jain and Singh (2011) considered deteriorating inventory in a three echelon system consisting of supplier, distribution center and customer. In this case they have considered poor storage conditions as the reason for deterioration, and deterioration starts only at the inventory and not at the suppliers' end as they have favourable storage conditions. They also considered that product demand reduces with the deterioration and is proportional to the same



and the product has no demand after a certain time. They developed a model for optimal ordering quantity to make the system stable.

Limvorasak and Xu (2013) analyzed the effect of a fulfillment center where fresh produce is to meet the demand variability through a risk pooling effect, which reduces the safety stock and thereby reduces the chances of fresh produce expiring. They also developed enhanced coefficient ratios for the fulfillment center to maintain the freshness of the product with various replenishment frequencies. De Keizer, Haijema, Bloemhof, and van der Vorst (2015) developed an approach to design an optimal logistic network using mixed integer linear programming and hybrid optimization simulation. The problem includes product flow, location of hubs for buffer, and maximum stay time for multiple types of perishable products. These perishable products are cut flowers, and the hubs are bouquet making centers or bundling centers.

Taleizadeh and Rasuli-baghiban (2015) used a time-based consolidation policy to develop a model for determining the selling price, dispatch cycle length, and replenishment quantity for perishable products. They considered the product has a constant deteriorating rate and also developed sensitivity analysis for practical applicability. Even Khurana and Chaudhary (2016) considered a rate of cost in their model for deteriorating products to find optimal pricing and ordering quantity where demand is price and stock dependent. Nguyen et al. (2014) and Nguyen et al. (2013) also considered the deterioration of the products that need to be consolidated; however, both enforce a hard time constraint on the products. Lim and Hur (2015) considered the problem of optimal shipping quantities of perishable products in their article.

## Joint Replenishment Models, Economic Lot Sizing Models and Echelon Systems

Other than the consolidation of the shipments, the research on multi-echelon systems and warehousing techniques mostly includes economic lot sizing, cost allocation and joint replenishment models. Below are the most recent and relevant articles.

The research on economic lot sizing models has been prevalent since the Wagner and Whitin (1958) article on a dynamic version of the economic lot size model. Patrizia, Gianpaolo, Antonio, and Emanuela (2006) tried to optimize the set-up costs and machine idle times for machining different types of products on identical parallel machines. They used a decomposition heuristic (fix and relax heuristic) to solve the stochastic lot sizing problem where they reduced the original problem to sub problems, which requires limited integer variables.

Anily and Haviv (2007) optimize the cost allocation of a joint replenishment system through the power of two (POT) optimal policy for a scenario where a set of retailers assign a third party logistics (3PL) services for for reordering or transportation of their supplies. They consider major and minor setup costs first at the 3PL provider and later at the respective retailer. These costs are incurred whenever there is order for replenishment.

Ben-Daya, Darwish, and Ertogral (2008) reviewed the joint economic lot sizing problems and summarized them into a generic model, lot for lot policy, delayed and non-delayed equal sized shipments, geometric, geometric then equal policies and optimal policy. Khouja and Goyal (2008) reviewed the literature on joint replenishment problems and compared different approaches and algorithms. Glock (2012) also reviewed lot-sizing of two-stage and multi-stage models. In which he compared models with stochastic demand or stochastic lead time, setup cost reduction or lead time reduction and product quality ,deterioration and decay.

Cha, Moon, and Park (2008) focused on a two stage supply chain involving one warehouse and n-retailer for developing an optimal joint replenishment and delivery scheduling policy. In this paper they compared a simple heuristic, a RAND algorithm, to the hybrid genetic algorithm. Hong and Kim (2009) also developed a genetic algorithm for JRP and compared it with a RAND algorithm with respect to computational time and the optimal costs. Abdul-Jalbar, Segerstedt, Silicia, and Nilsson (2010) developed a heuristic that can be used in spreadsheet applications for optimal replenishment quantities in a multi-echelon system, which consists of one warehouse and n-retailers.

Guan and Liu (2010) developed a dynamic programming model from two stochastic models for inventory bound constraints and the second with inventory bound and order capacity constraints. Köchel and Thiem (2011) introduced particle swarm optimization and threshold accepting approach in single-warehouse, multi-retailer (SWMR) systems. Gopaladesikan, Uhan, and Zou (2012) developed a primal-dual algorithm for cost allocation in economic lot sizing. Kang and Lee (2013) constructed a dynamic programming heuristic for stochastic lot-sizing model from a multi objective programming model and mixed integer programming model. Amaya, Carvajal, and Castaño (2013) created a linear programming model for joint replenishment problem with resource constraint, which provided better total cost result than other heuristics that were compared. Samouei, Kheirkhah, and Fattahi (2015) used a network approach to solve multi-echelon spare-part inventory system.

### Research Gap

Most of the literature available on multi-echelon systems is on joint replenishment problems, lot sizing problems, or freight consolidation. But there is no research available for

transportation of perishable products from multiple suppliers to multiple destinations through a consolidation center while considering the deterioration rate of the products being shipped.

These models also do not consider the deterioration during travel and inventory from supplier to customer. There is clearly a gap in the current literature in this area. Therefore, any research in this area will be new and we hope will add value to the current literature. With this motivation, we are going forward with our problem in hand. The following section describes the problem and the mathematical formulation.

