Supply Chain Network Optimization for Efficient and Resilient Network Pooling

Summary of Project

The work done over the course of this project can be split into two parts: a research portion exploring transshipment games and different ways of analyzing them, as well as a programming portion, where one of these analysis strategies was implemented to provide results for a certain scenario of a transshipment game.

Research

The focus of this project revolves around the concept of a transshipment game. A transshipment game can be defined as a scenario where several players in a game (for example, several retail stores) each either have an excess supply of a certain good, or an excess demand of that good. Players with excess supply must find a way to sell the stock or turn it in for its much lower salvage value, and players with demand must find a way to realize this potential profit or lose out on the sales (Huang and Sošić, 4). Each player in this game must work with the other players to trade excess stock or demand to utilize its potential value, but players will also take actions to maximize their own profits as much as possible. In this way, the transshipment game is both competitive and cooperative, with each player having an incentive to work with others to increase their own gains (Anupindi, Bassok, and Zemel, 8).

In this situation, the players often form coalitions among themselves to secure pairings of excess supply and excess demand. Each of these coalitions will have a total value associated with it, as well as a payoff for each player, unless the coalition has no more possible pairings of supply and demand (Anupindi, Bassok, and Zemel, 8). While each individual player will look at which coalitions will be best for them, we can also objectively calculate how valuable each player is to the other players in the game. One of the ways of calculating these values is with the Shapley Value.

$$\frac{1}{N!}\sum_{S\subseteq N\setminus\{i\}}|S|!(|N|-|S|-1)!\Big[v(S\cup\{i\})-v(S)\Big]$$

This Shapley Value equation will calculate the value that a single player has within the set of players. The components of the equation are as follows (Ferguson, 15):

i: The subject player, for which we are finding the Shapley Value

N : The total number of players in the game

S : The current set of players (current coalition)

v(S): Value of the current set of players

v(S ∪ {i}): Value of current set of players including subject player i

The Shapley Value calculation essentially visits every coalition that does not include the player i, and records how much value is added to the coalition by including player i (if any). This value is weighted based on the size of the current coalition and the total number of players, and each value is summed together to generate the Shapley Value for the player i. This value represents how valuable this player is to other coalitions in general and can be considered as a factor for how much profit should be allocated from the total profit pool to the player in question.

Another strategy for interpreting a transshipment game is the Nucleolus. The Nucleolus focuses on minimizing the excesses that each player receives from the game, with the purpose of stabilizing the differences in excesses between players, so that no player feels like they are the worst off in the game. This technique uses the assumption that a player’s satisfaction comes from comparing their excesses to the excesses of others, so satisfaction can be maximized when the excesses for the game are minimized (Ferguson, 22).

Program Implementation of the Shapley Value

One of the initial goals for this research project was to develop a program in order to generate Shapley Values from an input sets of players. The program creation was important because when finding the Shapley Value by hand, the number of possible coalitions in a game experiences factorial growth as more players are added, so larger games can quickly get out of hand for performing this calculation. We decided to use Matlab for this implementation, as it should be able to perform calculations on large amounts of data fairly easily.

In the scenario that we would be exploring for this program, each player will also have a selling price per unit when they are able to match a supply with a demand, and each pair of players involved will have a transshipment cost per unit shipped between them. The program itself essentially will find every single possible coalition in a n-player game, apply the sales prices and transshipment costs to calculate the values for each coalition, and then find the Shapley Value of each player by checking what value each player would be bringing to every other coalition. With this program, we can run scenarios of much larger and more complex games, see how changing different aspects of the game affects the Shapley Values. We can also start to compare how the Shapley Value interprets the game with how something like the Nucleolus would interpret it.

Results/Outcomes

The Shapley Value transshipment program provided many valuable insights into how this type of game plays out. One of the main takeaways is that players with higher transshipment costs are less valuable to other players, as other players would prefer to source their supply or demand from players with lower costs in order to increase profits. In the situation that all the players have the same transshipment costs and no other contributing factors to profit, any unit if supply or demand from a player is just as good as any other player. A player with excess supply and a higher salvage profits than any other supplier will have a slightly lower Shapley value than one with lower salvage profits, as they will still gain something if they are not able to sell off their supply, so they are somewhat less reliant on the other players.

One of the more interesting outcomes of the Shapley Value calculation is how the game values the flexibility of players. A player with a larger amount of supply or demand will have more potential for profit; however, that player will be a part of many coalitions where not all the units of supply or demand can be utilized. A player with a lower supply or demand has less potential for profit, but will be a member of many more coalitions where they will be able to utilize all of that potential. Due to these conditions, players with relatively high supply or demand will often have Shapley Values lower than their potential profit, while players with relatively low supply or demand will often have Shapley Values higher than their potential profit.

Discussion of how project was accomplished

This project began with the goal of exploring different ways to approach transshipment games. Initially, I was tasked with researching articles provided to me by my mentor, as well as searching for more information on the topics. Once I had a fair understanding of the topic and the relevant equations, I began to learn Matlab in order to implement one method of calculating the results of a transshipment game. The process of creating this program involved thinking through how the equation would function in Matlab, and included several unsuccessful attempts of that implementation. However, I was able to create a functional program which can generate the Shapley Values of a transshipment game with user input parameters.

Reflections

SEF was an experience that was and will continue to be very valuable to me in the future. It provided me with an opportunity to work with a professor and learn about topics I may not have been able to experience through my normal coursework. In my experience, SEF was also an effective introduction into the world of academic research, exposing me to how the process works and what is expected from everyone involved.

In this specific project, I became more involved in topic relating to my degree field of Industrial and Systems Engineering, as well as having opportunities to learn new skills that will likely benefit me through my academic and professional careers, such as Matlab and logical implementations of math in programming.

References

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