## CHAPTER 3

### PROBLEM DESCRIPTION

In this problem, the shipments from the suppliers are consolidated at the consolidation center and shipped to their respective destinations. The shipments can either be sent through LTL or FTL units based on the shipment size and origin destination pair. In addition to that, the inventory has to be maintained according to the capacities of respective areas (suppliers, consolidation center). Since we are dealing with the perishable products, the problem needs to consider the deterioration rate of the products.

Currently, suppliers need to directly ship the orders to their respective destinations, so the cost incurred is high as explained in the introduction section. The consolidation center can be established where all suppliers ship their orders to the consolidation center and combined shipments are then shipped to the respective destinations. The consolidation center plays a major role in cost reduction, as mentioned above. But establishing a consolidation center alone will not solve the problem, as there are many other constraints like time, shipment size, truck capacity and inventory capacity. To accommodate all these constraints, a mixed integer programming (MIP) model has been developed for finding the optimal number of LTL and FTL units to ship from suppliers to the destinations via the consolidation center. The MIP model includes the transportation of different units and corresponding costs, holding costs at all the stages, capacities of the trucks as well as inventories and time for the shipping and storage. The

following section describes various parameters, the objective function, and decision variables used in this model.

### Parameters

$S$ : Set of suppliers

$D$ : Set of destinations

$C_i^F$ : Transportation cost from supplier  $i$  to consolidation center in FTL units,  $\forall i \in S$  (\$/Truck)

$C_i^L$ : Transportation cost from supplier  $i$  to consolidation center in LTL units,  $\forall i \in S$  (\$/foot<sup>3</sup>)

$C_j^F$ : Transportation cost from consolidation center to destination  $j$  in FTL units,  $\forall j \in D$  (\$/Truck)

$C_j^L$ : Transportation cost from consolidation center to destination  $j$  in LTL units,  $\forall j \in D$  (\$/foot<sup>3</sup>)

$C_i^H$ : Holding cost at supplier  $i$ ,  $\forall i \in S$  (\$/foot<sup>3</sup>)

$C_o^H$ : Holding cost at consolidation center (\$/foot<sup>3</sup>)

$\tau_i$ : Transportation time from supplier  $i$  to consolidation center,  $\forall i \in S$  (days)

$\tau_j$ : Transportation time from consolidation center to destination  $j$ ,  $\forall j \in D$  (days)

$K^o$ : Maximum storage capacity at consolidation center (foot<sup>3</sup>)

$K_i^S$ : Maximum storage capacity at supplier  $i$ ,  $\forall i \in S$  (foot<sup>3</sup>)

$K^F$ : Maximum capacity of FTL unit (foot<sup>3</sup>)

$K^L$ : Maximum capacity of LTL unit (foot<sup>3</sup>)

$d_{ij}$ : Demand from destination  $j$  to supplier  $i$  shipped at time  $t$  (foot<sup>3</sup>)

$\alpha$ : Deterioration constant

## Decision Variables

$y_{ijst}^F$ : Volume shipped from supplier  $i$  to consolidation center in FTL, that corresponds to the demand ready at time  $s$ , shipped at time  $t$ , for destination  $j$ ,  $\forall i \in S, \forall j \in D, s = 1 \dots T, t = s \dots T$

$y_{ijst}^L$ : Volume shipped from supplier  $i$  to consolidation center in LTL, that corresponds to the demand ready at time  $s$ , shipped at time  $t$ , for destination  $j$ ,  $\forall i \in S, \forall j \in D, s = 1 \dots T, t = s \dots T$

$z_{ijst}^F$ : Volume shipped from consolidation center to destination  $j$  in FTL, that corresponds to the demand ready at time  $s$  at supplier  $i$ , shipped from consolidation center at time  $t$ ,  $\forall i \in S, \forall j \in D, s = 1 \dots T, t = (s + \tau_i) \dots T$

$z_{ijst}^L$ : Volume shipped from consolidation center to destination  $j$  in LTL, that corresponds to the demand ready at time  $s$  at supplier  $i$ , shipped from consolidation center at time  $t$ ,  $\forall i \in S, \forall j \in D, s = 1 \dots T, t = (s + \tau_i) \dots T$

$w_{it}^F$ : Number of FTL units from supplier  $i$  to consolidation center at time  $t$ ,  $\forall i \in S$

$w_{it}^L$ : Number of LTL units from supplier  $i$  to consolidation center at time  $t$ ,  $\forall i \in S$

$x_{jt}^F$ : Number of FTL units from consolidation center to destination  $j$  at time  $t$

$x_{jt}^L$ : Number of LTL units from consolidation center to destination  $j$  at time  $t$

$I_{ijt}^S$ : Inventory at supplier  $i$  to destination  $j$  at time  $t$ ,  $\forall i \in S \ \& \ \forall j \in D$  (foot<sup>3</sup>)

$I_{ijt}^{CC}$ : Inventory at consolidation center from supplier  $i$  to destination  $j$  at time  $t$ ,  $\forall i \in S \ \& \ \forall j \in D$  (foot<sup>3</sup>)

## Objective Function

Minimize

$$\sum_i \sum_j \sum_{s=1}^T \sum_{t=s}^T (z_{ijst}^F + z_{ijst}^L) \times Q(s, t, \tau_j) \quad (1a)$$

$$+ C_o^H \sum_i \sum_j \sum_t I_{ijt}^{CC} + \sum_i \sum_j \sum_t I_{ijt}^S C_i^H \quad (1b)$$

$$+ \sum_{i \in S} \sum_t w_{it}^F C_i^F + \sum_{i \in S} \sum_t w_{it}^L C_i^L + \sum_{j \in D} \sum_t x_{jt}^F C_j^F + \sum_{j \in D} \sum_t x_{jt}^L C_j^L \quad (1c)$$

$$\text{where } Q(s, t, \tau_j) = (t + \tau_j - s) \alpha$$

In the objective function, Expression 1a represents the cost for deterioration since the value of the product decreases with time. This deterioration starts immediately after the product is ready to be shipped. The cost of deterioration of the product is applied from time  $s$  (when the order is ready) until it reaches destination  $j$  at time  $(t + \tau_j)$ , where  $t$  is the time where the product left the consolidation center. Expression 1b represents the holding cost incurred at consolidation center and at respective supplier locations. 1c represents the cost of transportation in LTL and FTL units both from suppliers to consolidation center and consolidation center to the respective destinations.

### Constraints

$$I_{ij1}^S = d_{ij1} - y_{ij11}^F - y_{ij11}^L \quad \forall i \in S, j \in D \quad (2a)$$

$$I_{ijt}^S = I_{ij(t-1)}^S + d_{ijt} - \sum_{s=1}^t y_{ijst}^F - \sum_{s=1}^t y_{ijst}^L \quad \forall i \in S, j \in D, t = 2 \dots T \quad (2b)$$

$$I_{ijt}^{CC} = I_{ij(t-1)}^{CC} + \sum_{s=1}^{t-\tau_i} y_{ijs(t-\tau_i)}^F + \sum_{s=1}^{t-\tau_i} y_{ijs(t-\tau_i)}^L - \sum_{s=1}^{t-\tau_i} Z_{ijst}^F - \sum_{s=1}^{t-\tau_i} Z_{ijst}^L \quad \forall i \in S, j \in D, t = s + \tau_i \dots T \quad (3)$$

$$\sum_{i \in S} \sum_{j \in D} I_{ijt}^{CC} \leq K^O \quad \forall t = 1 \dots T \quad (4)$$



$$\sum_{j \in D} I_{ijst}^S \leq K_i^S \quad \forall i \in S \& t = 1 \dots T \quad (5)$$

$$\sum_{j \in D} \sum_{s=1}^t y_{ijst}^F \leq w_{it}^F K^F \quad \forall i \in S, t = 1 \dots T \quad (6)$$

$$\sum_{j \in D} \sum_{s=1}^t y_{ijst}^L \leq w_{it}^L K^L \quad \forall i \in S, t = 1 \dots T \quad (7)$$

$$\sum_{i \in S} \sum_{s=1}^{\max(0, t - \tau_i)} z_{ijst}^F \leq x_{jt}^F K^F \quad \forall j \in D, t = \min_{i \in S}(\tau_i) \dots T \quad (8)$$

$$\sum_{i \in S} \sum_{s=1}^{\max(0, t - \tau_i)} z_{ijst}^L \leq x_{jt}^L K^L \quad \forall j \in D, t = \min_{i \in S}(\tau_i) \dots T \quad (9)$$

$$d_{ijs} = \sum_{t=s+\tau_i}^{T-\tau_j} (z_{ijst}^F + z_{ijst}^L) \quad \forall i \in S, j \in D, s = 1 \dots T - \tau_i - \tau_j \quad (10)$$

$$d_{ijs} = \sum_{t=s}^T y_{ijst}^F + \sum_{t=s}^T y_{ijst}^L \quad \forall i \in S, j \in D, s = 1 \dots T \quad (11)$$

Constraint 2a is the inventory balance constraint at suppliers at time 1. Since there will be no inventory before time 1, the inventory is equated to supply newly generated (which equals the demand) minus the volume shipped through LTL and FTL units dispatched at time 1. Constraint 2b is also inventory balance at each supplier, but in this constraint, the inventory of the previous day is also taken into consideration for time 2 through  $T$ . Constraint 3 corresponds to the inventory balance constraint at the consolidation center, where current inventory is equal to previous inventory added to inbound shipments from suppliers on that day minus the outbound shipments from consolidation to different destinations.

Constraints 4 and 5 correspond to the maximum holding capacities of the inventories at the consolidation centers and different suppliers, respectively. Constraints 6 to 9 restrict the maximum loading capacities of FTL and LTL units shipped from suppliers and from the consolidation center. Constraint 10 makes sure that all demand is shipped either by LTL or FTL shipments from the consolidation center to meet demand. Constraint 11 ensures that the demand is going to the right destination from the right supplier.

CHAPTER 4  
CPLEX MODEL

The above model is programmed in C++ using concert technology in the IBM ILOG CPLEX 12.6 solver and run with the parameters shown in Table 1. The results for some test runs are shown in Table 2 and Table 3. Table 2 was obtained by running the program for constant time and destinations while varying the number of destinations in the system with a deterioration constant  $\alpha = 0.01$ . The original MIP is relaxed into linear programming model and the results are added for comparison as LP relaxed. Similarly, Table 3 ( $\alpha = 0.01$ ) was obtained with suppliers and time as constants and by varying destinations in each run.

Table 1  
Parameter Value Table

Capacity of FTL	2,600 cubic ft	Cost of holding at CC	\$0.2/day/cubic ft
Capacity of LTL	1 cubic ft	Cost of FTL unit	\$2,080/truck
Inventory capacity at CC	5,000 cubic ft	Cost of LTL unit	\$1.5/cubic ft
Inventory capacity at suppliers	1,000 cubic ft	Shipment size	UNIF (0,1000)
Cost of holding at suppliers	\$0.1/day/cubic ft	Time horizon	30 days

Table 2  
Results While Varying Number of Suppliers

<b>Supplier</b>	<b>Destination</b>	<b>Days</b>	<b>Feasible/Optimal</b>	<b>Gap%</b>	<b>Objective Function Value</b>	<b>LP Relaxed Objective Function Value</b>
30	30	30	Feasible	2.44	20,945,500	20,378,161.54
40	30	30	Feasible	2.85	28,039,100	27,170,377.65
50	30	30	Feasible	96.51	970,024,000	33,898,190.40
60	30	30	Feasible	96.51	1,636,300,000	40,662,694.52
70	30	30	Failed to optimize	-	-	47,377,194.97

Table 3  
Results While Varying Number of Destinations

<b>Supplier</b>	<b>Destination</b>	<b>Days</b>	<b>Feasible/Optimal</b>	<b>Gap%</b>	<b>Objective Function Value</b>	<b>LP Relaxed Objective Function Value</b>
30	30	30	Feasible	2.44	20,945,500	20,378,161.54
30	40	30	Feasible	1.97	27,788,400	27,170,376.27
30	50	30	Feasible	5.46	35,923,000	33,898,188.91
30	60	30	Failed to optimize	-	-	40,662,695.46
30	70	30	Failed to optimize	-	-	47,377,199.64

All of the results of the MIP model were taken after running the program for four hours with random inputs of demand. Four hours is very long in practical situations. Even though the LP relaxed results are much faster to achieve (all the relaxed results were obtained within two minutes), the solution value was much lower compared to the results of the MIP model. From the results, we can also say that the gap percentage was increasing with the increase in number of destinations or suppliers, which means it will take a much longer time to solve if there are more destinations or suppliers. For higher inputs of either number of destinations or supplier, the gap percentage is as high as 97%, which is unacceptable in practical applications. This gap represents the difference between the lower bound and the feasible solution found by CPLEX. The goal is to find a feasible solution with a small gap percentage. An optimal solution has a gap percentage of 0%. To solve this problem, we should find alternate solution approaches or heuristics to solve the model. These experiments will serve as the benchmark to compare the proposed methodology, which is explained next. We will compare the proposed approach with the LP relaxation and the MIP results from CPLEX.

## CHAPTER 5

### PROPOSED METHODOLOGY

As discussed in the earlier section, with the increase in number of suppliers or destinations, the percentage of gap increases. If the number of suppliers or destinations is above 70, the gap percentage is above 97. To create an alternate approach for arriving at a practical solution to the model, the results of the model are compared when the gap is increased drastically. It is found that, for a lesser number of suppliers or destinations, the inventory capacity at the consolidation center was not completely utilized, whereas with larger values, the utilization of inventory capacity increased. At a point with the increase in the number of destinations or suppliers, the inventory capacity at the consolidation center was full. The difficult decision was determining which shipments to hold and consolidate and which volume to ship. To proceed further with this problem, we proposed using a Lagrangian relaxation.

#### Lagrangian Relaxation

Beasley (2017) suggested that Lagrangian relaxation is very good when there are hard constraints in the mixed integer programming model. In a Lagrangian relaxation model, hard constraints are multiplied by Lagrangian multipliers and are relaxed by adding them to the objective function. By doing this, we get different lower bounds for different Lagrangian multipliers in minimizing objective function. After obtaining a Lagrangian relaxed objective

function, we need to find the best Lagrangian multipliers such that it gives the maximum lower bound that is feasible.

Tragantalerngsak, Holt, and Ro (1997) used Lagrangian relaxation and created a heuristic to solve a two echelon facility problem. They used a subgradient optimization problem to solve the Lagrangian dual problem. Beasley (2017) also mentioned that there are two types of problems with Lagrangian relaxation: strategic issue and tactical issue. A strategic problem is when we need choose which constraints have to be relaxed. A tactical problem is when we find numerical values for the multipliers. Beasley also suggested two methods to find the multipliers, which are subgradient optimization and multiplier adjustment.

### Subgradient Optimization

Subgradient optimization is done using different methods for different kinds of problems, but the core of the method does not change. In this method, we arbitrarily assumed first the set of multipliers and solve the relaxed problem. Then based on the solution, we updated the multipliers using the subgradients in systematic fashion until we were satisfied with the solution. The steps followed in Subgradient method are as follows.

- Step 1: Assuming arbitrary values for Lagrangian multipliers and solving the Lagrangian relaxed problem.
- Step 2: Defining a step size based on the upper bound, lower bound of the current solution and the subgradients. (Subgradient is defined by  $b - Ax_i$ , where  $A$  and  $b$  are the coefficients of the relaxed constraint  $Ax \geq b$  and  $x_i$  is the current solution)
- Step 3: Update the Lagrangian multipliers using the step size and subgradients

$\lambda_i = \max(0, \lambda_i + TG_i)$  where  $\lambda_i$  is Lagrangian multipliers,  $T$  is step size,  $G_i$  is the subgradient,  $i$  represents iteration number (Beasley, 2017).

- Step 4: Solve the problem using the updated multipliers.
- Step 5: Go to Step 2.

Termination of this procedure can be done in different ways. One way is to terminate after a few iterations or after a certain percentage of gap. The step size can be altered based on the solution values at each iteration such that we get the desired solution. Termination depends on the expectation of the user. All the multipliers usually change after every iteration in subgradient optimization, whereas in multiplier adjustment method not all the multipliers change.

### Multiplier Adjustment

Multiplier adjustment is like subgradient optimization, but all the multipliers are not updated after every iteration. In this method, initially the problem is solved by arbitrary multipliers. Then the multipliers were updated systematically based on the solution. After every iteration, based on the improvement of the solution, the multipliers were decided creatively. A single multiplier is usually changed in this method. Usually the multiplier method is computationally cheaper since we need not calculate subgradients and update the all the multipliers after every iteration. The solution value becomes better after almost every iteration in the multiplier adjustment method, unlike the subgradient method where the solution can sometimes be poorer than the previous solution. But the subgradient method can be applied to most kinds of problems unlike the multiplier adjustment method, and the lower bound obtained from the subgradient method is usually better than that of the multiplier method (Beasley, 2017).



## Decomposition

A problem can be solved using different optimization techniques, and decomposition of the problem is one such technique. In this technique, the problem is decomposed into two or more smaller problems and each is solved separately. Decomposition of the problem helps reduce time for solving the problem as well as the computational effort compared to the original problem in most cases. Most of the problems can be decomposed based on several characteristics related to the problem, and they can be decomposed in different ways. In this case, the problem can be decomposed by considering one supplier at a time or one destination at a time, or it can be decomposed at the consolidation center based on inbound and outbound shipments. This problem can be solved in either ways, but this problem was decomposed at the consolidation center. The original problem was decomposed into two smaller problems. The first part was to solve for when to ship volumes of product from the suppliers to the consolidation center. The second part used the results of first part to solve for outbound shipments from consolidation center using Lagrangian relaxation. The following sections explain the two parts of the decomposed problem: inbound shipping and outbound shipping.

### Inbound Shipping Problem

The original formulation is decomposed into two problems: the inbound problem, which includes all the suppliers shipping product to the consolidation center, and the outbound problem, which includes shipments from the consolidation center to all the destinations. Below are details of the inbound problem

Objective Function (Z<sub>1</sub>)

Minimize

$$\sum_i \sum_j \sum_t I_{ijt}^S C_i^H \quad (1a)$$

$$+ \sum_{i \in S} \sum_t w_{it}^F C_i^F + \sum_{i \in S} \sum_t w_{it}^L C_i^L \quad (1b)$$

The objective function of the first part is a minimizing function with two expressions. Expression 1a represents the holding cost at each supplier from the time when the demand is ready to be shipped to the time when it is shipped from the supplier. Expression 1b represents the cost of transporting in different truckloads from the supplier to the consolidation center.

Constraints

$$I_{ij1}^S = d_{ij1} - y_{ij11}^F - y_{ij11}^L \quad \forall i \in S, j \in D \quad (2a)$$

$$I_{ijt}^S = I_{ij(t-1)}^S + d_{ijt} - \sum_{s=1}^t y_{ijst}^F - \sum_{s=1}^t y_{ijst}^L \quad \forall i \in S, j \in D, t = 2 \dots T \quad (2b)$$

$$\sum_{j \in D} I_{ijt}^S \leq K_i^S \quad \forall i \in S \& t = 1 \dots T \quad (3)$$

$$\sum_{j \in D} \sum_{s=1}^t y_{ijst}^F \leq w_{it}^F K^F \quad \forall i \in S, t = 1 \dots T \quad (4)$$

$$\sum_{j \in D} \sum_{s=1}^t y_{ijst}^L \leq w_{it}^L K^L \quad \forall i \in S, t = 1 \dots T \quad (5)$$

$$d_{ijs} = \sum_{t=s}^T y_{ijst}^F + \sum_{t=s}^T y_{ijst}^L \quad \forall i \in S, j \in D, s = 1 \dots T \quad (6)$$

The constraints of the first part of the decomposed problem involve holding the capacities at the suppliers, truck capacities, and balance constraints. Constraint 2a is the inventory balance at the supplier on day 1 of the process when there will not be any inventory at suppliers except for the demand for day 1. Constraint 2b represents the inventory balance from day 2 to the end of process. Constraint 3 controls the volume of shipments held at suppliers at all times to not exceed the maximum holding capacity of respective supplier. Constraint 4 and Constraint 5 are the truck capacity constraints of FTL and LTL, respectively, which originates from the suppliers. Constraint 6 makes sure that all demand is being shipped.

The result of the first part of the problem is volume shipped in LTL units and FTL units to the consolidation center from the respective suppliers where the demand originated. These results are then used as input to solve the second part of decomposed problem. The volume that is being shipped from the suppliers in LTL and FTL is consolidated at the consolidation center. Along with the volume of the LTL and FTL units shipped from suppliers, we also tracked when the order for shipment was made, time it shipped from the supplier, the destination, and the supplier of the order. This information was used to solve the second part of the decomposed problem. The following section contains the formulation of the second part of the decomposed problem.

Objective Function (Z<sub>2</sub>)

Minimize

$$\sum_i \sum_j \sum_{s=1}^T \sum_{t=s}^T (z_{ijst}^F + z_{ijst}^L) \times Q(s, t, \tau_j) \quad (7a)$$

$$+ C_o^H \sum_i \sum_j \sum_t I_{ijt}^{CC} \quad (7b)$$

$$+ \sum_{j \in D} \sum_t x_{jt}^F C_j^F + \sum_{j \in D} \sum_t x_{jt}^L C_j^L \quad (7c)$$

$$+ \sum_t \lambda_t \left( \sum_{i \in S} \sum_{j \in D} I_{ijt}^{CC} - K^0 \right) \quad (7d)$$

$$\text{where } Q(s, t, \tau_j) = (t + \tau_j - s) \alpha$$

The objective function contains the equivalent cost for deterioration of the shipment (7a), holding cost at the consolidation center (7b), transportation costs from the consolidation center to the respective destinations (7c), and the relaxed inventory capacity constraint of the consolidation center (7d). Expression 7d is a constraint in the original problem, which is relaxed and added as a cost in the objective function. The relaxation allows the problem to be solved much faster than the original problem.

Constraints

$$I_{ijt}^{CC} = I_{ij(t-1)}^{CC} + \sum_{s=1}^{t-\tau_i} y_{ijs(t-\tau_i)}^F + \sum_{s=1}^{t-\tau_i} y_{ijs(t-\tau_i)}^L - \sum_{s=1}^{t-\tau_i} z_{ijst}^F - \sum_{s=1}^{t-\tau_i} z_{ijst}^L \quad \forall i \in S, \quad (8)$$

$$j \in D, t = s + \tau_i \dots T$$

$$\sum_{i \in S} \sum_{s=1}^{\max(0, t-\tau_i)} z_{ijst}^F \leq x_{jt}^F K^F \quad \forall j \in D, t = \min_{i \in S}(\tau_i) \dots T \quad (9)$$

$$\sum_{i \in S} \sum_{s=1}^{\max(0, t-\tau_i)} z_{ijst}^L \leq x_{jt}^L K^L \quad \forall j \in D, t = \min_{i \in S}(\tau_i) \dots T \quad (10)$$

$$\sum_{i \in S} \sum_{j \in D} \sum_{s=1}^{T-\tau_i-\tau_j} \sum_{t=s}^{T-\tau_i-\tau_j} (y_{ijst}^F + y_{ijst}^L) = \sum_{i \in S} \sum_{j \in D} \sum_{s=1}^{T-\tau_j} \sum_{t=s+\tau_i}^{T-\tau_j} (z_{ijst}^F + z_{ijst}^L) \quad (11)$$

Constraint 8 represents the inventory balance constraint at the consolidation center. The FTL and LTL volumes from suppliers are the results of part 1 of the decomposed problem. Constraints 9 and 10 are the truck capacity constraints at the consolidation center. Constraint 11 makes sure that all the demands coming into the consolidation center leave for the respective destinations. The second part of the decomposed problem has lambda ( $\lambda$ ) in its objective function. The feasibility of the solution depends on the lambda values. There are a few predefined algorithms to find the lambda values, like the subgradient method and the multiplier adjustment method. In this research we used a subgradient optimization method to solve for lambda. Steps involving the subgradient method to find the lambda value are in the following section.

### Finding the Multipliers

The heuristic for finding lambda is inspired by the Lorena, Antonio, and Narciso (1996) article on relaxation heuristics for a generalized assignment problem. For our decomposed problem, the first part of the problem was solved for the volume from each supplier, and the

number of FTL and LTL units leaving the suppliers or reaching the consolidation center are found. These results are now considered as inputs or demand for the second part of the problem. Then the following steps were followed to solve the second part of the problem: let the objective function of part 1 be  $Z_1$  and the objective function of part 2 as  $Z_2$ , and the relaxed objective function as  $Z_{LR}$ .

- Step 1: Consider a  $\lambda_t \geq 0$ , since our objective function is a minimizing function and the relaxed constraint is  $(\sum_{i \in S} \sum_{j \in D} I_{ijt}^{CC} - K^0) \leq 0 \forall t \in T$ .
- Step 2 : Set a high upperbound value ( $+\infty$ ) and a low lower bound value ( $-\infty$ ).
- Step 3: Solve the relaxed problem to find out  $Z_{LR}$ .
- Step 4 : Check for the feasibility of the solution for the relaxed constraint i.e. check whether  $\sum_{i \in S} \sum_{j \in D} I_{ijt}^{CC} \leq K^0, \forall t \in T$ . If the solution is feasible for all time periods then stop and find the  $Z_2$  by subtracting relaxed value from  $Z_{LR}$  and the final solution is  $Z_1 + Z_2$ . If the solution is not feasible, then update lambda by finding the step size from updating the upper bound and lower bound and go to Step 3.

### Updating Upper Bound and Lower Bound

Since our goal was to reduce the gap between the upper bound and lower bound; the upper bound was updated after every iteration if it was less than the previous upper bound. After every iteration, if the value  $Z_{LR} + Z_1 - (\sum_{i \in S} \sum_{j \in D} I_{ijt}^{CC} - K^0)$  was less than the previous upperbound, then the upper bound value was updated to  $Z_{LR} + Z_1 - (\sum_{i \in S} \sum_{j \in D} I_{ijt}^{CC} - K^0)$ . The upper bound value is the sum of objective function values  $Z_1$  and  $Z_{LR}$  minus the relaxed part in the  $Z_{LR}$ . Similarly, the lower bound value was also updated after every iteration if it was higher than the previous lower bound. The lower bound is  $Z_1 + Z_{LR}$  without removing the relaxed constraint. This way the step size was updated using the formula in subgradient optimization, and the lambda values were updated using step size. After updating all the lambda values, the problem

was solved again for the  $Z_{LR}$  for 50 iterations. This process ended if an objective function value of an iteration stayed best for at least 15 consecutive iterations. If the feasible solution was found within the above mentioned conditions, the program was stopped and the results were obtained. If the program was infeasible after the stopping criteria, new constraints were added based on the results of the best iteration.

### Finding Feasible Solution

After the stopping the criteria mentioned above, if the solution was not feasible, the best iteration (which had lowest objective function value) was made feasible by adding more constraints to the  $Z_{LR}$  (second part of the problem after decomposition). The only relaxed constraint in the problem was the holding capacity of the consolidation center.

$$\sum_{i \in S} \sum_{j \in D} I_{ijt}^{CC} \leq K^0, \forall t \in T \quad (12)$$

So, the problem can be made feasible if the above constraint was satisfied was done by finding the time periods in which are the inventory of the consolidation center exceeded the holding capacity of the consolidation center and shipping the extra shipments without affecting other time periods. But the extra shipments cannot be shipped directly because it would affect the objective function value. To accommodate that, the same model that was used to find the solution was modified by adding constraints such that the volume being shipped on violating time periods was increased to a required amount. The steps to make the solution feasible are mentioned below.

- Step 1: Find the best iteration using the objective function value ( $\min Z_{LR}$ ).

- Step 2: Find the time in which the first violation ( $t\_violate$ ) of the inventory capacity of the consolidation center occurs and calculate the volume ( $Vol\_extra$ ) exceeding the holding capacity.
- Step 3: Add volume that is exceeding in Step 2 to the volume being shipped on  $t\_violate$  day by FTL units ( $Vol\_FTL$ ) and LTL units ( $Vol\_LTL$ ).
- Step 4: Calculate the volume that needs to be shipped such that the volume held in the inventory is within the capacity i.e.,  $Vol\_extra + Vol\_FTL + Vol\_LTL = Vol\_to\_send$ .
- Step 5: Calculate the volumes that are being shipped by FTL and LTL units for each time before  $t\_violate$  as  $Vol\_shipped\_FTL_t$  and  $Vol\_shipped\_LTL_t$ .
- Step 6: Add Constraints 13, 14 and 15 to the existing model such that the volumes being sent before  $t\_violate$  are not changed and the volume being shipped on  $t\_violate$  is greater than  $Vol\_to\_send$ .

$$\sum_{i \in S} \sum_{j \in D} \sum_{s=1}^t (z_{ijst}^F) \geq Vol\_shipped\_FTL_t, \forall t \in 1 \dots (t\_violate - 1) \quad (13)$$

$$\sum_{i \in S} \sum_{j \in D} \sum_{s=1}^t (z_{ijst}^L) \geq Vol\_shipped\_LTL_t, \forall t \in 1 \dots (t\_violate - 1) \quad (14)$$

$$\sum_{i \in S} \sum_{j \in D} \sum_{s=1}^t (z_{ijst}^F + z_{ijst}^L) \geq Vol\_to\_send, \forall t \in (t\_violate - 1) \quad (15)$$

- Step 7: The volume stored in the inventory of the consolidation center will not exceed the capacity until  $t\_violate$  because of the constraints above. But after  $t\_violate$  there may be more violations. So, check for violations after  $t\_violate$ . If no violations found stop, since the solution is feasible. If any violations found go to step 2.

Following the above steps, we get a feasible solution. The following section contains the comparison between the numerical results of the relaxed model with the results of the CPLEX model.



## CHAPTER 6

### NUMERICAL RESULTS AND DISCUSSION

The experiments using the CPLEX model were repeated with the proposed model, and it was observed that the proposed methodology had better results than the CPLEX model. The CPLEX model has a gap of approximately 97% between best bound and objective function if the experiments have more than 50 suppliers or destinations. The same numbers give a feasible solution with a gap less than 3% when compared to best bound of the CPLEX model. The CPLEX model has failed to optimize if there are more than 60 suppliers or destinations, whereas the proposed model is able to give a feasible solution within four hours. Table 4 and Table 5 have the results of the experiments of the CPLEX model, best bound of the CPLEX model result while relaxing the integers of the CPLEX model, and the last column has the solution of the proposed model. Results of Table 4 were run with 30 destinations and for 30 days while varying the number of suppliers and Table 5 was run with 30 suppliers while varying the number of destinations.

We can observe that in both the tables the solution of the CPLEX model is higher than the proposed model and the model has failed to optimize in some instances. On the contrary, the solution of the proposed methodology is close to the LP relaxed solution. This can be easily seen in graph a and graph b of Figure 1. Figure 1 has the results of Table 4 plotted on a bar chart where the difference between the CPLEX model and the proposed model can be easily observed. Figure 2 shows the same results, but the solution of the CPLEX model is removed for better

visualization. In Figure 2, we can clearly observe that the solution of proposed model is closer to the LP relaxed solution.

Table 4  
Results While Varying Number of Suppliers

Supplier	Objective function Value CPLEX model	LP Relaxation Solution	Proposed Model
30	20,945,500	20,378,161	20,735,649
40	28,039,100	27,170,377	27,995,200
50	970,024,000	33,898,190	34,976,000
60	1,636,300,000	40,662,694	41,714,700
70	Failed to optimize	47,377,194	48,609,300

Table 5  
Results While Varying Number of Destinations

Destination	Objective function Value CPLEX Model	LP Relaxation	Proposed Model
30	20,945,500	20,378,161	20,735,649
40	27,788,400	27,170,376	27,906,700
50	35,923,000	33,898,188	34,749,200
60	Failed to optimize	40,662,695	42,513,200
70	Failed to optimize	47,377,199	49,884,183

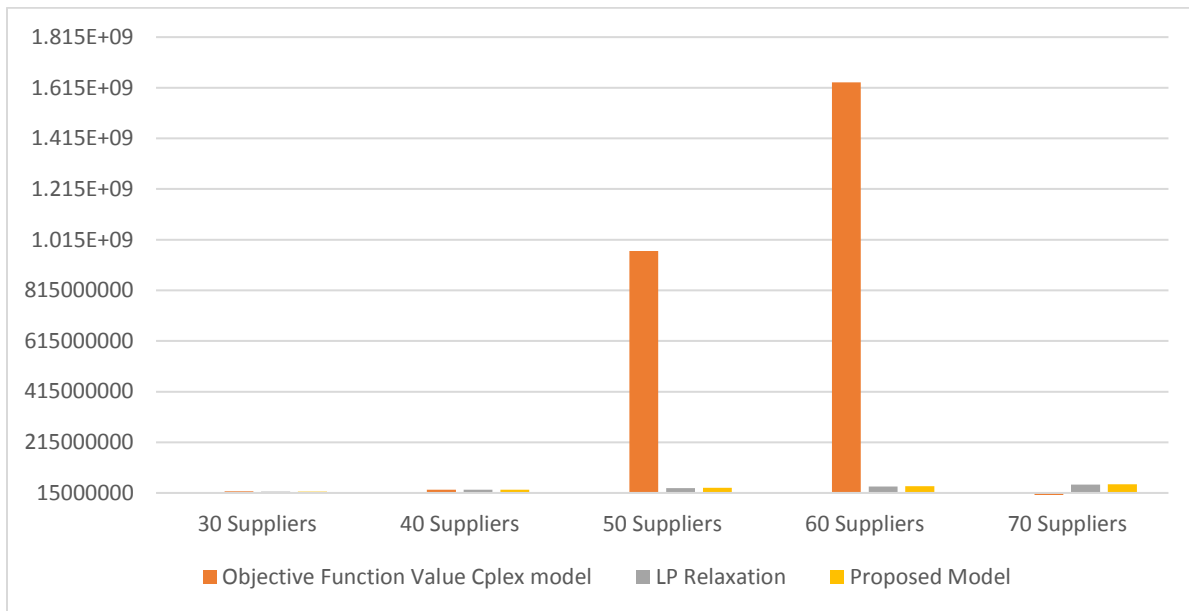


Figure 1. Comparison between CPLEX model and proposed model.

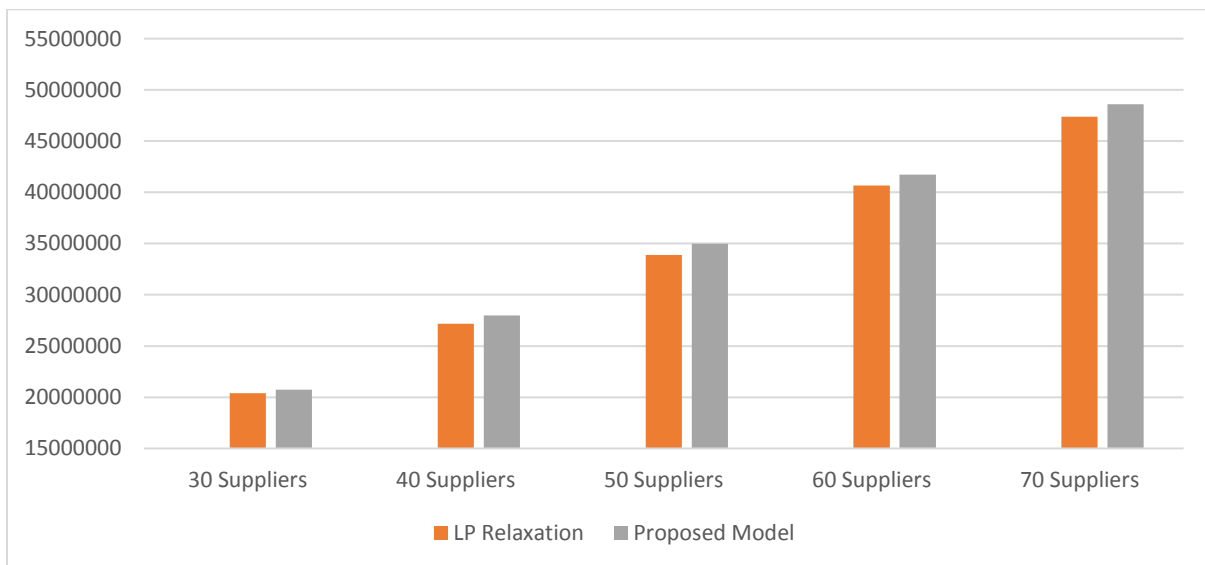


Figure 2. Comparison between LP relaxation and proposed model.

## CHAPTER 7

### CONCLUSION AND FUTURE RECOMMENDATIONS

In this research, we developed a mixed integer programming model for consolidation of perishable products from a set of suppliers shipping to various destinations. The model helps to optimize the transportation costs and holding costs while minimizing the deterioration of the products. The deterioration of the products was controlled using a parameter called alpha value, which is included in the cost of the model's objective function. The alpha value can be changed according to the deterioration rate of the products being shipped. More research can be done on how to test the sensitivity of the alpha value on the model.

We also proposed a new methodology by decomposing the problem and using Lagrangian relaxation to generate better feasible solutions than the IBM ILOG CPLEX solver. But the proposed model can still be improved by further research to make it use less memory since the proposed model ran out of memory while solving for 70 destinations with 30 suppliers in 30 days. More research can be done using this model to make it applicable for practical purposes. This will help not only reduce cost and time for the suppliers but also control the deterioration of the products, ensuring better cost and quality for the customers. Other decomposition techniques can be used, such as by destination.

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